Questions from Homework

④ b) $f(x)=\frac{2x-1}{4x}$ $f(x+h)=\frac{2(x+h)-1}{4(x+h)}=\frac{2x+2h-1}{4x+4h}$

$$f'(x)=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}=\lim_{h\to 0}\frac{\frac{2x+2h-1}{4x+4h}-\left[\frac{2x-1}{4x}\right]}{h}$$

$$f'(x)=\lim_{h\to 0}\frac{4x(2x+2h-1)-(2x-1)(4x+4h)}{4xh(4x+4h)}$$

$$f'(x)=\lim_{h\to 0}\frac{8x^2+8xh-4x-(8x^2+8xh-4x-4h)}{4xh(4x+4h)}$$

$$f'(x)=\lim_{h\to 0}\frac{4h}{4xh(4x+4h)}=\frac{1}{x(4x)}=\frac{1}{4x^2}$$

⑥ b) $y=2x^3-6\sqrt{x}$ at $(4,20)$

$y=2x^3-6x^{1/2}$

① $y'=4x-3x^{-1/2}$

$y'=4x-\frac{3}{\sqrt{x}}$

③ $y'(4)=4(4)-\frac{3}{\sqrt{4}}$

$=16-\frac{3}{2}$

$=\frac{29}{2}$ ← m

③ $y-y_1=m(x-x_1)$

$y-20=\frac{29}{2}(x-4)$

$y-20=\frac{29x}{2}-58$

$2y-40=29x-116$

$0=29x-2y-76$

① a) $\lim_{x\to 0}\frac{\frac{2}{x+2}-1}{x}$ CD: $x+2$

$\lim_{x\to 0}\frac{2-x-2}{x(x+2)}$

$\lim_{x\to 0}\frac{-x}{x(x+2)}=\frac{-1}{2}$

⑦ d) $y=(3x^2-5)^3$ (Chain Rule)

$y'=2(3x^2-5)^1(6x)$

$y'=12x(3x^2-5)$

d) $y=(3x^2-5)^3=9x^4-30x^2+25$ (Power Rule)

$y'=36x^3-60x$

Functions Toolkit 2

1. Solve the following $|3x-2|>7$

$3x-2>7$ $\quad$ $3x-2<-7$

$3x>9$ $\quad$ $3x<-5$

$x>3$ $\quad$ $x<-\frac{5}{3}$

2. Solve the following $-9\leq|2x-5|\leq 13$

$-9\leq 2x-5\leq 13$ $\quad$ $9\geq 2x-5\geq -13$

$-4\leq 2x\leq 18$ $\quad$ $14\geq 2x\geq -8$

$-2\leq x\leq 9$ $\quad$ $7\geq x\geq -4$

$-4\leq x\leq 7$

3. Solve for x $1+\sqrt{x-3}=x-2$

$\sqrt{x-3}=x-3$ ← Square both sides

$x-3=x^2-6x+9$

$0=x^2-7x+12$

$0=(x-4)(x-3)$

$x-4=0$ | $x-3=0$

✓ $x=4$ | ✓ $x=3$

Both are solutions

6. Simplify the following rational expressions.

a) $\frac{4}{x^2-x-30}-\frac{2}{x^2+8x+15}$

b) $\frac{2x}{3x+5}+\frac{x}{3x^2-x-10}$

c) $\frac{3x+6}{x^2}\times\frac{x}{x^2+2x}$

d) $\frac{\frac{2}{x}+\frac{3}{y}}{\frac{2}{xy}+\frac{3}{y}}$ $\cdot xy$ $\cdot xy$ $= \boxed{\frac{2y+3}{2+3x}}$ $x\neq 0,-\frac{2}{3}$ $y\neq 0$

a) $\frac{4}{(x-6)(x+5)}-\frac{2}{(x+5)(x+3)}$

$\frac{4(x+3)-2(x-6)}{(x-6)(x+5)(x+3)}$

$\frac{4x+12-2x+12}{(x-6)(x+5)(x+3)}$

$\boxed{\frac{2x+24}{(x-6)(x+5)(x+3)}}$

$x\neq -5,-3,6$

c) $\frac{3(x+2)}{x^2}\times\frac{x}{x(x+2)}$

$\frac{3x(x+2)}{x^3(x+2)}$

$\boxed{\frac{3}{x^2}}$

$x\neq 0,-2$

9. Sketch the following rational function

a) $f(x)=\dfrac{x^2-2x-3}{x+1}$ $=\dfrac{(x-3)(x+1)}{(x+1)}=x-3$

① Roots: $x=3$

② y int: $y=-3$

③ VA: None

④ OA: $y=x-3$

⑤ Holes: $x=-1$

$f(-1)=(-1)-3$
$=-4$
$(-1,-4)$