Questions from Homework

① c) $\frac{3x+6}{x^2} \cdot \frac{x}{x^2+2x}$

$\frac{3(x+2)}{x^2} \cdot \frac{x}{x(x+2)}$

$\boxed{\frac{3}{x^2}}$ $x \neq 0, -2$

d) $\frac{\frac{2}{x}+\frac{3}{xy}}{\frac{2}{xy}+\frac{3}{y}} \cdot \frac{xy}{xy}$

$\boxed{\frac{2y+3}{2+3x}}$ $x \neq -\frac{2}{3}, 0$ $y \neq 0$

② c) $\sqrt{3x+15} = 1+\sqrt{18+x}$

$3x+15 = 19+2\sqrt{18+x}+x$

$2x-4 = 2\sqrt{18+x}$

$x-2 = \sqrt{18+x}$

$x^2-4x+4 = 18+x$

$x^2-5x-14=0$

$(x-7)(x+2)=0$

$\boxed{x=7}$ is a solution. | $\boxed{x=-2}$ ← extraneous root

④ b) $f(x) = \dfrac{x^2-2x-3}{x^2+6x+8} = \dfrac{(x-3)(x+1)}{(x+2)(x+4)}$

① Roots:

$(x-3)(x+1)=0$

$x=-1,3$

② y int:

$y=-\frac{3}{8}$

③ VA:

$x+2=0 \mid x+4=0$

$x=-2 \quad x=-4$

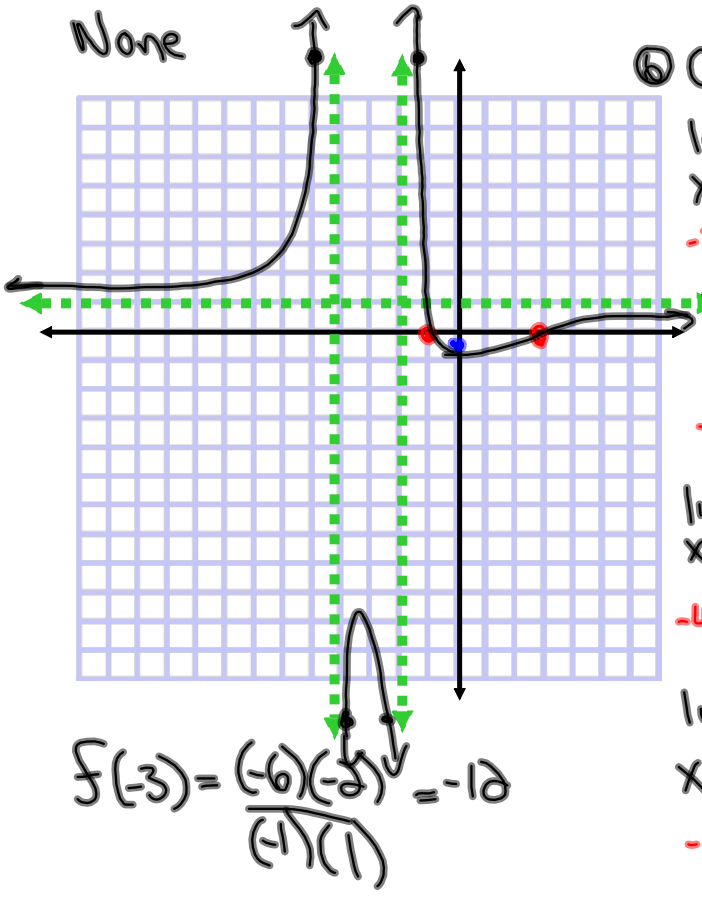
④ HA:

$\lim_{x\to\infty} \dfrac{x^2-2x-3}{x^2+6x+8} = \dfrac{1}{1}$

$y=1$

⑤ Holes:

None



⑥ Check VA:

$\lim_{x\to -2^-} \dfrac{(-)(-)}{(-)(+)} = -\infty$

-2.01

$\lim_{x\to -2^+} \dfrac{(-)(-)}{(+)(+)} = +\infty$

-1.99

$\lim_{x\to -4^-} \dfrac{(-)(-)}{(-)(-)} = +\infty$

-4.01

$\lim_{x\to -4^+} \dfrac{(-)(-)}{(-)(+)} = -\infty$

-3.99

$f(-3) = \dfrac{(-6)(-2)}{(-1)(1)} = -12$

Series and Sequence

Arithmetic (common difference "d")	Geometric (Common Ratio "r")
$t_n = a + (n-1)d$	$t_n = ar^{n-1}$
$S_n = \frac{n}{2}(2a + (n-1)d)$	$S_n = \frac{a(r^n - 1)}{r - 1}$
$S_n = \frac{n}{2}(a + t_n)$	$\boxed{S_n = \frac{a}{1-r}}$ $-1 < r < 1$

1. Identify as Arithmetic or Geometric and then find the number of terms "n"

a) -5, -2, 1, 4, …103. **b)** 2, 6, 18, …486. **c)** $\frac{1}{4}$, $\frac{1}{2}$, 1, … 64.

