Extreme Values Final Review

① a) $f(x) = x^4 - 2x^2 + 16$

$f'(x) = 4x^3 - 4x$

$f'(x) = 4x(x^2 - 1)$

$f'(x) = 4x(x+1)(x-1)$

CV: $x = -1, 0, 1$

$-\quad +\quad -\quad +$

$(-2)\ \ -1\ \ \left(-\tfrac{1}{2}\right)\ \ 0\ \ \left(\tfrac{1}{2}\right)\ \ 1\ \ (2)$

b) $y = 2x^3 - 3x^2 - 36x + 62$

$y' = 6x^2 - 6x - 36$

$y' = 6(x^2 - x - 6)$

$y' = 6(x-3)(x+2)$

CV: $x = -2, 3$

$+\quad -\quad +$

$(-3)\ \ -2\ \ (0)\ \ 3\ \ (4)$

c) $y = x^5 + 8x^3 + x$

$y' = 5x^4 + 24x^2 + 1$

y' is always (+) for all x.

Inc: $(-\infty, \infty)$

②

$V = \pi r^2 h$

$1000 = \pi r^2 h$

$\frac{1000}{\pi r^2} = h$

minimize the surface area of the can!

$A = 2\pi r^2 + 2\pi r h$ ← Express with single variable

$A = 2\pi r^2 + 2\pi r\left(\frac{1000}{\pi r^2}\right)$

$A = 2\pi r^2 + 2000 r^{-1}$

$A' = 4\pi r - 2000 r^{-2}$

$0 = 4\pi r - \frac{2000}{r^2}$

$\frac{2000}{r^2} = 4\pi r$

$4\pi r^3 = 2000$

$\boxed{r = \sqrt[3]{\frac{500}{\pi}} \doteq 5.4 \text{ cm}}$

③ a) $f(x) = x^4 - 4x^3 - 8x^2 - 1$

$f'(x) = 4x^3 - 12x^2 - 16x$

$f'(x) = 4x(x^2 - 3x - 4)$

$f'(x) = 4x(x-4)(x+1)$

$\boxed{\text{CV: } x = -1, 0, 4}$

b) $f(x) = 2x^3 - 9x^2 - 60x + 82$

$f'(x) = 6x^2 - 18x - 60$

$f'(x) = 6(x^2 - 3x - 10)$

$f'(x) = 6(x-5)(x+2)$

$\boxed{\text{CV: } x = -2, 5}$

4 a) $f(x) = 5x^4 + 20x^3 - 40x^2 + 8$ $\quad -5 \le x \le 2$

$f'(x) = 20x^3 + 60x^2 - 80x$

$f'(x) = 20x(x^2 + 3x - 4)$

$f'(x) = 20x(x+4)(x-1)$

CV: $x = -4, 0, 1$

$f(-5) = -367$

$f(-4) = -632$ abs min

$f(0) = 8$

$f(1) = -7$

$f(2) = 88$ abs max

b) $f(x) = x^4 - 4x^3 - 8x^2 - 1$ $\quad -3 \le x \le 5$

$f'(x) = 4x^3 - 12x^2 - 16x$

$f'(x) = 4x(x^2 - 3x - 4)$

$f'(x) = 4x(x-4)(x+1)$

CV: $x = -1, 0, 4$

$f(-3) = 116$ abs max

$f(-1) = -4$

$f(0) = -1$

$f(4) = -129$ abs min

$f(5) = -76$

5 a) $f(x) = 3x^4 - 16x^3 + 18x^2 + 1$

$f'(x) = 12x^3 - 48x^2 + 36x$

$f'(x) = 12x(x^2 - 4x + 3)$

$f'(x) = 12x(x-3)(x-1)$

CV: $x = 0, 1, 3$.

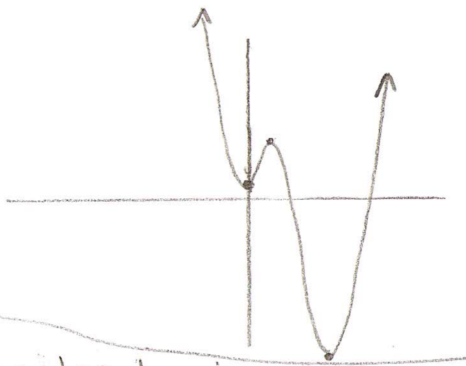
− + − +

(−1) 0 $\left(\frac{1}{2}\right)$ 1 (2) 3 (4)

$f(0) = 1$ local min

$f(1) = 6$ local max

$f(3) = -26$ local min



b) $f(x) = 1 + 3x^2 - 2x^3$

$f'(x) = 6x - 6x^2$

$f'(x) = 6x(1 - x)$

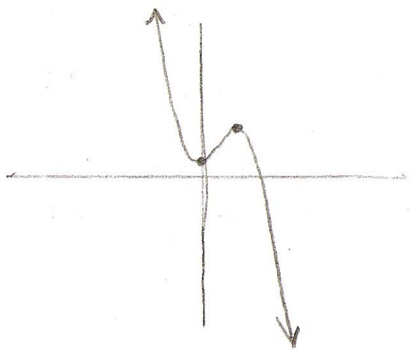
CV: $x = 0, 1$

− + −

(−1) 0 $\left(\frac{1}{2}\right)$ 1 (2)

