1a) $y=(x-2)(x-4)(x+3)^2$

(i) Degree: 4

(ii) Roots ($y=0$): $x=-3,-3,2,4$

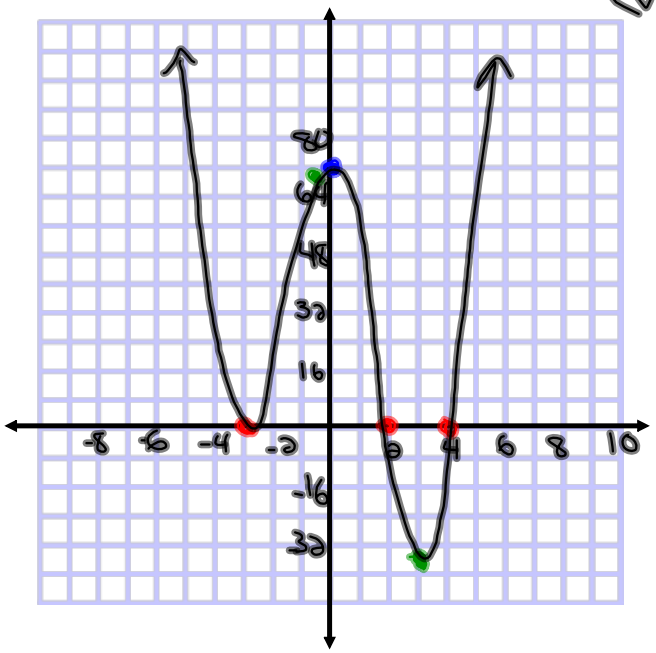
Double root

(iii) y int ($x=0$) $y=(-2)(-4)(3)^2$

$y=72$

$(0,72)$

(iv) Stretch: $a=1$



(v) max ($x=-0.5$)

$y=(-2.5)(-4.5)(2.5)^2$

$y=70.3$

min ($x=3$)

$y=(1)(-1)(6)^2$

$y=-36$

(1b) $y = (x-5)(x-2)(x+3)(x+1)$

(i) Degree: 4

(ii) Roots ($y=0$): $x = -3, -1, 2, 5$

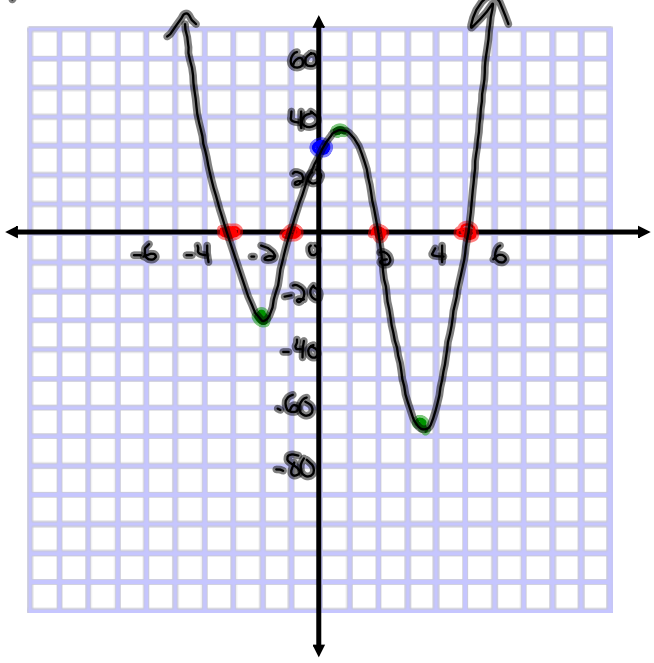
$(-3,0)$ $(-1,0)$ $(2,0)$ $(5,0)$

(iii) y int ($x=0$) $y = (-5)(-2)(3)(1)$

$y = 30$

$(0,30)$

(iv) Stretch: $a=1$



(v) min ($x=-2$)

$y = (-7)(-4)(1)(-1)$

$y = -28$

$(-2,-28)$

---

max (0.5)

$y = (-4.5)(-1.5)(3.5)(1.5)$

$y = 35.4$

$(0.5, 35.4)$

---

min ($x=3.5$)

$y = (-1.5)(1.5)(6.5)(4.5)$

$y = -65.8$

$(3.5, -65.8)$

① c) $y = -(x-3)^2(x+1)^2$

$y = -(x-3)(x-3)(x+1)(x+1)$

(i) 4th Degree

(ii) Roots ($y=0$)

$x = -1, -1, 3, 3$

Double Roots (Just Touches)

(iii) y intercept ($x=0$)

$y = -(0-3)^2(0+1)^2$

$y = -(9)(1)$

$y = -9$

(iv) Local min ($x=1$)

