

Equations in Standard Form

$$y = a \sin[b(x - c)] + d$$

a = **Amplitude** → influences how tall the sine curve is.

$b = \frac{360}{P}$ → influences how often the pattern repeats.

c = **Horizontal Translation** → Influences how far to the left or the right that the graph will shift.

- If c is positive → Shift Right
- If c is negative → Shift Left

d = **Vertical Translation** → influences how far up and down the graph will shift.

- If d is positive → Shift Up
- If d is negative → Shift Down

Sketching Sinusoidal Functions using Mapping

Development of a standard form for sinusoidal functions...

Standard Form $\longrightarrow y = a \sin[b(x - c)] + d$

1. Reflection: If $a < 0$ the graph will be reflected in the x-axis.
2. Amplitude: The amplitude of the graph will be equal to $|a|$.
3. Period: The period of the graph will be equal to $\frac{360^\circ}{b}$
4. Horizontal Phase Shift: The graph will shift "c" units to the right.
5. Vertical Translation: The graph will shift "d" units up.

The Mapping Rule: $(x, y) \rightarrow \left[\frac{x}{b} + c, ay + d \right]$

Questions from Worksheet

$$\frac{2y}{2} = \frac{6 \sin(\theta - 60^\circ) - 8}{2}$$

$$y = 3 \sin(\theta - 60^\circ) - 4$$

$$a = 3$$

$$b = 1$$

$$c = 60^\circ$$

$$d = -4$$

$$P = 360^\circ$$

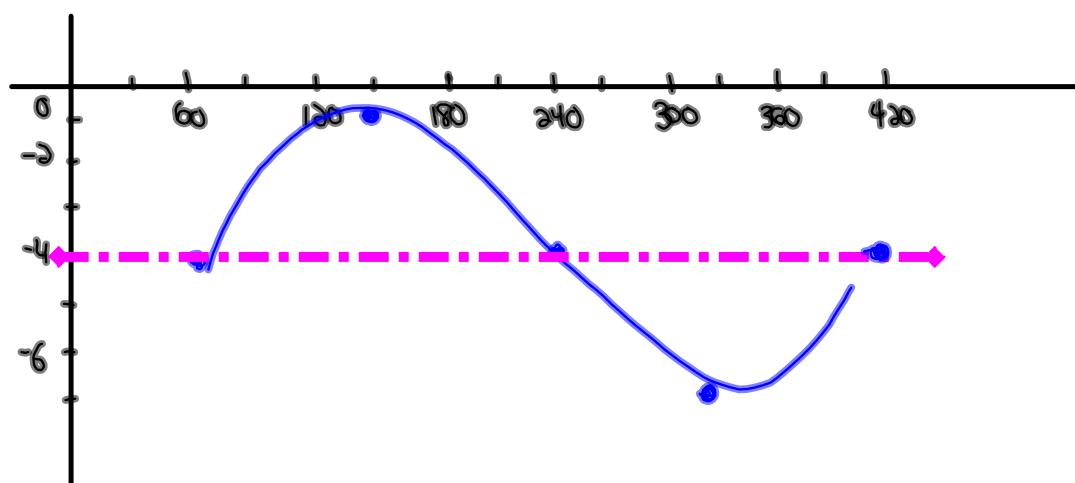
$$y = \sin \theta$$

θ	y
0	0
90	1
180	0
270	-1
360	0



New points after mapping

θ	y
60°	-4
150°	-1
240°	-4
330°	-7
420°	-4



