

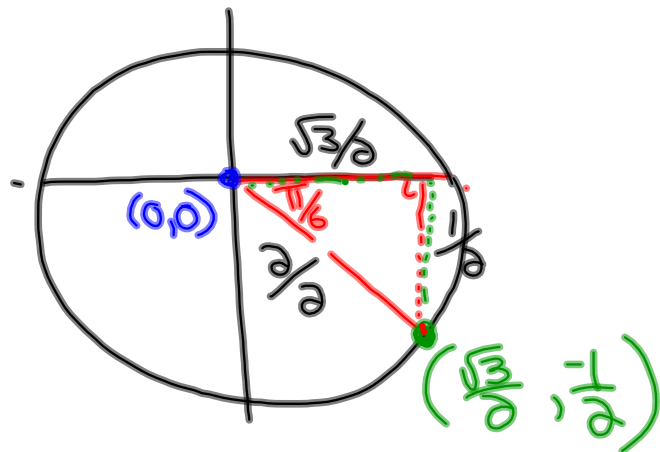
Questions from Homework

- Show the integral multiples of $\frac{\pi}{6}$ on the unit circle and list the coordinates for each.
- Show the integral multiples of $\frac{\pi}{4}$ on the unit circle and list the coordinates for each.
- Workbook #1-6 pages 123-126

④ $f) \frac{23\pi}{6}$

$\frac{22\pi}{6}, \frac{23\pi}{6}, \frac{24\pi}{6}$

4π

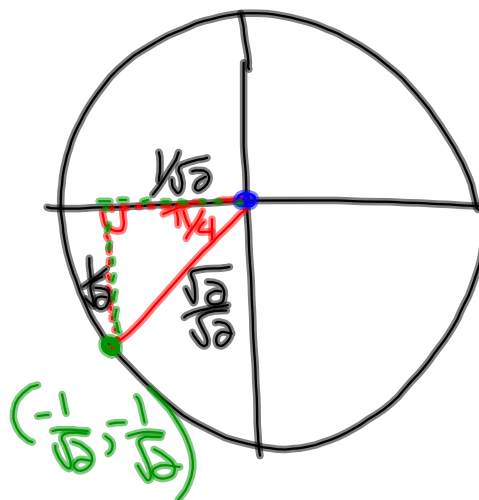


$\frac{5\pi}{4}$

$\frac{4\pi}{4}, \frac{5\pi}{4}, \frac{6\pi}{4}$

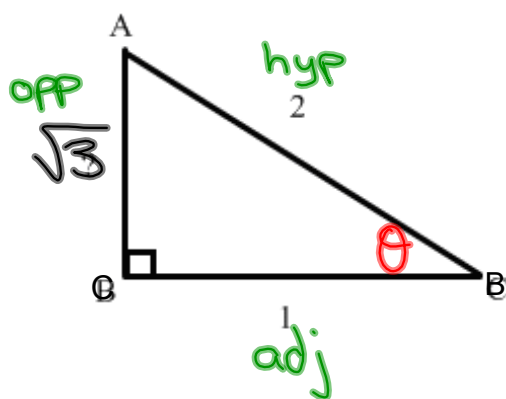
π

$\frac{3\pi}{2}$



Trig Ratios:

$$\sin = \frac{\text{opp}}{\text{hyp}} \quad \cos = \frac{\text{adj}}{\text{hyp}} \quad \tan = \frac{\text{opp}}{\text{adj}} \quad \csc = \frac{\text{hyp}}{\text{opp}} \quad \sec = \frac{\text{hyp}}{\text{adj}} \quad \cot = \frac{\text{adj}}{\text{opp}}$$



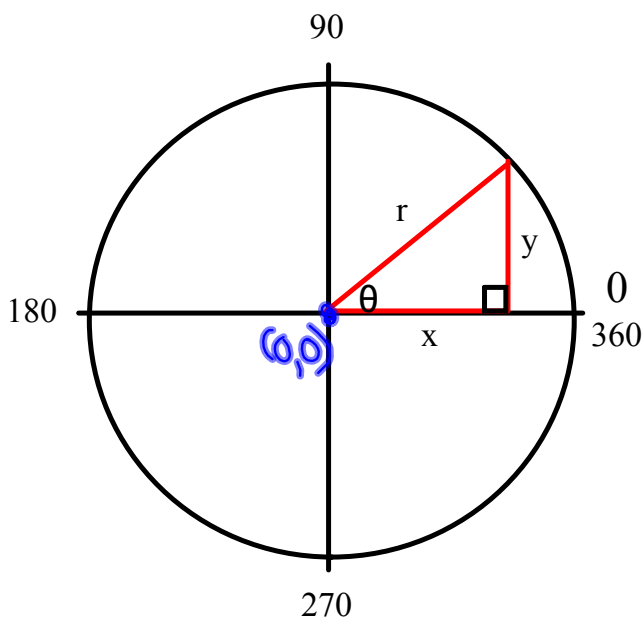
Determine the 6 trig ratios for angle B, expressed as radicals in simplest form.

