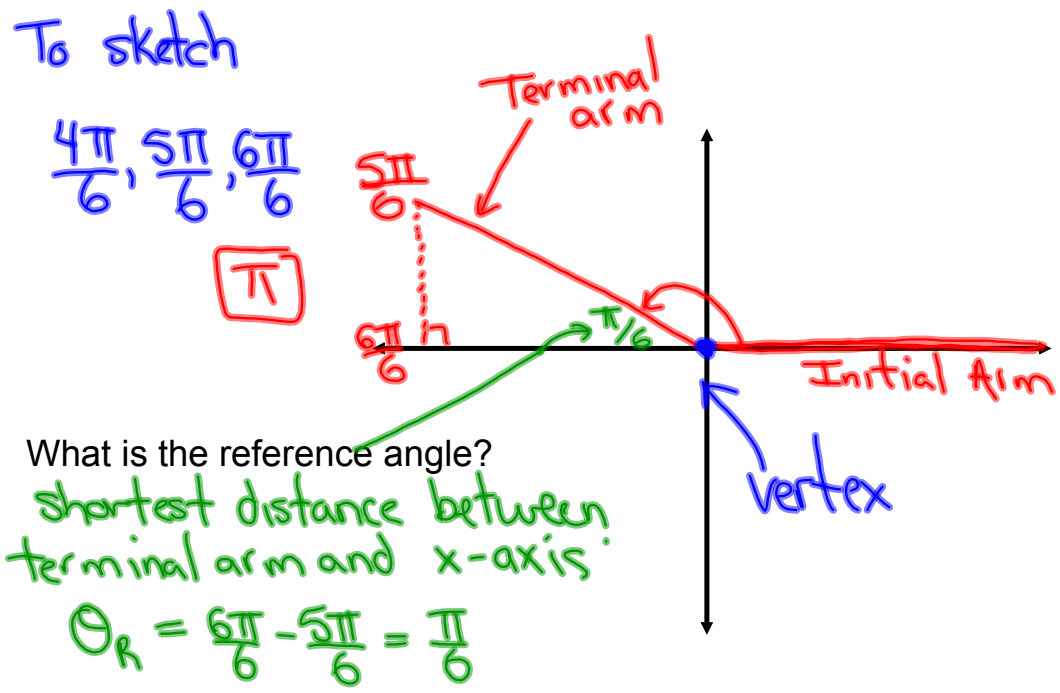
Warm-up

Draw an angle of $\frac{5\pi}{6}$ in standard position. Label the initial arm, vertex, and terminal arm.



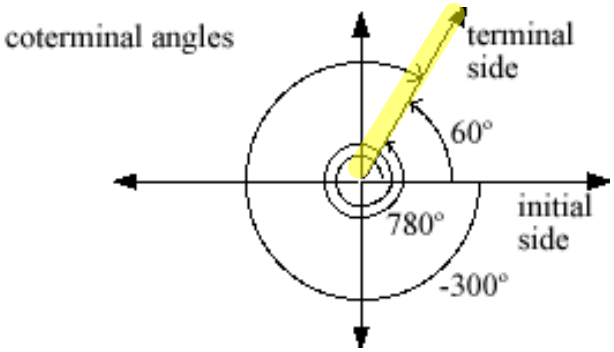
What is the reference angle?

shortest distance between terminal arm and x-axis.

$\theta_R = \frac{6\pi}{6} - \frac{5\pi}{6} = \frac{\pi}{6}$

Find one positive and one negative angle that is coterminal to $\frac{5\pi}{6}$.

Remember coterminal angles share the same terminal arm.



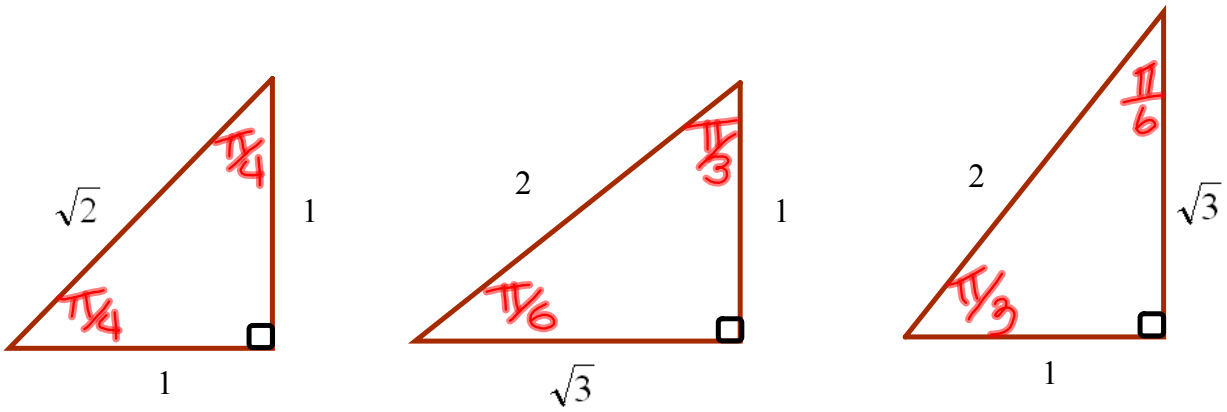
Positive:

