Questions from Homework

② $y = 2x^3 - 5x^2 - 4x + 3$

$= 2(-1)^3 - 5(-1)^2 - 4(-1) + 3$

$= -2 - 5 + 4 + 3$

$= 0$

if $x = -1$ makes $y = 0$ the $(x+1)$ is a factor.

$$\begin{array}{r} 2x^2 - 7x + 3 \\ x+1 \overline{)\,2x^3 - 5x^2 - 4x + 3} \\ -(2x^3 + 2x^2) \downarrow \quad\quad \\ -7x^2 - 4x \quad \\ -(-7x^2 - 7x) \quad \\ \hline 3x + 3 \\ -(3x+3) \\ \hline 0 \end{array}$$

Decomp. $-6 \times -1 = 6$, $-6 + -1 = -7$

$(x+1)(2x^2 - 7x + 3)$

$(x+1)(2x^2 - 6x - 1x + 3)$

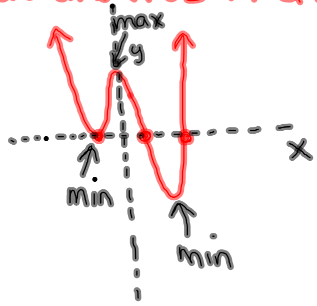
$(x+1)[2x(x-3) - 1(x-3)]$

$\boxed{(x+1)(2x-1)(x-3)}$

③ $y = (x+3)^2(x-2)(x-3)$ $a=1$

Roots: $x = -3, -3, 2, 3$

Quartic with a positive stretch factor starts in Q2 and ends in Q1.



c) App. Local Max $(x = -0.5)$

$y = (x+3)^2(x-2)(x-3)$

$y = (2.5)^2(-2.5)(-3.5)$

$y = 54.7$

$(-0.5, 54.7)$

d) App. Local min $(x = 2.5)$

$y = (x+3)^2(x-2)(x-3)$

$y = (5.5)^2(0.5)(-0.5)$

$y = -7.6$

$(2.5, -7.6)$

Questions from Homework

⑥ $x^3 - x^2 < 12x$

$x^3 - x^2 - 12x < 0$

$y = x^3 - x^2 - 12x$

$y = x(x^2 - x - 12)$

$y = (x)(x-4)(x+3)$

Roots: $x = -3, 0, 4$

$-$ $+$ $-$ $+$

(-4) -3 (-1) 0 (1) 4 (5)

$$\boxed{x \in (-\infty, -3) \cup (0, 4)}$$

Limits Review

1. Evaluate the following limits if they exist.

(a) $\lim_{x\to -5} \frac{x^2-25}{x^2+5x}$

$$\lim_{x\to -5} \frac{(x-5)\cancel{(x+5)}}{x\cancel{(x+5)}}$$

$$\lim_{x\to -5} \frac{(x-5)}{x} = \frac{-10}{-5} = \boxed{2}$$

(b) $\lim_{x\to \infty} \frac{2x^2-x-6}{(3x^2-1)^2}$

$$\lim_{x\to \infty} \frac{2x^2-x-6}{9x^4-6x^2+1}$$

$$= \boxed{0}$$

(c) $\lim_{x\to 1} \frac{(x+3)^3-64}{x-1}$

$$\lim_{x\to 1} \frac{\cancel{((x+3)-4)}((x+3)^2+4(x+3)+16)}{\cancel{x-1}}$$

$$= 16+16+16$$

$$= \boxed{48}$$

(d) $\lim_{x\to 5} \frac{(\sqrt{x+4}-3)}{(x-5)} \cdot \frac{(\sqrt{x+4}+3)}{(\sqrt{x+4}+3)}$

$$\lim_{x\to 5} \frac{\cancel{x+4-9}}{\cancel{(x-5)}(\sqrt{x+4}+3)}$$

$$= \boxed{\frac{1}{6}}$$

2. Given the function ... $f(x)=\begin{cases}(x+3)^2 & \text{if } x<-2\\ -x-1 & \text{if } -2\le x<1\\ 1 & \text{if } x=1\\ (x-2)^2-3 & \text{if } x>1\end{cases}$

Using the three conditions for continuity examine *f(x)* for any points of discontinuity. Draw a sketch of *f(x)* and list any point(s) of discontinuity