$$\begin{aligned} a^2 + b^2 &= c^2 \\ (1)^2 + b^2 &= (2)^2 \\ 1 + b^2 &= 4 \\ b^2 &= 3 \\ b &= \sqrt{3} \end{aligned}$$

$\sin \theta = \frac{\sqrt{3}}{2}$	$\csc \theta = \frac{2 \cdot \sqrt{3}}{\sqrt{3} \cdot \sqrt{3}} = \frac{2\sqrt{3}}{3}$
$\cos \theta = \frac{1}{2}$	$\sec \theta = 2$
$\tan \theta = \sqrt{3}$	$\cot \theta = \frac{1 \cdot \sqrt{3}}{\sqrt{3} \cdot \sqrt{3}} = \frac{\sqrt{3}}{3}$

The Unit Circle

The unit circle has a radius of 1 unit and its center lies on the origin (0,0)



$$\begin{aligned} \sin \theta &= \frac{y}{r} & \csc \theta &= \frac{r}{y} \\ \cos \theta &= \frac{x}{r} & \sec \theta &= \frac{r}{x} \\ \tan \theta &= \frac{y}{x} & \cot \theta &= \frac{x}{y} \end{aligned}$$

Since the radius is equal to 1, then:

$$\sin \theta = \frac{y}{r} = \frac{y}{1} = y$$

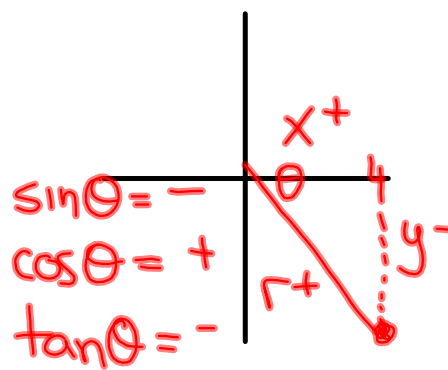
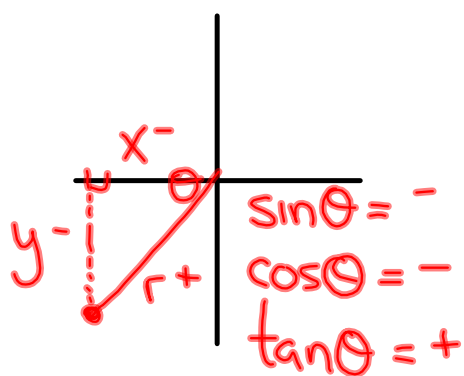
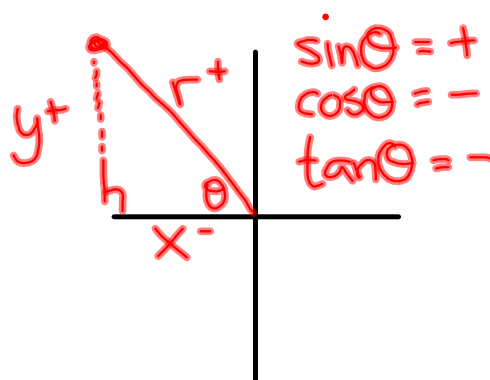
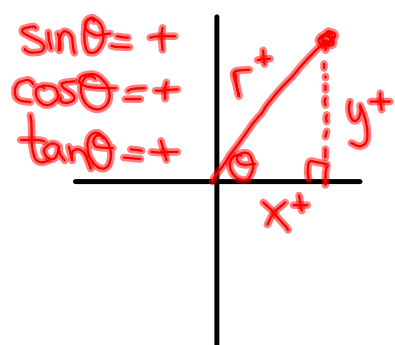
$$\csc \theta = \frac{1}{y}$$