$\frac{5\pi}{6} + \frac{2\pi}{1} = \frac{5\pi + 12\pi}{6} = \frac{17\pi}{6}$

Negative:

$\frac{5\pi}{6} - \frac{2\pi}{1} = \frac{5\pi - 12\pi}{6} = \frac{-7\pi}{6}$

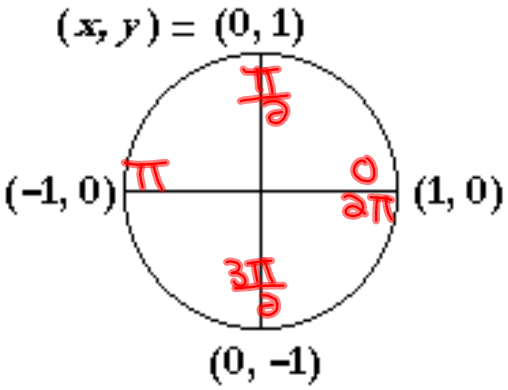
The Special Angles (Remember These)



Unit Circle

Quadrantal Angles (Multiples of 90 or $\frac{\pi}{2}$)

radius = 1 and center of circle is at (0,0)



$\sin\theta = y$ $\quad \csc\theta = \frac{1}{y}$

$\cos\theta = x$ $\quad \sec\theta = \frac{1}{x}$

$\tan\theta = \frac{y}{x}$ $\quad \cot\theta = \frac{x}{y}$

Determine the Trigonometric Ratios for Angles in the Unit Circle

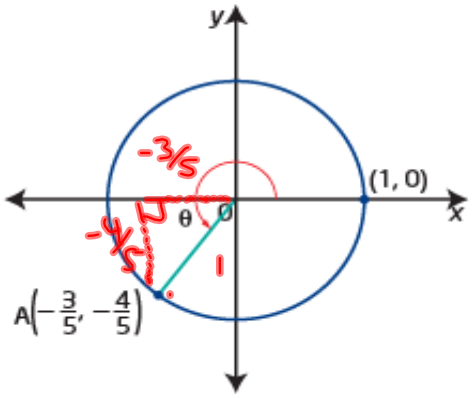
The point $A\left(-\frac{3}{5},-\frac{4}{5}\right)$ lies at the intersection of the unit circle and the terminal arm of an angle θ in standard position.

a) Draw a diagram to model the situation.

b) Determine the values of the six trigonometric ratios for θ. Express answers in lowest terms.

Solution

a)



b) $\sin\theta = y = \boxed{-\frac{4}{5}}$

$\cos\theta = x = \boxed{-3/5}$

$\tan\theta = \frac{y}{x} = -\frac{4}{5} \div -\frac{3}{5}$

$= -\frac{4}{5} \times \frac{5}{-3} = \boxed{\frac{4}{3}}$

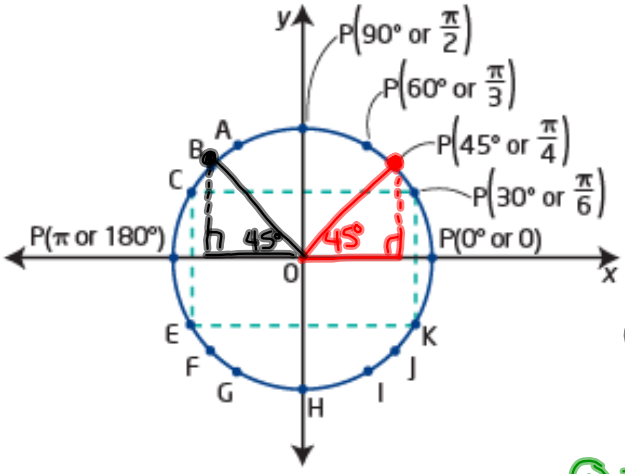
$\csc\theta = \frac{1}{y} = \frac{1}{-4/5} = \boxed{\frac{5}{4}}$

$\sec\theta = \frac{1}{x} = \frac{1}{-3/5} = \boxed{\frac{5}{3}}$

$\cot\theta = \boxed{\frac{3}{4}}$

Exact Values of Trigonometric Ratios

Exact values for the trigonometric ratios can be determined using special triangles (30°-60°-90° or 45°-45°-90°) and multiples of $\theta = 0, \frac{\pi}{6}, \frac{\pi}{4}, \frac{\pi}{3}$, and $\frac{\pi}{2}$ or $\theta = 0°, 30°, 45°, 60°$, and $90°$ for points $P(\theta)$ on the unit circle.



