Questions from Homework

Complex Numbers

Convert to Polar coordinates

Find the radius ***r,*** using the Pythagorean relationship $r = \sqrt{x^2 + y^2}$

Find the related angle, **α,** using $\alpha = \tan^{-1}\left(\frac{|y|}{|x|}\right)$

Find the angle, θ, by determining the quadrant in which the terminal arm is located and using the related angle.

★	180-α	α
180+α	360-α

Remember from last semester

The polar coordinates are ***(r, θ)***

Convert to Rectangular coordinates

$x = r\cos\theta$

$y = r\sin\theta$

De Moivre's Theorem

$$(rcis\theta)^n = r^n cisn\theta$$

1. Simplify the following expressions. Express solution in the form $a + bi$.

(a) $3i^7 - i^{10} + (2i)^5$

$3i^7 - i^{10} + 32i^5$

$3(i^4)(i^3) - (i^8)(i^2) + 32(i^4)(i)$

$-3i + 1 + 32i$

$\boxed{1 + 29i}$

(b) $\dfrac{(1+i)(2+3i)}{-3+2i}$

$\dfrac{2 + 5i + 3i^2}{-3+2i}$

$\dfrac{(-1+5i)(-3-2i)}{(-3+2i)(-3-2i)}$

$\dfrac{3 - 13i - 10i^2}{9 - 4i^2}$

$\dfrac{13 - 13i}{13}$

$\boxed{1 - i}$

3. Use polar coordinates and De Moivre's Theorem to evaluate the following expression:

(***Express solution in the form*** a + b*i*)

$$\frac{\left(-\sqrt{3}+i\right)^4\left(2-2i\right)^6}{\left(-1-i\sqrt{3}\right)^{10}}$$

$a=-\sqrt{3}$ $b=1$

$r=\sqrt{(-\sqrt{3})^2+(1)^2}$

$r=\sqrt{3+1}$

$r=2$

$\alpha=\tan^{-1}\left(\frac{1}{\sqrt{3}}\right)$

$\alpha=30°$

Quad 2

$\theta=180-30°$

$\theta=150°$

$2\text{cis}150°$

$a=2$ $b=-2$

$r=\sqrt{(2)^2+(-2)^2}$

$r=\sqrt{4+4}$

$r=2\sqrt{2}$

$\alpha=\tan^{-1}\left(\frac{2}{2}\right)$

$\alpha=\tan^{-1}(1)$

$\alpha=45°$

Quad 4

$\theta=360-45$

$\theta=315°$

$2\sqrt{2}\text{cis}315°$

$a=-1$ $b=\sqrt{3}$

$r=\sqrt{1+3}$

$r=2$

$\alpha=\tan^{-1}\left(\frac{\sqrt{3}}{1}\right)$

$\alpha=\tan^{-1}(\sqrt{3})$

$\alpha=60°$

Quad 3

$\theta=180+60°$

$\theta=240°$

$2\text{cis}240°$

$$\frac{(2\text{cis}150°)^4(2\sqrt{2}\text{cis}315°)^6}{(2\text{cis}240°)^{10}}$$

$$\frac{(16\text{cis}600°)(512\text{cis}1890°)}{1024\text{cis}2400°}$$

$$\frac{8192\text{cis}2490°}{1024\text{cis}2400°}$$

$8\text{cis}90°$ as a polar number

$a=8\cos90°$

$=8(0)$

$=0$

$b=8\sin90°$

$=8(1)$

$=8$

$0+8i$ as a rectangular number.

Complex Numbers Review:

① b) $(1-i)^{10}$

$a=1$, $b=-1$ } Quad 4

② $r=\sqrt{a^2+b^2}$

$r=\sqrt{(1)^2+(-1)^2}$

$r=\sqrt{2}$

③ $\alpha=\tan^{-1}\left(\frac{|b|}{|a|}\right)$

$\alpha=\tan^{-1}\left(\frac{1}{1}\right)$

$\alpha=45^\circ$

④ Q4

$\theta=360-\alpha$

$\theta=360-45^\circ$

$\theta=315$

⑤ $\sqrt{2}\,\text{cis}\,315^\circ$

$(\sqrt{2}\,\text{cis}\,315)^{10}$

$=\sqrt{2}^{10}\,\text{cis}(10\cdot 315)$

$=32\,\text{cis}\,3150^\circ$

$=32\,\text{cis}\,270^\circ$ Polar

$=0-32i$ Rectangular

① c) $(1+i)^{11}(1-i)^{15}$

$a=1, b=1$ } Q1 $\quad a=1, b=-1$ } Q4

① $r=\sqrt{a^2+b^2}$
$r=\sqrt{(1)^2+(1)^2}$
$r=\sqrt{2}$

② $\alpha=\tan^{-1}\left(\frac{|b|}{|a|}\right)$
$\alpha=\tan^{-1}\left(\frac{1}{1}\right)$
$\alpha=45°$

③ Q1
$\theta=\alpha$
$\theta=45°$

④ $\sqrt{2}\,\text{cis}\,45°$

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① $r=\sqrt{(1)^2+(-1)^2}$
$r=\sqrt{2}$

② $\alpha=\tan^{-1}\left(\frac{1}{1}\right)$
$\alpha=45°$

③ Q4
$\theta=360-\alpha$
$\theta=315°$

④ $\sqrt{2}\,\text{cis}\,315°$

$(\sqrt{2}\,\text{cis}\,45°)^{11}(\sqrt{2}\,\text{cis}\,315°)^{15}$

$(\sqrt{2}^{11}\cdot\text{cis}\,11\times45)(\sqrt{2}^{15}\cdot\text{cis}\,15\times315)$

$(32\sqrt{2}\,\text{cis}\,495°)(128\sqrt{2}\,\text{cis}\,4725°)$

$8192\,\text{cis}\,5220°$

$8192\,\text{cis}\,180°$ Polar

$-8192+0i$ Rectangular