Questions from Worksheet

$$y = -3 \cos \left[2\left(\theta + \frac{\pi}{6}\right) \right] + 1$$

$$a = 3$$

$$b = 2$$

$$c = -\frac{\pi}{6}$$

$$d = 1$$

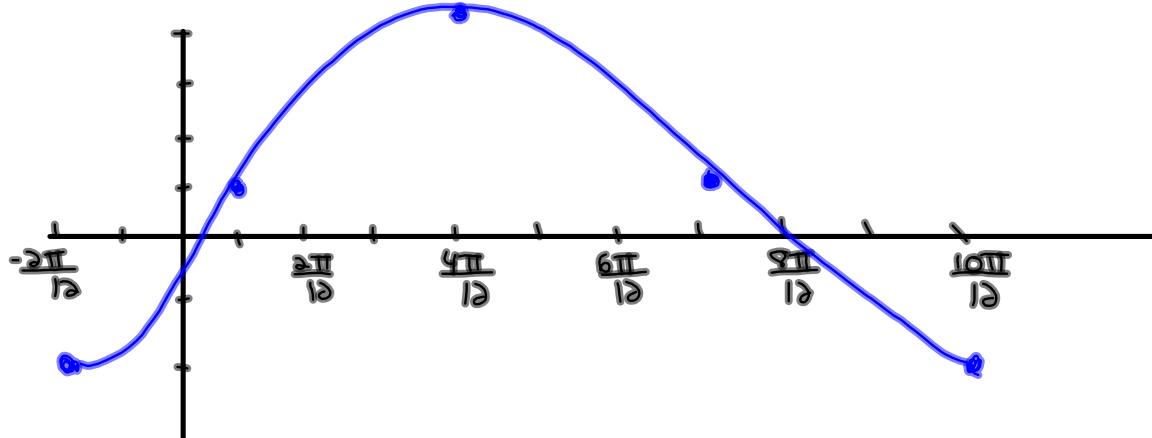
$$P = \frac{2\pi}{2} = \pi$$

$$y = -\cos \theta$$

θ	y
0	-1
$\frac{\pi}{2}$	0
π	-1
$\frac{3\pi}{2}$	0
2π	-1

New points after mapping

θ	y
$-\frac{\pi}{6}$	-2
$\frac{\pi}{12}$	1
$\frac{\pi}{3}$	4
$\frac{7\pi}{12}$	1
$\frac{5\pi}{6}$	-2

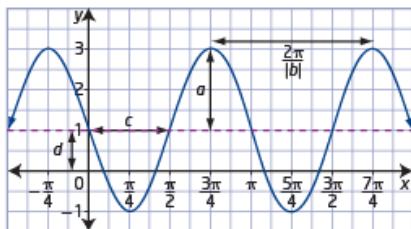


Key Ideas

- You can determine the amplitude, period, phase shift, and vertical displacement of sinusoidal functions when the equation of the function is given in the form $y = a \sin b(x - c) + d$ or $y = a \cos b(x - c) + d$.

For: $y = a \sin b(x - c) + d$
 $y = a \cos b(x - c) + d$

How does changing each parameter affect the graph of a function?



Vertical stretch by a factor of $|a|$

- changes the amplitude to $|a|$
- reflected in the x-axis if $a < 0$

Horizontal stretch by a factor of $\frac{1}{|b|}$

- changes the period to $\frac{360^\circ}{|b|}$ (in degrees) or $\frac{2\pi}{|b|}$ (in radians)
- reflected in the y-axis if $b < 0$

Horizontal phase shift represented by c

- to right if $c > 0$
- to left if $c < 0$

Vertical displacement represented by d

- up if $d > 0$
- down if $d < 0$

$$d = \frac{\text{maximum value} + \text{minimum value}}{2}$$

- You can determine the equation of a sinusoidal function given its properties or its graph.