$$\cos \theta = \frac{x}{r} = \frac{x}{1} = x$$

$$\sec \theta = \frac{1}{x}$$

$$\tan \theta = \frac{y}{x}$$

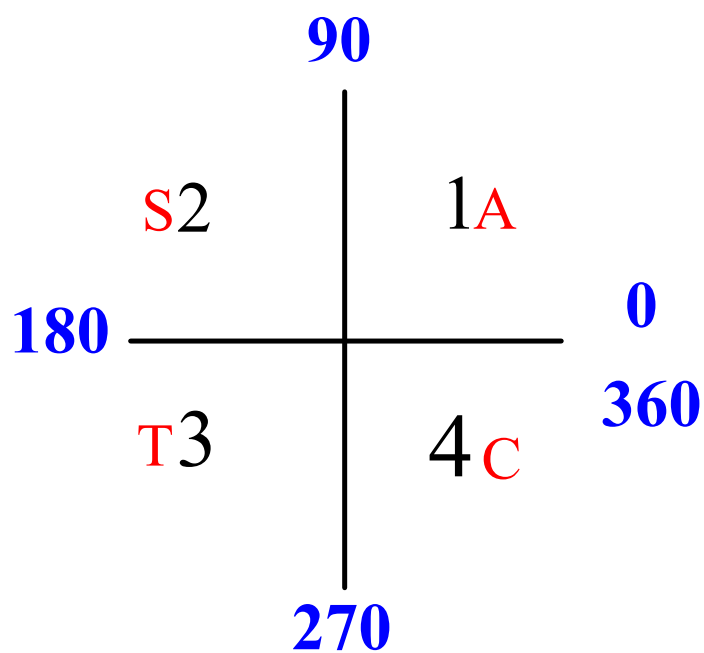
$$\cot \theta = \frac{x}{y}$$

Review signs of the trig ratios in the 4 quadrants

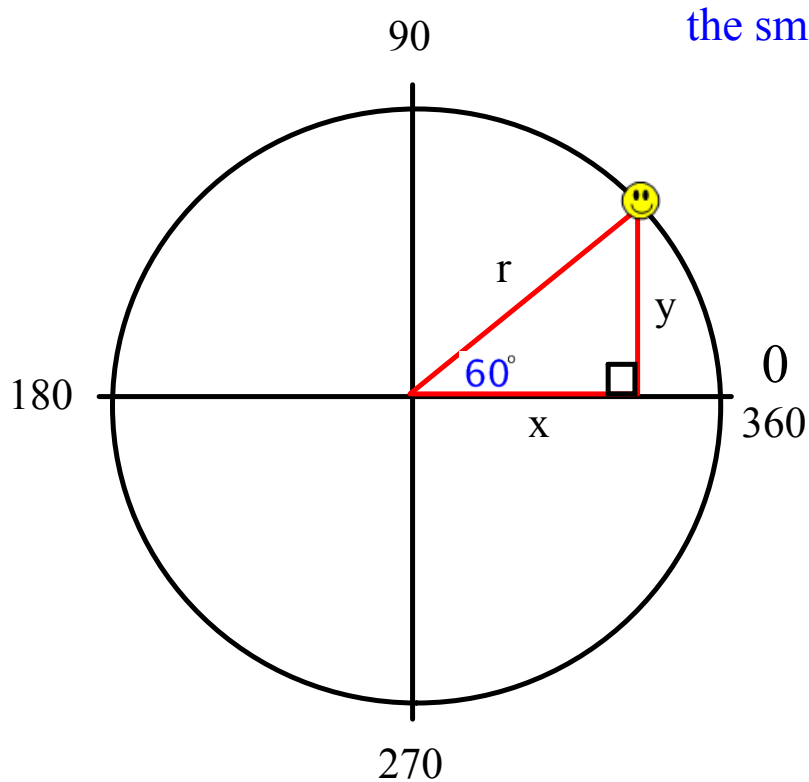
Cast Rule

Quadrants

Remember 4CAST



To find the coordinates (x,y) of the smiley face given $\theta = 60^\circ$



$$\cos\theta = x = \frac{1}{2}$$

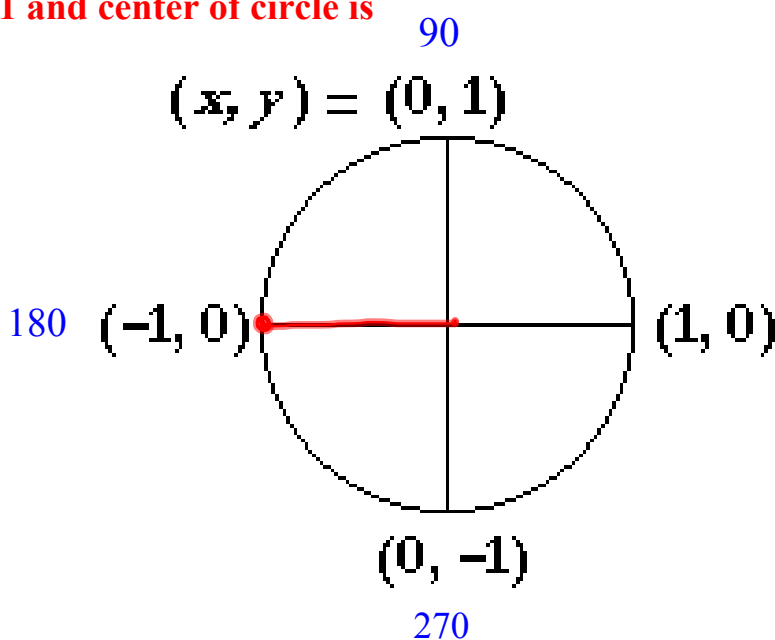
$$\sin\theta = y = \frac{\sqrt{3}}{2}$$

Unit Circle

Quadrantal Angles (Multiples of 90)

or $\frac{\pi}{2}$

radius = 1 and center of circle is at (0,0)



$$\begin{aligned} \sin \theta &= y \\ \cos \theta &= x \\ \tan \theta &= \frac{y}{x} \\ \csc \theta &= \frac{1}{y} \\ \sec \theta &= \frac{1}{x} \\ \cot \theta &= \frac{x}{y} \end{aligned}$$