$(x+3)^2$

x	y
-2	1
-3	0
-4	1
-5	4

$-x-1$

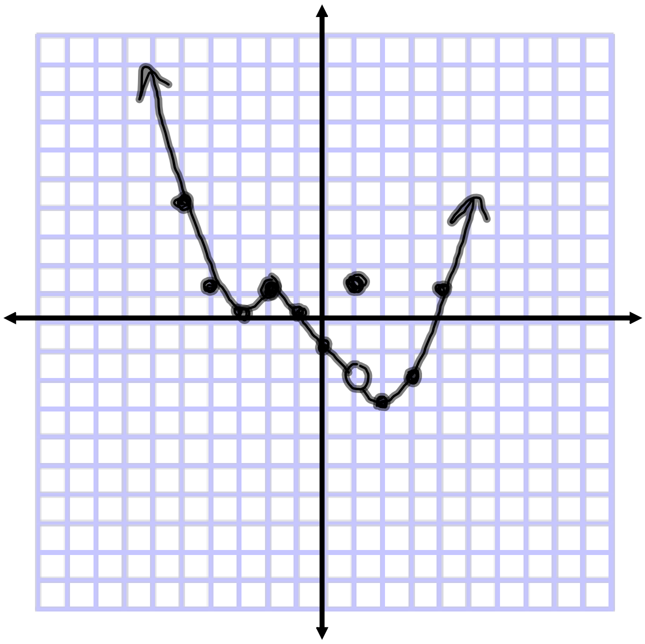
x	y
-2	1
-1	0
0	-1
1	-2

1

x	y
1	1

$(x-2)^2-3$

x	y
1	-2
2	-3
3	-2
4	1



Discontinuous at x=1

4. Differentiate the following functions using the *limit definition of the derivative:*

$$f'(x) = \lim_{h \to 0} \frac{\boxed{f(x+h)} - \boxed{f(x)}}{h}$$

a) $f(x) = \boxed{x^2 + 4x + 2}$

$$f(x+h) = (x+h)^2 + 4(x+h) + 2$$

$$f(x+h) = \boxed{x^2 + 2xh + h^2 + 4x + 4h + 2}$$

$$f'(x) = \lim_{h \to 0} \frac{\cancel{x^2} + 2xh + h^2 \cancel{+4x} + 4h \cancel{+2} - (\cancel{x^2} \cancel{+4x} \cancel{+2})}{h}$$

$$f'(x) = \lim_{h \to 0} \frac{2xh + h^2 + 4h}{h}$$

$$f'(x) = \lim_{h \to 0} \frac{\cancel{h}(2x + h + 4)}{\cancel{h}} = \boxed{2x + 4}$$

Slope of the tangent

5. Find the equation of the tangent line to the curve at the given point.

a) $y = (x^2 + 1)^2$ at $(-1, 4)$

$y = x^4 + 2x^2 + 1$

① Find derivative:

$y' = 4x^3 + 4x$

② Find Slope (Sub in "x")

$y' = 4(-1)^3 + 4(-1)$

$y' = -4 - 4$

$y' = \boxed{-8}$ → Slope "m"

③ Find the equation

$y - y_1 = m(x - x_1)$

$y - 4 = -8(x + 1)$

$y - 4 = -8x - 8$

$\boxed{8x + y + 4 = 0}$

7. Find the derivative: *Express answers with positive exponents!*

a) $f(x) = 3x^5 + \sqrt[3]{x}$

$f(x) = 3x^5 + x^{1/3}$

$f'(x) = 15x^4 + \frac{1}{3}x^{-2/3}$

$\boxed{f'(x) = 15x^4 + \frac{1}{3x^{2/3}}}$

b) $f(x) = \sqrt[5]{x^2}$

$f(x) = x^{2/5}$

$f'(x) = \frac{2}{5}x^{-3/5}$

$\boxed{f'(x) = \frac{2}{5x^{3/5}}}$

Homework.

① a) $\lim_{x\to 0} \dfrac{\dfrac{2}{\cancel{x+2}}(\cancel{x+2}) - \dfrac{1}{1}(x+2)}{x\,(x+2)}$ CD: $(x+2)$

$\lim_{x\to 0} \dfrac{2-x-2}{x(x+2)}$

$\lim_{x\to 0} \dfrac{-\cancel{x}}{\cancel{x}(x+2)} = \boxed{\dfrac{-1}{2}}$

④ b) $f(x) = \boxed{\dfrac{2x-1}{4x}}$ $\quad f(x+h) = \dfrac{2(x+h)-1}{4(x+h)} = \boxed{\dfrac{2x+2h-1}{4x+4h}}$

$$f'(x) = \lim_{h\to 0} \frac{\boxed{f(x+h)} - \boxed{f(x)}}{h}$$

$$f'(x) = \lim_{h\to 0} \frac{\dfrac{2x+2h-1}{\cancel{4x+4h}} - \dfrac{2x-1}{\cancel{4x}}}{h\,(4x)(4x+4h)} \quad (4x)\cancel{(4x+4h)} \quad \cancel{(4x)}(4x+4h)$$

$$f'(x) = \lim_{h\to 0} \frac{4x(2x+2h-1) - (2x-1)(4x+4h)}{4xh(4x+4h)}$$

$$f'(x) = \lim_{h\to 0} \frac{\cancel{8x^2} + \cancel{8xh} - \cancel{4x} - (\cancel{8x^2} + \cancel{8xh} - \cancel{4x} - 4h)}{4xh(4x+4h)}$$

$$f'(x) = \lim_{h\to 0} \frac{\cancel{4h}}{\cancel{4xh}(4x+4h)} = \frac{1}{x(4x)} = \frac{1}{4x^2}$$

Using the Quotient Rule: $\dfrac{f'g - fg'}{g^2}$

$f(x) = \dfrac{2x-1}{4x}$

$$f'(x) = \frac{2(4x) - (2x-1)(4)}{(4x)^2} = \frac{8x - 8x + 4}{16x^2} = \frac{4}{16x^2} = \frac{1}{4x^2}$$