

CH.3 REVIEW

Please complete pgs. 198 - 200

Questions 1bc, 4ab, 5, 9, 10b, & 14

Solutions

1. Use transformations to explain how the graph of each quadratic function compares to the graph of $f(x) = x^2$. Identify the vertex, the axis of symmetry, the direction of opening, the maximum or minimum value, and the domain and range without graphing.

b) $f(x) = -2x^2 + 19$

The graph of $f(x) = -2x^2 + 19$ will have a shape that is narrower than the graph of $f(x) = x^2$ and be reflected in the x -axis. Since $p=0$ and $q=19$, this represents a vertical translation of 19 units up relative to the graph of $f(x) = x^2$.

Vertex: $(0, 19)$

Axis of Symmetry: $x = 0$

Direction of Opening: Downward

Maximum Value: $y = 19$ or $(0, 19)$

Domain: $\{x \mid x \in \mathbb{R}\}$

Range: $\{y \mid y \leq 19, y \in \mathbb{R}\}$

c) $f(x) = \frac{1}{5}(x-10)^2 + 100$

The graph of $f(x) = \frac{1}{5}(x-10)^2 + 100$ will

have a shape that is wider than the graph of $f(x) = x^2$. Since $p=10$ and $q=100$, this represents a horizontal translation of 10 units to the right and a vertical translation of 100 units up relative to the graph of $f(x) = x^2$.

Vertex: $(10, 100)$

Axis of Symmetry: $x = 10$

Direction of Opening: Upward

Minimum Value: $y = 100$ or $(10, 100)$

Domain: $\{x \mid x \in \mathbb{R}\}$

Range: $\{y \mid y \geq 100, y \in \mathbb{R}\}$

Solutions

4. Determine a quadratic function with each set of characteristics.

{a&b} a) vertex at (0,0), passing through the point (20,-150)

$$y = a(x-p)^2 + q$$

$$y = a(x-0)^2 + 0$$

$$y = a(x)^2$$

$$-150 = a(20)^2$$

$$-150 = a(400)$$

$$-150 = \frac{400a}{400}$$

$$-\frac{3}{8} = a$$

$$y = -\frac{3}{8}x^2$$

b) vertex at (8,0), passing through the point (2,54).

$$y = a(x-p)^2 + q$$

$$y = a(x-8)^2 + 0$$

$$y = a(x-8)^2$$

$$54 = a(2-8)^2$$

$$54 = a(-6)^2$$

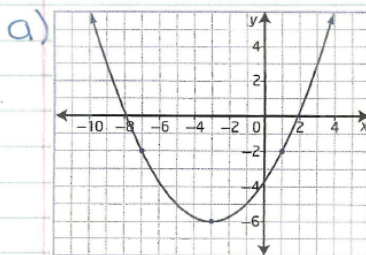
$$54 = a(36)$$

$$\frac{54}{36} = \frac{36a}{36}$$

$$\frac{3}{2} = a$$

$$y = \frac{3}{2}(x-8)^2$$

5. Write a quadratic function in vertex form for each graph.



Vertex: (-3,-6)
Point: (1,-2)