① How are P(30°), C, E, and K related?

② What points have the same coordinates as $P\left(\frac{\pi}{3}\right)$ except for their signs?

③ For P(45°), what are the coordinates and in which quadrant is θ?

④ Which special triangle would you use and where would it be placed for θ = 135°?

① They have the same coordinates except for their signs

② A, G, I

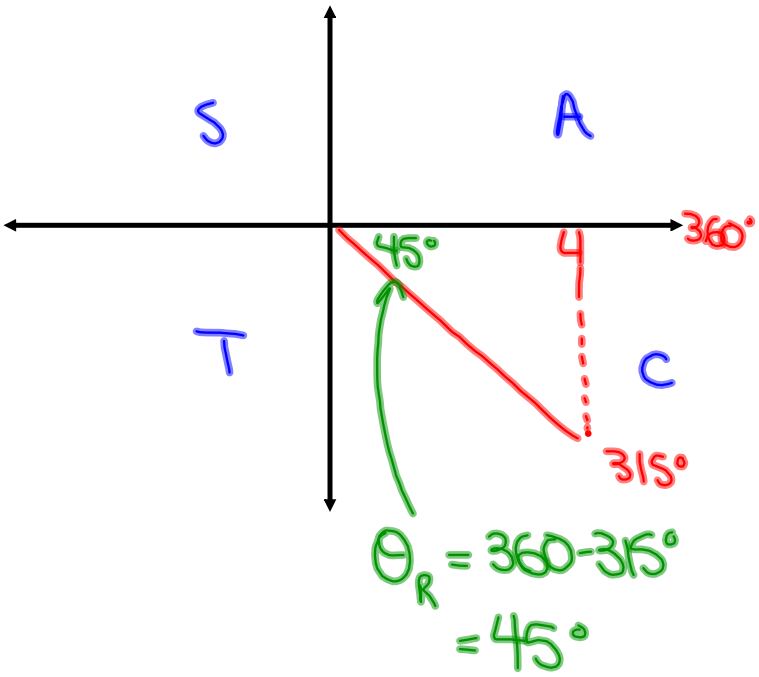
③ Quad I $\left(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$