	0°	90°	180°	270°	360°
sin	0	1	0	-1	0
cos	1	0	-1	0	1
tan	0	undefined	0	undefined	0

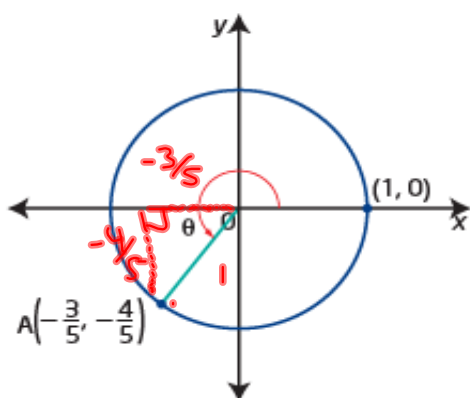
Determine the Trigonometric Ratios for Angles in the Unit Circle

The point $A\left(-\frac{3}{5}, -\frac{4}{5}\right)$ lies at the intersection of the unit circle and the terminal arm of an angle θ in standard position.

- Draw a diagram to model the situation.
- Determine the values of the six trigonometric ratios for θ . Express answers in lowest terms.

Solution

a)



$$b) \sin \theta = y = \boxed{-\frac{4}{5}}$$

$$\cos \theta = x = \boxed{-\frac{3}{5}}$$

$$\tan \theta = \frac{y}{x} = \frac{-\frac{4}{5}}{-\frac{3}{5}}$$

$$= -\frac{4}{5} \times \frac{5}{-3} = \boxed{\frac{4}{3}}$$

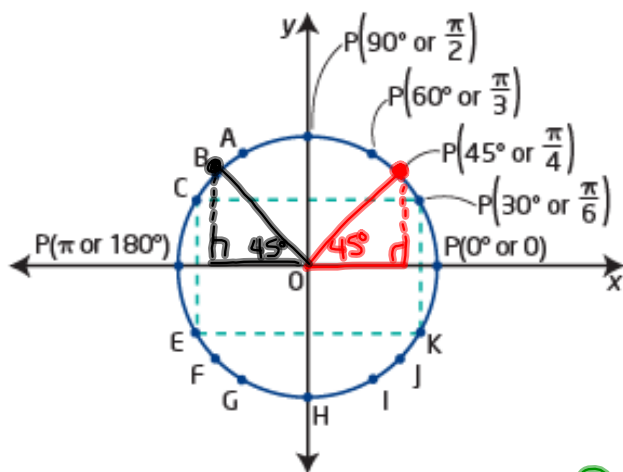
$$\csc \theta = \frac{1}{y} = \frac{1}{-\frac{4}{5}} = \boxed{-\frac{5}{4}}$$

