Equations in Standard Form

$$y = a\sin\left[b(x-c)\right]+d$$

a = ***Amplitude*** → influences how tall the sine curve is.

$b = \frac{360}{P}$ → influences how often the pattern repeats.

c = ***Horizontal Translation*** → Influences how far to the left or the right that the graph will shift.

- If C is positive → Shift Right
- If C is negative → Shift Left

d = ***Vertical Translation*** → influences how far up and down the graph will shift.

- If d is positive → Shift Up
- If d is negative → Shift Down

Questions from Worksheet

State ***a, b, c, d, and P*** from the following sinusoidal equations:

$$2y+6=4\sin(4x+\frac{\pi}{2})-2$$

$$\frac{2y}{2}=\frac{4}{2}\sin\left(4x+\frac{\pi}{2}\right)-\frac{8}{2}$$

Factor out a 4

$$y=2\sin\left(4x+\frac{\pi}{2}\right)-4$$

$$y=2\sin\left[4\left(x+\frac{\pi}{8}\right)\right]-4$$

a = 2 b = 4 c = $-\frac{\pi}{8}$ d = -4

P = $\frac{2\pi}{b}$

sinusoidal axis: $y=-4$

$$P=\frac{2\pi}{4}$$

$$P=\frac{\pi}{2}$$

Sketching Sinusoidal Functions using Mapping

Development of a standard form for sinusoidal functions...