Homework

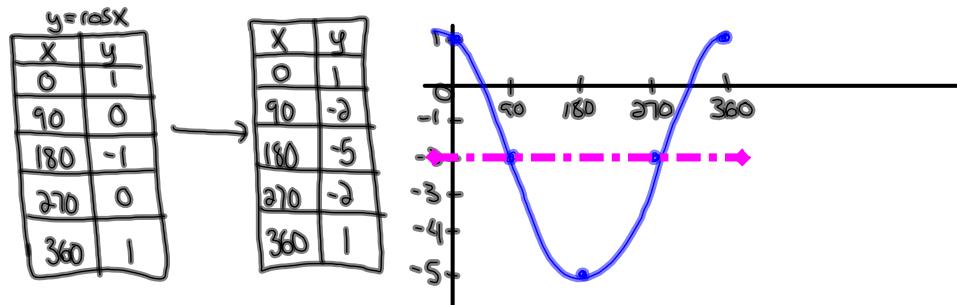
Finish worksheet



Solutions to Homework

$$\textcircled{1} \quad y = 3\cos(x) - 2$$

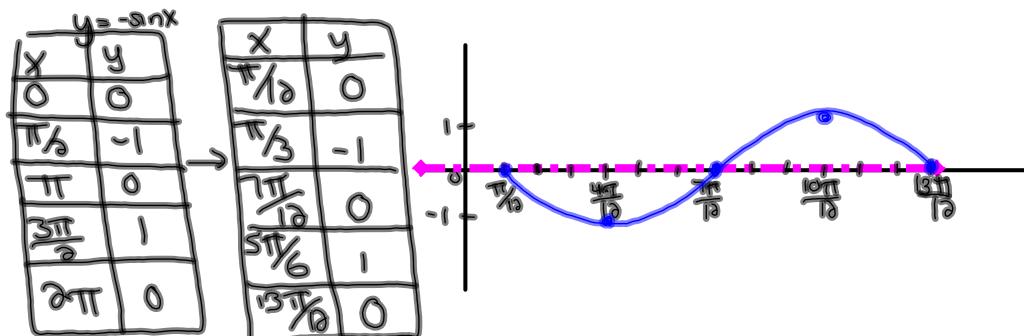
$$A=3 \quad b=1 \quad C=0 \quad D=-2 \quad P=360^\circ$$



$$\textcircled{2} \quad y = -\sin(2x - \frac{\pi}{6})$$

$$y = -\sin[2(x - \frac{\pi}{12})]$$

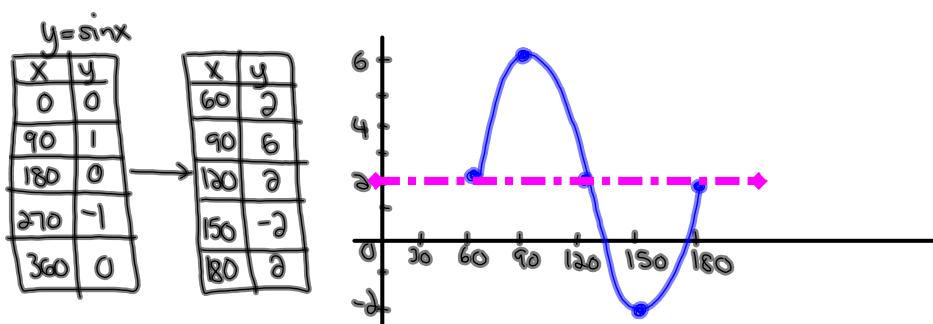
$$A=1 \quad b=2 \quad C=\frac{\pi}{12} \quad D=0 \quad P=\pi$$



$$\textcircled{3} \quad y = 4\sin(3x - 180^\circ) + 2$$

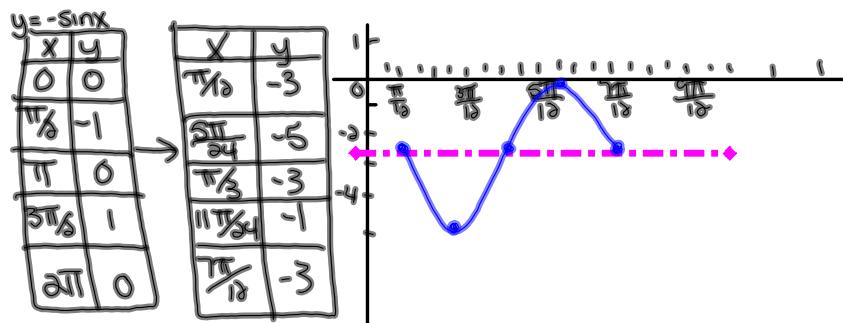
$$y = 4\sin[3(x - 60^\circ)] + 2$$

$$A=4 \quad b=3 \quad C=60 \quad D=2 \quad P=120^\circ$$



$$\textcircled{5} \quad \begin{aligned} 2y+3 &= -4\sin(4x - \frac{\pi}{3}) - 3 \\ 2y &= -4\sin[4(x - \frac{\pi}{12})] - 6 \\ y &= -2\sin[4(x - \frac{\pi}{12})] - 3 \end{aligned}$$