$f(0) = 1$ local min

$f(1) = 2$ local max



Extreme Values Final Review

⑥ $y = 3x - 2$ closest to $(0,0)$ $\Rightarrow$ minimize distance

* d is smallest when d^2 is smallest

$$d = \sqrt{(x-x_1)^2 + (y-y_1)^2}$$

$$d = \sqrt{(x)^2 + (y)^2}$$

$$d = \sqrt{x^2 + (3x-2)^2}$$

$$d = \sqrt{x^2 + 9x^2 - 12x + 4}$$

* $$d = \sqrt{10x^2 - 12x + 4}$$

$$f(x) = 10x^2 - 12x + 4$$

$$f'(x) = 20x - 12$$

$$0 = 20x - 12$$

$$12 = 20x$$

$$\boxed{\frac{3}{5} = x}$$

$$y = 3x - 2$$

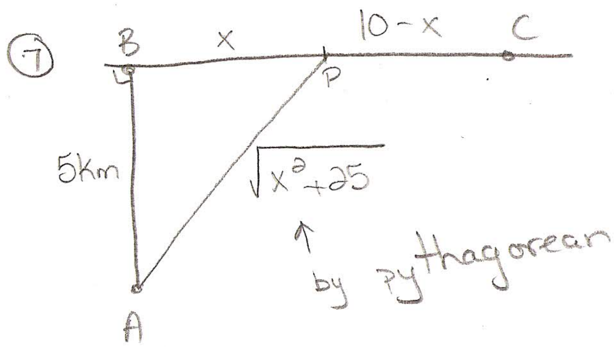
$$y = 3\left(\frac{3}{5}\right) - 2$$

$$y = \frac{9}{5} - \frac{10}{5}$$

$$\boxed{y = \frac{-1}{5}}$$

$\therefore$ The point is $\left(\frac{3}{5}, \frac{-1}{5}\right)$

⑦



minimize the time $\Rightarrow$ $t = \frac{d}{s}$

Let x = distance from B to P

$$t = \frac{\sqrt{x^2+25}}{3} + \frac{10-x}{5}$$

$$t = \frac{1}{3}(x^2+25)^{1/2} + \frac{10}{5} - \frac{1x}{5}$$

$$t' = \frac{1}{6}(x^2+25)^{-1/2}(2x) - \frac{1}{5}$$

$$0 = \frac{x}{3\sqrt{x^2+25}} - \frac{1}{5}$$

$$\frac{1}{5} = \frac{x}{3\sqrt{x^2+25}}$$

$$\left[3\sqrt{x^2+25}\right]^2 = \left[5x\right]^2$$

$$9(x^2+25) = 25x^2$$

$$9x^2 + 225 = 25x^2$$

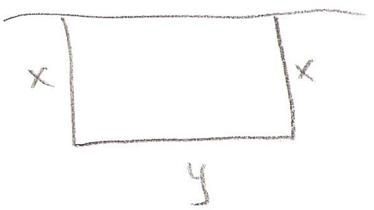
$$225 = 16x^2$$

$$\frac{225}{16} = x^2$$

$$\frac{15}{4} = x$$

$$\boxed{3.75\text{Km} = x}$$

(8)



Let x = width
Let y = length

maximize Area:

$A = xy$

$A = x(500-2x)$

$A = 500x - 2x^2$

$A' = 500 - 4x$

$0 = 500 - 4x$

$4x = 500$

$\boxed{x = 125\text{ m}}$

$P = 2x + y$

$500 = 2x + y$

$500 - 2x = y$

$500 - 2(125) = y$

$\boxed{250\text{ m} = y}$

(9)

Let x = First
Let y = second

$P = xy$
$16 = xy$

a) $S = x + y$

$S = x + \frac{16}{x}$

$S = x + 16x^{-1}$

$S' = 1 - 16x^{-2}$

$0 = 1 - \frac{16}{x^2}$

$\frac{16}{x^2} = 1$

$x^2 = 16$

$x = \pm 4$

$\boxed{x = 4}$

$xy = 16$

$4y = 16$

$\boxed{y = 4}$

b) $S = x + y^2$

$S = \frac{16}{y} + y^2$

$S = 16y^{-1} + y^2$

$S' = -16y^{-2} + 2y$

$S' = \frac{-16}{y^2} + 2y$

$0 = \frac{-16}{y^2} + 2y$

$\frac{16}{y^2} = 2y$

$2y^3 = 16$

$\boxed{y = 2}$

$xy = 16$

$x(2) = 16$

$\boxed{x = 8}$