$$\sec \theta = \frac{1}{x} = \frac{1}{-\frac{3}{5}} = \boxed{-\frac{5}{3}}$$

$$\cot \theta = \boxed{\frac{3}{4}}$$

Exact Values of Trigonometric Ratios

Exact values for the trigonometric ratios can be determined using special triangles (30° - 60° - 90° or 45° - 45° - 90°) and multiples of $\theta = 0, \frac{\pi}{6}, \frac{\pi}{4}, \frac{\pi}{3},$ and $\frac{\pi}{2}$ or $\theta = 0^\circ, 30^\circ, 45^\circ, 60^\circ,$ and 90° for points $P(\theta)$ on the unit circle.



- ① How are $P(30^\circ)$, C, E, and K related?
- ② What points have the same coordinates as $P(\frac{\pi}{3})$ except for their signs?
- ③ For $P(45^\circ)$, what are the coordinates and in which quadrant is θ ?
- ④ Which special triangle would you use and where would it be placed for $\theta = 135^\circ$?

① They have the same coordinates except for their signs

② A, G, I

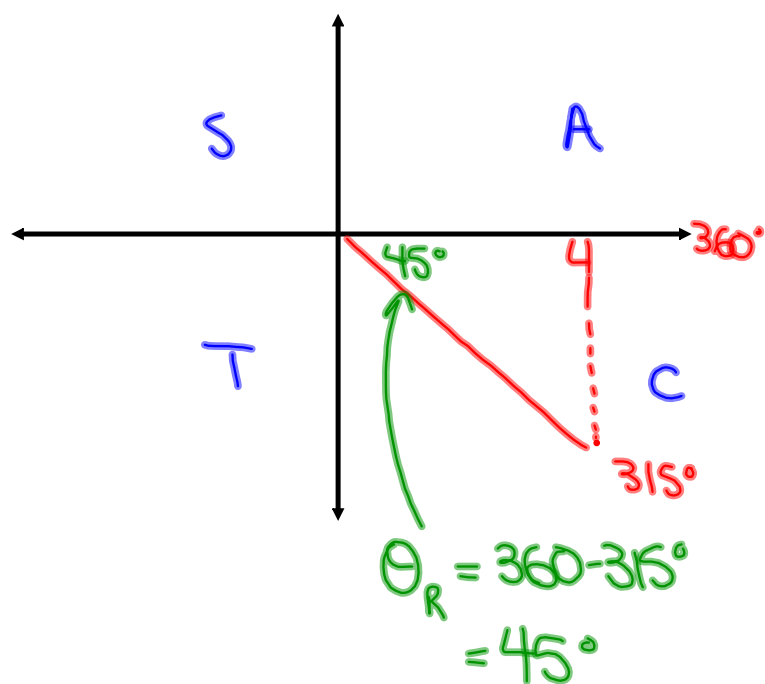
③ Quad I $(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}})$

④ It would be placed at point B

Extend the special angles into all FOUR quadrants

Without a calculator determine the value of $\tan 315^\circ \rightarrow -\frac{1}{1} \rightarrow -1$