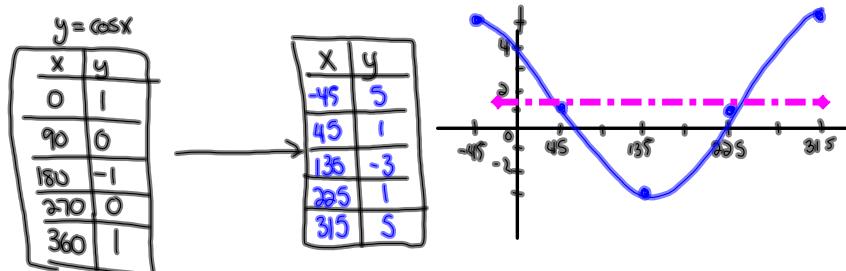
$$A=2 \quad b=4 \quad C=\frac{\pi}{12} \quad D=-3 \quad P=\frac{\pi}{2}$$



$$\textcircled{6} \quad \cancel{\frac{y-1}{2}} = \cancel{2} \cos(\theta + 45^\circ) + \cancel{0}$$

$$\begin{aligned} y-1 &= 4 \cos(\theta + 45^\circ) + 0 \rightarrow 1 \\ y &= 4 \cos(\theta + 45^\circ) + 1 \end{aligned}$$

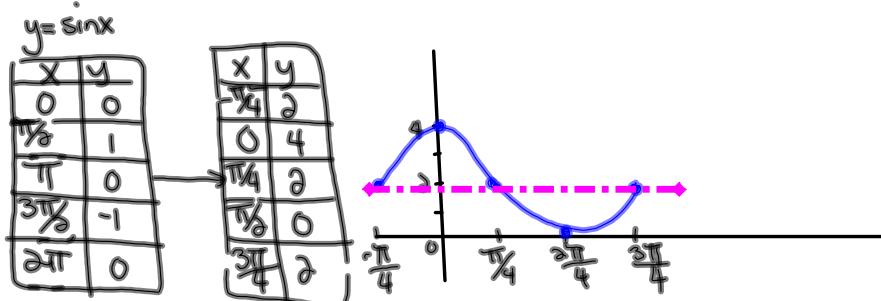
$$A=4 \quad b=-1 \quad C=-45 \quad D=1 \quad P=360$$



$$\textcircled{1} \quad \begin{aligned} \frac{1}{2}y-1 &= \sin[2(x+\frac{\pi}{4})] \\ \cancel{\frac{1}{2}y} &= \sin[2(x+\frac{\pi}{4})] + 1 \end{aligned}$$

$$y = 2\sin[2(x+\frac{\pi}{4})] + 2$$

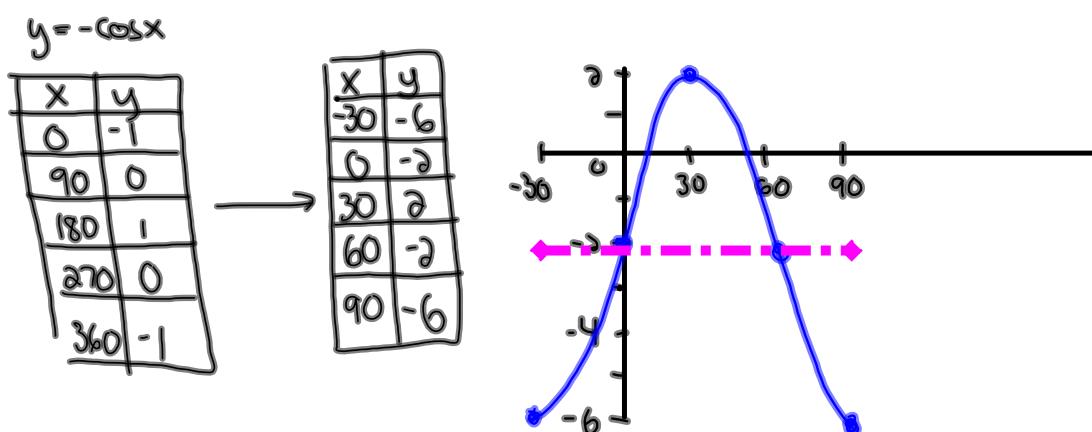
$$A=2 \quad b=2 \quad C=-\frac{\pi}{4} \quad D=2 \quad P=\pi$$



$$\textcircled{8} \quad y = -4 \cos(3x + 90^\circ) - 2$$

$$y = -4 \cos[3(x + 30^\circ)] - 2$$

$$A = 4 \quad b = 3 \quad c = -30 \quad D = -2 \quad P = 120$$



Attachments

[Sketching Sinusoidal Functions #2.pdf](#)