Questions from Homework

Ex. 2

④ h) $10^{5^x} = 3$

$\underbrace{\log_{10} 3}_{\text{ans}} = \underset{\text{base}}{5}^{x \leftarrow \text{exp.}}$

$\log_5(\log_{10} 3) = x$

g) $\log_{\underset{\text{base}}{2}}(\underset{\text{ans.}}{\log_3 x}) = \underset{\text{exp}}{4}$

$2^4 = \log_3 x$

$\underset{\text{exp.}}{16} = \log_{\underset{\text{base}}{3}} \underset{\text{ans}}{x}$

$3^{16} = x$

Logarithms

exponential form

$$x = b^y$$

Say "the base ***b*** to the exponent ***y*** is ***x***."

logarithmic form

$$y = \log_b x$$

Say "***y*** is the exponent to which you raise base ***b*** to get the answer ***x***."

$$x = b^y \longleftrightarrow y = \log_b x$$

When you work with equations involving logarithms you need to use the laws of logarithms, which are summarized below:

$$\log_b M + \log_b N = \log_b(MN)$$

$$\log_b M - \log_b N = \log_b\left(\frac{M}{N}\right)$$

$$\log_b(N^p) = p\log_b(N)$$

$$\log_b(N^{\frac{p}{q}}) = \frac{p}{q}\log_b(N)$$

The base of a logarithm can be any real number. However, a logarithm to the base 10 is especially useful because the decimal system, and as a result your calculator, is also based on the number 10. Logarithms to the base 10 are called *common logarithms* and are written as

$$\log_{10} x \quad \text{or} \quad \log x$$

Example 1

Find $\log 56 = 1.748$

$10^{1.748} = 56$

Common logarithms appear in many formulas as shown in the following example.

Example 2

The approximate distance above sea level,*d,* in kilometers, is given by the formula:

$$d = \frac{500(\log P - 2)}{27}$$

where *P* is the pressure in kilopascals.

a) If the reading on a barometer is 750 *kPa,* then how far above sea level are you?

b) What is the barometric pressure 1 *km* above sea level?

a) $P = 750\,kPa$

$d = ?$

$$d = \frac{500(\log P - 2)}{27}$$

$$d = \frac{500(\log 750 - 2)}{27}$$

$$d = \frac{500(2.875 - 2)}{27}$$

$$\boxed{d = 16.2\,km}$$

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b) $d = 1\,km$

$P = ?$

$$d = \frac{500(\log P - 2)}{27}$$

$$1 = \frac{500(\log P - 2)}{27}$$

$$\frac{\cancel{500}(\log P - 2)}{\cancel{500}} = \frac{27}{500}$$

$$\log P - 2 = 0.054$$

$$\log P = 2.054$$

$$10^{2.054} = P$$

$$\boxed{113.24\,kPa = P}$$

The irrational number *"e"* which is approximately 2.71828... plays an important role in the development of mathematics. The value of *e* can be approximated by the following expression:

$$\left(1+\frac{1}{n}\right)^n$$

As *"n"* gets larger, the expression approaches the number 2.71828.. which is an approximation of *e.* This value is called *"Euler's Constant"* named after Leonard Euler.

$$\boxed{e=\lim_{n\to\infty}\left(1+\frac{1}{n}\right)^n}$$

Logarithms with the base of *e* are often used in advanced mathematics and are called *natural logarithms.* The notation ln *x* is used to indicate logarithms to the base *e.* Thus,

$$\ln x = \log_e x$$

Example 3

Solve

a) $y = \ln 3 = 1.098$

$e^y = 3$

$\cancel{e}^y = \cancel{e}^{1.0986}$

$y = 1.0986$

b) $2.685 = \ln x$

$e^{2.685} = x$

$14.66 = x$

Homework