$$y = a(x-p)^2 + q$$

$$y = a(x+3)^2 - 6$$

$$-2 = a(1+3)^2 - 6$$

$$-2 = a(4)^2 - 6$$

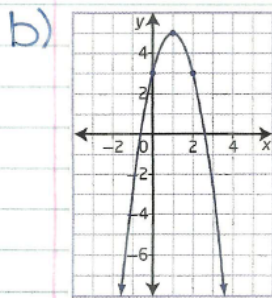
$$-2 + 6 = a(16)$$

$$\frac{4}{16} = \frac{16a}{16}$$

$$\frac{1}{4} = a$$

$$y = \frac{1}{4}(x+3)^2 - 6$$

Solutions



$$y = a(x-p)^2 + q$$

$$y = a(x-1)^2 + 5$$

$$3 = a(0-1)^2 + 5$$

$$3 = a(-1)^2 + 5$$

$$3 - 5 = a(1)$$

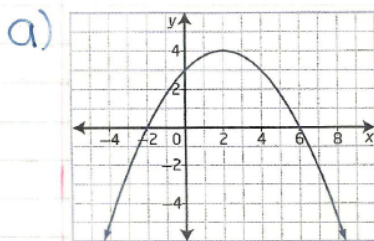
$$\frac{-2}{1} = \frac{a}{x}$$

$$-2 = a$$

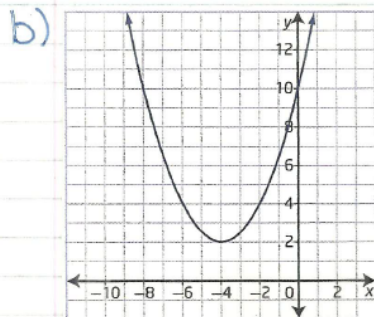
Vertex: (1, 5)
Point: (0, 3)

$$y = -2(x-1)^2 + 5$$

9. For each graph, identify the vertex, axis of symmetry, maximum or minimum value, direction of opening, domain and range, and any intercepts.



Vertex: (2, 4)
Axis of Symmetry: $x = 2$
Maximum Value: $y = 4$ or $(2, 4)$
Direction of Opening: Downward
Domain: $\{x \mid x \in \mathbb{R}\}$
Range: $\{y \mid y \leq 4, y \in \mathbb{R}\}$
x-intercepts: $x = -2$ and $x = 6$
y-intercept: $y = 3$



Vertex: (-4, 2)
Axis of Symmetry: $x = -4$
Minimum Value: $y = 2$ or $(-4, 2)$
Direction of Opening: Upward
Domain: $\{x \mid x \in \mathbb{R}\}$
Range: $\{y \mid y \geq 2, y \in \mathbb{R}\}$
x-intercepts: NONE
y-intercept: $y = 10$

Solutions

10. Show why each function fits the definition of a quadratic function.

$$\begin{aligned} \text{b) } y &= (2x+7)(10-3x) \\ &= 20x - 6x^2 + 70 - 21x \\ &= -6x^2 + 20x - 21x + 70 \\ &= -6x^2 - 1x + 70 \end{aligned}$$

This is a quadratic function since the highest exponent is 2.

14. Write each function in vertex form.

$$\begin{aligned} \text{a) } y &= x^2 - 24x + 10 \\ &= (x^2 - 24x) + 10 \\ &= (x^2 - 24x + 144 - 144) + 10 \\ &= (x^2 - 24x + 144) - 144 + 10 \\ &= (x - 12)^2 - 134 \end{aligned}$$

$$\begin{aligned} \text{b) } y &= 5x^2 + 40x - 27 \\ &= 5(x^2 + 8x) - 27 \\ &= 5(x^2 + 8x + 16 - 16) - 27 \\ &= 5[(x^2 + 8x + 16) - 16] - 27 \\ &= 5[(x + 4)^2 - 16] - 27 \\ &= 5(x + 4)^2 - 80 - 27 \\ &= 5(x + 4)^2 - 107 \end{aligned}$$

Solutions

$$\begin{aligned} \text{c) } y &= -2x^2 + 8x \\ &= -2(x^2 - 4x) \\ &= -2(x^2 - 4x + 4 - 4) \\ &= -2[(x^2 - 4x + 4) - 4] \\ &= -2[(x-2)^2 - 4] \\ &= -2(x-2)^2 + 8 \end{aligned}$$

$$\begin{aligned} \text{d) } y &= -30x^2 - 60x + 105 \\ &= -30(x^2 + 2x) + 105 \\ &= -30(x^2 + 2x + 1 - 1) + 105 \\ &= -30[(x^2 + 2x + 1) - 1] + 105 \\ &= -30[(x+1)^2 - 1] + 105 \\ &= -30(x+1)^2 + 30 + 105 \\ &= -30(x+1)^2 + 135 \end{aligned}$$