Questions from Homework

Exercise #2

① f) $\log_2\left(\frac{1}{16}\right) = -4 \longleftrightarrow 2^{-4} = \frac{1}{16}$

base, ans., exp.

② d) $81^{\frac{1}{2}} = 9 \longleftrightarrow \log_{81} 9 = \frac{1}{2}$

base, exp., ans.

④ e) $2^{1-x} = 3$

$\log_2 3 = 1 - x$

$x = 1 - \log_2 3$

g) $\log_2(\log_3 x) = 4$

base, ans., exp

$2^4 = \log_3 x$

$16 = \log_3 x$

exp., base, ans.

$3^{16} = x$

$43\ 046\ 721 = x$

h) $10^{5^x} = 3$

$\log_{10} 3 = 5^x$

ans., base, exp.

$\log_5(\log_{10} 3) = x$

Exercise # 3

① f) $\log_a (xy)^{10}$

$10[\log_a (xy)]$

$10(\log_a x + \log_a y)$

$10\log_a x + 10\log_a y$

h) $\log_b \frac{x^2}{yz^3}$

$\log_b x^2 - \log_b y - \log_b z^3$

$2\log_b x - \log_b y - 3\log_b z$

② a) $\log_5 \sqrt{125}$

$5^x = (125)^{1/2}$

$5^x = (5^3)^{1/2}$

$5^x = 5^{3/2}$

$x = \frac{3}{2}$

③ d) $4\log_a x - \frac{1}{3}\log_a (x^2+1) + \log_a (x-1)$

$\log_a x^4 - \log_a (x^2+1)^{1/3} + \log_a (x-1)$

$\log_a \frac{x^4(x-1)}{(x^2+1)^{1/3}}$

$\log_a \frac{x^4(x-1)}{\sqrt[3]{x^2+1}}$

e) $\frac{1}{2}[\log_5 x + 2\log_5 y - 3\log_5 z]$

$\frac{1}{2}[\log_5 x + \log_5 y^2 - \log_5 z^3]$

$\frac{1}{2}[\log_5 \frac{xy^2}{z^3}]$

$\log_5 \left(\frac{xy^2}{z^3}\right)^{1/2}$

$\log_5 \sqrt{\frac{xy^2}{z^3}} \rightarrow \log_5 y\sqrt{\frac{x}{z^3}}$

Logarithms

exponential form

$$x = a^y$$

Say "the base ***a*** to the exponent ***y*** is ***x***."

logarithmic form

$$y = \log_a x$$

Say "***y*** is the exponent to which you raise base ***a*** to get the answer ***x***."

$$x = a^y \longleftrightarrow y = \log_a x$$

When you work with equations involving logarithms you need to use the laws of logarithms, which are summarized below:

$$\log_a M + \log_a N = \log_a (M \times N)$$

$$\log_a M - \log_a N = \log_a \left(\frac{M}{N} \right)$$

$$\log_a (N^p) = p \log_a N$$

$$\log_a (N^{\frac{p}{q}}) = \frac{p}{q} \log_a N$$

The base of a logarithm can be any real number. However, a logarithm to the base 10 is especially useful because the decimal system, and as a result your calculator, is also based on the number 10. Logarithms to the base 10 are called *common logarithms* and are written as

$\log_{10} x$ or $\log x$

Example 1

Find $\log 56$

Common logarithms appear in many formulas as shown in the following example.

Example 2

The approximate distance above sea level, *d,* in kilometers, is given by the formula:

$$d = \frac{500(\log P - 2)}{27}$$

where *P* is the pressure in kilopascals.

a) If the reading on a barometer is 750 *kPa,* then how far above sea level are you?

b) What is the barometric pressure 1*km* above sea level?

The irrational number *"e"* which is approximately 2.71828... plays an important role in the development of mathematics. The value of *e* can be approximated by the following expression:

$$\left(1+\frac{1}{n}\right)^n$$

As *"n"* gets larger, the expression approaches the number 2.71828... which is an approximation of *e.* This value is called *"Euler's Constant"* named after Leonard Euler.

$$e = \lim_{n\to\infty}\left(1+\frac{1}{n}\right)^n$$

Logarithms with the base of *e* are often used in advanced mathematics are called *natural logarithms.* The notation ln *x* is used to indicate logarithms to the base *e.* Thus,

$$\ln x = \log_e x$$

Example 3

Solve

a) $y = \ln 3$

b) $2.685 = \ln x$

Homework