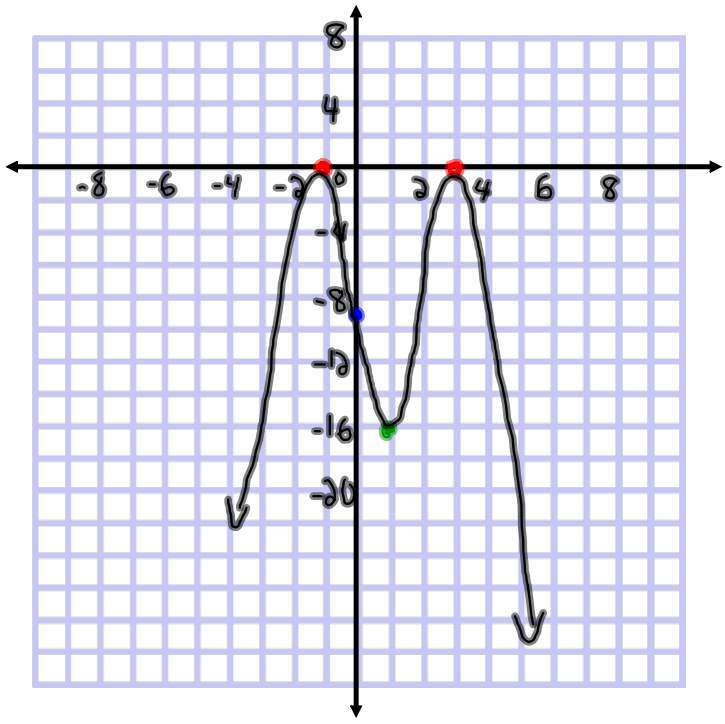
$y = -(x-3)^2(x+1)^2$

$y = -(1-3)^2(1+1)^2$

$y = -(4)(4)$

$y = -16$

$(1, -16)$



Specialized Factoring Techniques

- Common Factors
- The Sum and Difference of Cubes
- Grouping to Find a Common Factor
- Quartic Expressions Factored as Trinomials
- Grouping to get the Difference of Squares

Let's Start with a quick refresher!

Common Factor

$$12x^7y^8 + 24x^9y^4$$

$12x^7y^4(y^4 + 2x^2)$

Simple Trinomials

$x^2 - 5x + 6$

$-3 \times -2 = 6$

$-3 + -2 = -5$

$(x-3)(x-2)$

Trinomial Decomposition

$4x^2 + 5x - 6$

$8 \times -3 = -24$

$8 + -3 = 5$

$(4x^2 + 8x)(-3x - 6)$

$4x(x+2) - 3(x+2)$

$(4x-3)(x+2)$

Difference of Squares

$$a^2 - b^2 = (a - b)(a + b)$$

$$81x^2 - 49b^2$$

$$(9x - 7b)(9x + 7b)$$

Common Factor

$x^3 - x^2 - 12x$

$x(x^2 - x - 12)$ ← Simple Trinomial

$x(x-4)(x+3)$

Difference of Cubes

$$a^3 - b^3 = (a-b)(a^2 + ab + b^2)$$

$x^3 - 8$

$(x-2)(x^2+2x+4)$

$27x^3 - 64$

$(3x-4)(9x^2+12x+16)$

Sum of Cubes

$$a^3 + b^3 = (a+b)(a^2 - ab + b^2)$$

$x^3 + 27$

$(x+3)(x^2-3x+9)$

$64x^3 + 125$

$(4x+5)(16x^2-20x+25)$

Grouping to Find a Common Factor

A common factor can sometimes be found for specific groups of terms in a polynomial expression. The expression is written in the necessary order and each group of terms is then factored, leaving a common factor in brackets, which in turn is factored.

$(x^3 - 2x^2)(-16x + 32)$

$x^2(x-2) - 16(x-2)$

$(x-2)(x^2-16)$ ← Difference of Squares

$(x-2)(x-4)(x+4)$

$(x^3 + 4x^2)(-4x - 16)$

$x^2(x+4) - 4(x+4)$

$(x+4)(x^2-4)$

$(x+4)(x-2)(x+2)$

$(8x^5 - 40x^4 + 32x^3)(-x^2 + 5x - 4)$

$8x^3(x^2-5x+4) - 1(x^2-5x+4)$

Diff. of Cubes → $(8x^3-1)(x^2-5x+4)$ ← Simple Trinomial

$(2x-1)(4x^2+2x+1)(x-4)(x-1)$

Quartic Expressions Factored as Trinomials

$$x^4 - 5x^2 + 4$$

$$4x^4 - 37x^2 + 9$$

Grouping to Get the Difference of Squares

If a polynomial expression can be grouped in the form $(x+m)^2-n^2$, then it can be factored as the difference of squares.

$$x^4 + 5x^2 + 9$$

$$x^4 - 6x^2 + 1$$