Arithmetic

$a=-5$ $\quad t_n = a+(n-1)d$

$d=3$ $\quad 103 = -5+(n-1)(3)$

$t_n=103$ $\quad 108 = 3n-3$

$111 = 3n$

$\boxed{37=n}$

b) 2, 6, 18, …486.

Geometric

$a=2$ $\quad t_n = ar^{n-1}$

$r=3$ $\quad \frac{486}{2} = \frac{(2)(3)^{n-1}}{2}$

$t_n=486$

$243 = 3^{n-1}$

$3^5 = 3^{n-1}$

$5 = n-1$

$\boxed{6=n}$

c) $\frac{1}{4}$, $\frac{1}{2}$, 1, … 64.

Geometric

$a=\frac{1}{4}$ $\quad t_n = ar^{n-1}$

$r=2$ $\quad \frac{64}{\frac{1}{4}} = \frac{(\frac{1}{4})(2)^{n-1}}{\frac{1}{4}}$

$t_n=64$

$256 = 2^{n-1}$

$2^8 = 2^{n-1}$

$8 = n-1$

$\boxed{9=n}$

As it aged, a maple tree produced sap according to the pattern shown in the table below.

Year	2001	2002	2003	2004
Sap (Litres)	$t_1 = 60.000$	$t_2 = 57.000$	$t_3 = 54.150$	$t_4 = 51.4425$

$a = 60$

a) Does the data follow an arithmetic or geometric pattern?

$r = \frac{57}{60} = \frac{54.15}{57} = 0.95$ geometric

b) Write down a formula for t_n?

$t_n = ar^{n-1}$

$t_n = 60(0.95)^{n-1}$

c) Assuming the pattern continues, how long will it take for the sap production to be approximately 17.5L?

$t_n = 17.5L$.

$n = ?$

$\frac{17.5}{60} = \frac{60(0.95)^{n-1}}{60}$

$0.29\overline{16} = 0.95^{n-1}$

$0.95^{24} = 0.95^{n-1}$

$24 = n-1$

$25 = n$

d) If the tree lives for a very long time approximately how much sap will it produce from 2001 on? (Infinite Geometric Series)

$S_n = \frac{a}{1-r}$ * $-1 < r < 1$ $r = 0.95$ satisfies

$S_n = \frac{60}{1-0.95}$

$= \frac{60}{0.05}$

$= 1200\ L$

8. A computer software company formed a committee of 5 people to spread the word about a new feature. Each person on the committee emailed 3 individuals (cycle 1) who were each asked to email 3 more people (cycle 2). This pattern continued and new individuals were contacted at each cycle (i.e. nobody received two emails).

a) What is the value of t_1? $5 \times 3 = 15 \quad a = 15$ **[1]**

b) Write down a formula for t_n. $t_n = ar^{n-1} = (15)(3)^{n-1}$ **[1]**

$a = 15 \quad r = 3$

c) On which cycle were 32 805 people contacted? **[2]**

d) Find the total number of people contacted in 4 cycles. **[2]**

c) $\frac{32\,805}{15} = \frac{(15)(3)^{n-1}}{15}$

$2187 = 3^{n-1}$

$3^7 = 3^{n-1}$

$7 = n - 1$

$\boxed{8 = n}$

d) $S_n = \frac{a(r^n - 1)}{r - 1}$

$S_4 = \frac{15(3^4 - 1)}{3 - 1}$

$S_4 = \frac{1200}{2}$

$\boxed{S_4 = 600}$

① $a = 80000$

$t_5 = 117\,128$

$n = 5$

$r = 1.1$

$t_n = ar^{n-1}$

$\frac{117\,128}{80000} = \frac{80000r^{5-1}}{80000}$

$1.4641 = r^4$

$1.1 = r$

Annual rate of increase is 10%.

⑤ $\sum_{n=1}^{5} n^2 + 1$

$= 2 + 5 + 10 + 17 + 26$

$= 6$

③c) $a = 60$

$r = 0.95$

$t_n = 22.64$

$n = ?$

$t_n = ar^{n-1}$

$\frac{22.64}{60} = \frac{(60)(0.95)^{n-1}}{60}$

$0.3773 = 0.95^{n-1}$

$(0.95)^{19} = (0.95)^{n-1}$

$19 = n - 1$

$20 = n$

$\frac{\log(0.3773)}{\log(0.95)} = 19$