Standard Form $\longrightarrow$ $y = a\sin\left[b(x-c)\right]+d$

1. Reflection: If a < 0 the graph will be reflected in the x-axis.

2. Amplitude: The amplitude of the graph will be equal to | a |. Always positive

3. Period: The period of the graph will be equal to $\frac{360°}{|b|}$ or $\frac{2\pi}{|b|}$

4. Horizontal Phase Shift: The graph will shift "c" units to the right.

5. Vertical Translation: The graph will shift "d" units up.

The Mapping Rule: $(x,y) \rightarrow \left[\frac{x}{b}+c, ay+d\right]$

Use Mapping to Graph

$$y = -2\sin[2(\theta + 90°)] - 1$$

a = 2 b = 2 c = -90° d = -1

P = $\frac{360°}{2}$ = 180°

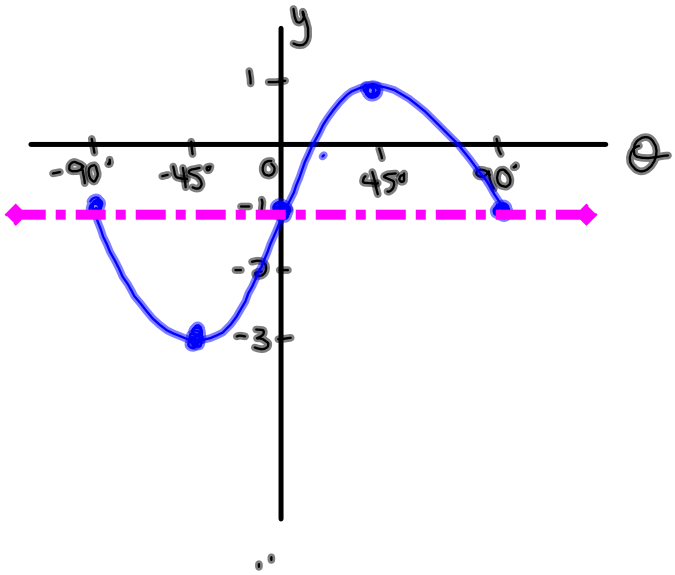
$y = -\sin\theta$

θ	y
0°	0
90°	-1
180°	0
270°	1
360°	0

$$(x, y) \rightarrow \left[\frac{x}{b} + c, ay + d\right]$$

New points after mapping

θ	y
-90°	-1
-45°	-3
0°	-1
45°	1
90°	-1



Use Mapping to Graph

$$3y = 6\cos[(3x - \pi)] - 9$$

$$y = 2\cos\left[3\left(x - \frac{\pi}{3}\right)\right] - 3$$

a = 2 b = 3 c = $\frac{\pi}{3}$ d = -3

P = $\frac{2\pi}{3}$

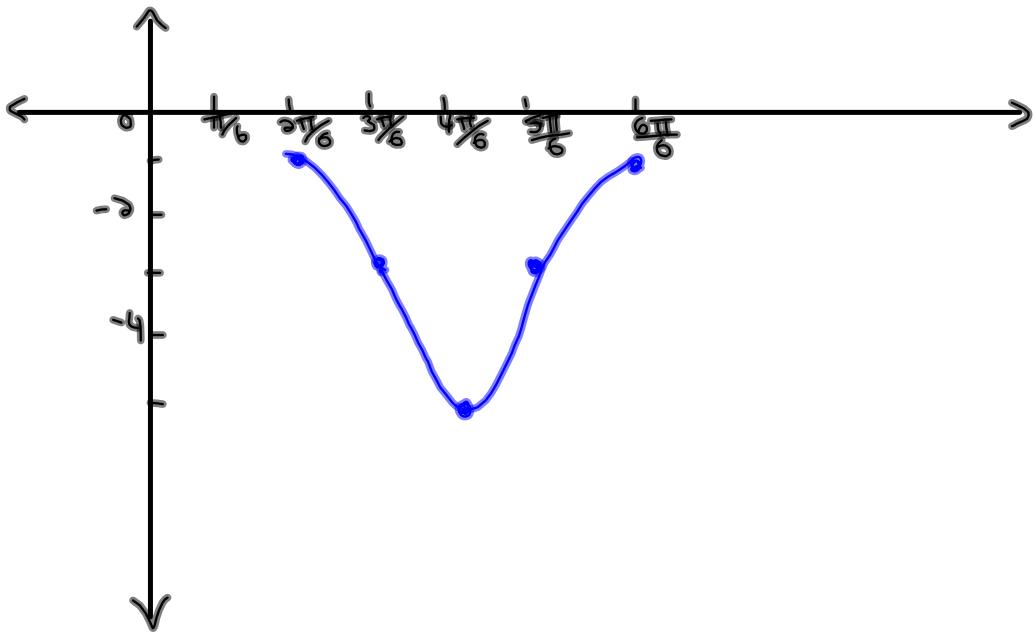
$y = \cos\theta$

θ	y
0	1
$\pi/2$	0
π	-1
$3\pi/2$	0
2π	1

$$(x, y) \rightarrow \left[\frac{x}{b} + c, ay + d\right]$$

New points after mapping

	θ	y
$2\pi/6$	$\pi/3$	-1
$3\pi/6$	$\pi/2$	-3
$4\pi/6$	$2\pi/3$	-5
$5\pi/6$	$5\pi/6$	-3
$6\pi/6$	π	-1

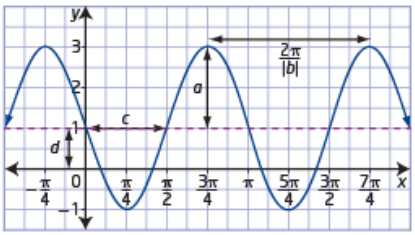


Key Ideas

- You can determine the amplitude, period, phase shift, and vertical displacement of sinusoidal functions when the equation of the function is given in the form $y = a \sin b(x - c) + d$ or $y = a \cos b(x - c) + d$.

For: $y = a \sin b(x - c) + d$
$y = a \cos b(x - c) + d$

How does changing each parameter affect the graph of a function?



Vertical stretch by a factor of $|a|$
- changes the amplitude to $|a|$
- reflected in the x-axis if $a < 0$

Horizontal stretch by a factor of $\frac{1}{|b|}$
- changes the period to $\frac{360°}{|b|}$ (in degrees) or $\frac{2\pi}{|b|}$ (in radians)
- reflected in the y-axis if $b < 0$

Horizontal phase shift represented by c
- to right if $c > 0$
- to left if $c < 0$

Vertical displacement represented by d
- up if $d > 0$
- down if $d < 0$

$$d = \frac{\text{maximum value} + \text{minimum value}}{2}$$

- You can determine the equation of a sinusoidal function given its properties or its graph.

Homework

Finish worksheet