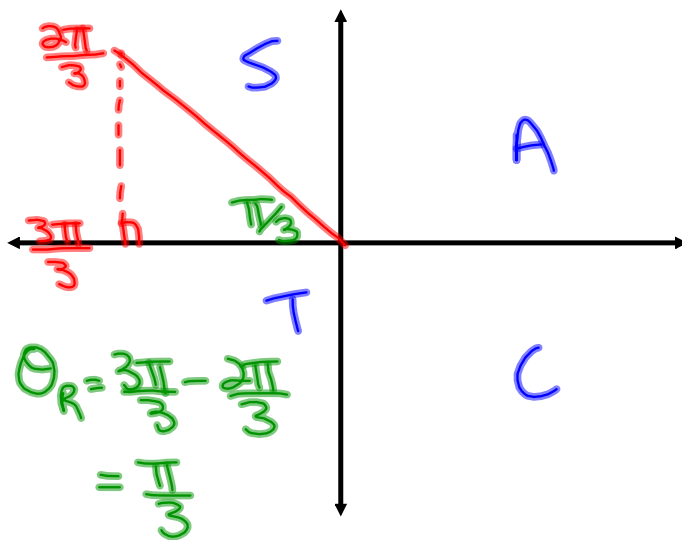
1. Start by sketching the angle



Extend the special angles into all FOUR quadrants

Without a calculator determine the value of $\sin\left(\frac{-4\pi}{3}\right)$

1. Start by sketching the angle



$$\sin \frac{2\pi}{3}$$

$$\rightarrow \frac{\sqrt{3}}{2}$$

$$\frac{-4\pi}{3} + \frac{2\pi}{1}$$

$$\frac{-4\pi + 6\pi}{3}$$

$$\frac{2\pi}{3}$$

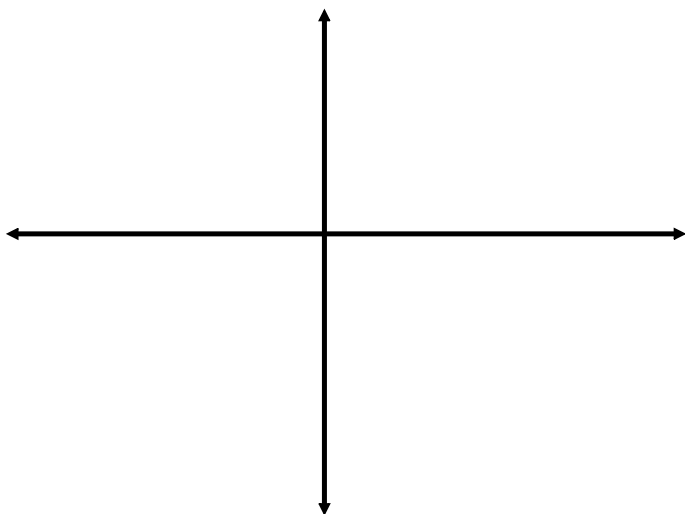
$$\frac{1\pi}{3}, \frac{2\pi}{3}, \frac{3\pi}{3}$$

π

Extend the special angles into all FOUR quadrants

Without a calculator determine the value of $\csc 630^\circ$

1. Start by sketching the angle



Homework

Attempt #1-4 on page 133 of workbook

Exact Values for Trigonometric Ratios

Determine the exact value for each. Draw diagrams to illustrate your answers.

a) $\cos \frac{5\pi}{6}$

b) $\sin \left(-\frac{4\pi}{3}\right)$

c) $\sec 315^\circ$

d) $\cot 270^\circ$

Approximate Values of Trigonometric Ratios

You can determine approximate values for sine, cosine, and tangent using a scientific or graphing calculator. Most calculators can determine trigonometric values for angles measured in degrees or radians. You will need to set the mode to the correct angle measure. Check using

$$\cos 60^\circ = 0.5 \text{ (degree mode)}$$

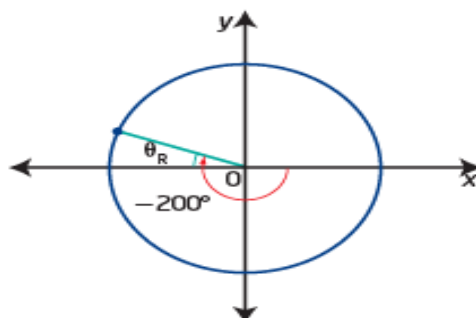
$$\cos 60 = -0.952\ 412\ 980\dots \text{ (radian mode)}$$

In which quadrant does an angle of 60 terminate?

Most calculators can compute trigonometric ratios for negative angles. However, you should use your knowledge of reference angles and the signs of trigonometric ratios for the quadrant to check that your calculator display is reasonable.

$$\cos (-200^\circ) = -0.939\ 692\ 620\dots$$

Is the negative value appropriate?
What is the reference angle for -200° ? What other trigonometric ratio could you compute as a check?



You can find the value of a trigonometric ratio for cosecant, secant, or cotangent using the correct reciprocal relationship.

$$\begin{aligned} \sec 3.3 &= \frac{1}{\cos 3.3} \\ &= -1.012\ 678\ 973\dots \\ &\approx -1.0127 \end{aligned}$$

Approximate Values for Trigonometric Ratios

Determine the approximate value for each trigonometric ratio. Give your answers to four decimal places.

a) $\tan \frac{7\pi}{5}$

b) $\cos 260^\circ$

c) $\sin 4.2$

d) $\csc (-70^\circ)$