④ It would be placed at point B

Extend the special angles into all FOUR quadrants

Without a calculator determine the value of $\tan 315°$ → $\frac{-1}{1}$ → -1

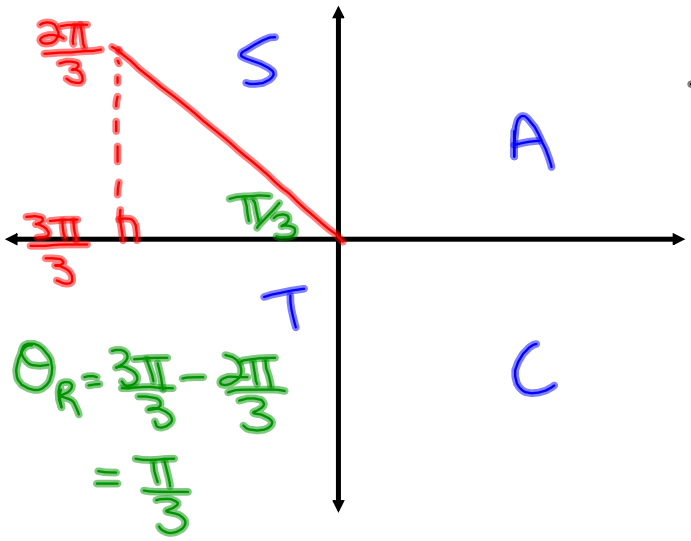
1. Start by sketching the angle



Extend the special angles into all FOUR quadrants

Without a calculator determine the value of $\sin\left(\frac{-4\pi}{3}\right)$

1. Start by sketching the angle



$\sin\frac{2\pi}{3}$

$\rightarrow +\frac{\sqrt{3}}{2}$

$-\frac{4\pi}{3}+\frac{2\pi}{1}$

$\frac{-4\pi+6\pi}{3}$

$\frac{2\pi}{3}$

$\frac{1\pi}{3}\ \frac{2\pi}{3}, \frac{3\pi}{3}$

π

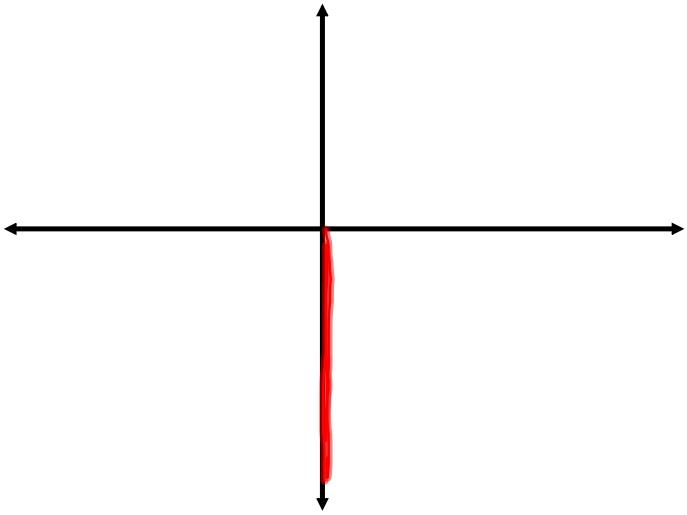
Extend the special angles into all FOUR quadrants

Without a calculator determine the value of csc630

P.A. = 630° − 360
= 270°

1. Start by sketching the angle

csc 270° → $\frac{1}{-1}$ → −1



Extend the special angles into all FOUR quadrants

Without a calculator determine the value of $\tan\frac{17\pi}{6}$

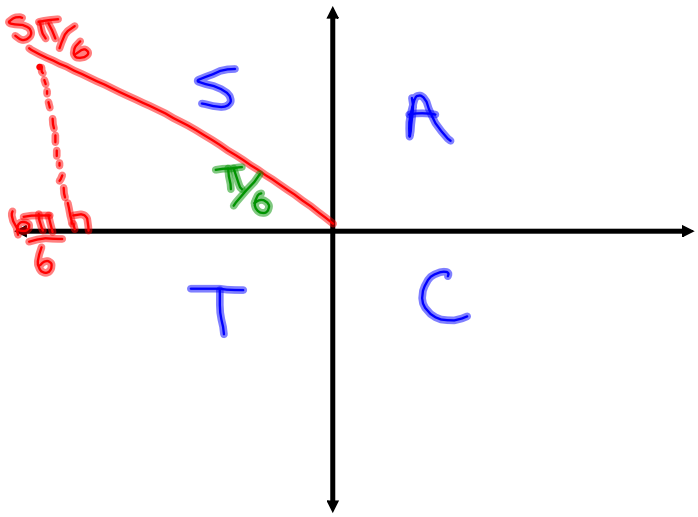
P.A. $\frac{17\pi}{6} - \frac{12\pi}{1}$

$\frac{17\pi - 12\pi}{6}$

$\frac{5\pi}{6}$

$\tan\frac{5\pi}{6} \rightarrow \frac{-1}{\sqrt{3}}$

1. Start by sketching the angle



Approximate Values of Trigonometric Ratios

You can determine approximate values for sine, cosine, and tangent using a scientific or graphing calculator. Most calculators can determine trigonometric values for angles measured in degrees or radians. You will need to set the mode to the correct angle measure. Check using

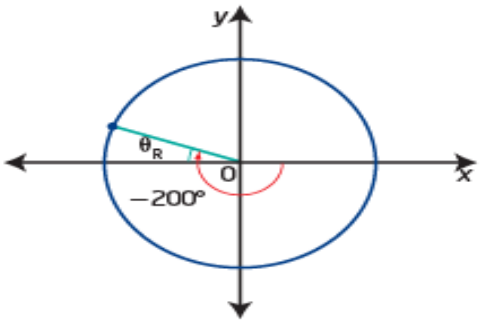
$\cos 60° = 0.5$ (degree mode)
$\cos 60 = -0.952\ 412\ 980\ldots$ (radian mode)

In which quadrant does an angle of 60 terminate?

Most calculators can compute trigonometric ratios for negative angles. However, you should use your knowledge of reference angles and the signs of trigonometric ratios for the quadrant to check that your calculator display is reasonable.

$\cos(-200°) = -0.939\ 692\ 620\ldots$

Is the negative value appropriate? What is the reference angle for $-200°$? What other trigonometric ratio could you compute as a check?



You can find the value of a trigonometric ratio for cosecant, secant, or cotangent using the correct reciprocal relationship.

$$\sec 3.3 = \frac{1}{\cos 3.3}$$
$$= -1.012\ 678\ 973\ldots$$
$$\approx -1.0127$$

Homework

Attempt #1-6 on page 133 of workbook

Exact Values for Trigonometric Ratios

Determine the exact value for each. Draw diagrams to illustrate your answers.

a) $\cos \frac{5\pi}{6}$

b) $\sin \left(-\frac{4\pi}{3}\right)$

c) $\sec 315°$

d) $\cot 270°$

Approximate Values for Trigonometric Ratios

Determine the approximate value for each trigonometric ratio. Give your answers to four decimal places.

a) $\tan \frac{7\pi}{5}$

b) $\cos 260°$

c) $\sin 4.2$

d) $\csc (-70°)$