Questions from Homework

Remember!

$$\boxed{f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}}$$

⑧ $f(x) = \boxed{\dfrac{4x^2}{3x+2}}$

$$f(x+h) = \frac{4(x+h)^2}{3(x+h)+2}$$

$$f(x+h) = \frac{4(x^2+2xh+h^2)}{3x+3h+2}$$

$$f(x+h) = \boxed{\frac{4x^2+8xh+4h^2}{3x+3h+2}}$$

$$f'(x) = \lim_{h \to 0} \frac{\dfrac{4x^2+8xh+4h^2}{3x+3h+2} - \dfrac{4x^2}{3x+2}}{h}$$

Multiply everything by $(3x+2)(3x+3h+2)$

$$= \lim_{h \to 0} \frac{(3x+2)(4x^2+8xh+4h^2) - 4x^2(3x+3h+2)}{h(3x+2)(3x+3h+2)}$$

$$= \lim_{h \to 0} \frac{\cancel{12x^3} + 24x^2h + 12xh^2 + \cancel{8x^2} + 16xh + 8h^2 - \cancel{12x^3} - 12x^2h - \cancel{8x^2}}{h(3x+2)(3x+3h+2)}$$

$$= \lim_{h \to 0} \frac{12x^2h + 12xh^2 + 16xh + 8h^2}{h(3x+2)(3x+3h+2)}$$

$$= \lim_{h \to 0} \frac{\cancel{h}(12x^2 + 12xh + 16x + 8h)}{\cancel{h}(3x+2)(3x+3h+2)} = \boxed{\frac{12x^2+16x}{(3x+2)^2}}$$

Slope of the tangent

Remember!

$$f'(x) = \lim_{h \to 0} \frac{\boxed{f(x+h)} - \boxed{f(x)}}{h}$$

If $f(x) = x^2 + 7x$, find $f'(3)$

Hint: find the derivative first then substitute 3 into that

$f(x) = \boxed{x^2 + 7x}$

$f(x+h) = (x+h)^2 + 7(x+h)$

$f(x+h) = \boxed{x^2 + 2xh + h^2 + 7x + 7h}$

$$f'(x) = \lim_{h \to 0} \frac{\cancel{x^2} + 2xh + h^2 \cancel{+7x} + 7h - (\cancel{x^2} \cancel{+7x})}{h}$$

$$f'(x) = \lim_{h \to 0} \frac{2xh + h^2 + 7h}{h}$$

$$f'(x) = \lim_{h \to 0} \frac{\cancel{h}(2x + h + 7)}{\cancel{h}}$$

$\boxed{f'(x) = 2x + 7}$ → Slope of the tangent

$f'(3) = 2(3) + 7$

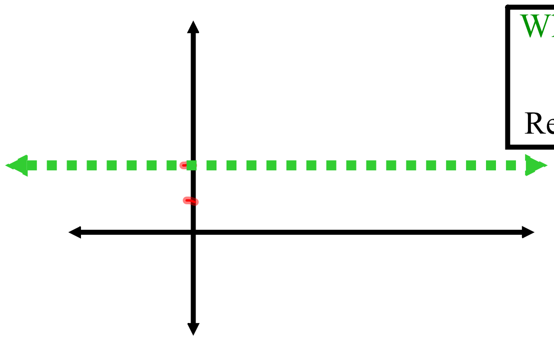
$= 6 + 7$

$= 13$

Differentiation Rules

I. Constant Functions

- Sketch the function $y = 2$



What is the slope of the tangent to this graph?

Recall: slope of the tangent is the derivative

The derivative of a constant will always be equal to "0".

$f(x) = 6$ $\quad$ $f(x) = 32$ $\quad$ $f(x) = 6x + 5$

$f'(x) = 0$ $\quad$ $f'(x) = 0$ $\quad$ $f'(x) = 6 + 0$

$f'(x) = 6$

Formal Proof:

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h \to 0} \frac{c - c}{h}$$

$$= \lim_{h \to 0} 0 = 0$$

II. Power Functions

We want to come up with a rule to differentiate functions of the $f(x) = x^n$, $x \in R$

Using the definition of a derivative to differentiate $f(x) = \boxed{x^4}$ $f(x+h) = \boxed{(x+h)^4}$
would lead to ...

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h \to 0} \frac{(x+h)^4 - x^4}{h}$$

$$= \lim_{h \to 0} \frac{\boxed{x^4 + 4x^3h + 6x^2h^2 + 4xh^3 + h^4} - \boxed{x^4}}{h}$$

$$= \lim_{h \to 0} \frac{4x^3h + 6x^2h^2 + 4xh^3 + h^4}{h}$$

$$= \lim_{h \to 0} (4x^3 + 6x^2h + 4xh^2 + h^3) = 4x^3$$

Other examples we have looked at so far

$f(x) = x^2$	$f(x) = x^3$
$f'(x) = 2x$	$f'(x) = 3x^2$

Do you see a pattern emerging?

The Power Rule (General Version) If n is any real number, then

$$\frac{d}{dx}(x^n) = nx^{n-1}$$

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Let's practice using the power rule...

Differentiate each of the following functions:

1. $f(x) = x^{25}$

$f'(x) = 25x^{24}$

2. $f(x) = x^{-5}$

$f'(x) = -5x^{-6}$

$f'(x) = \frac{-5}{x^6}$

3. $f(x) = \frac{1}{x^{10}}$

$f(x) = x^{-10}$

$f'(x) = -10x^{-11}$

4. $f(x) = \sqrt{x}$

$f(x) = x^{1/2}$

$f'(x) = \frac{1}{2}x^{-1/2}$

$f'(x) = \frac{1}{2x^{1/2}}$

$f'(x) = \frac{1}{2\sqrt{x}}$

Constant Multiples

- The following formula says that <u>the derivative of a constant multiplied by a function is the constant multiplied by the derivative of the function:</u>

The Constant Multiple Rule If c is a constant and f is a differentiable function, then

$$\frac{d}{dx}[cf(x)] = c\frac{d}{dx}f(x)$$

EXAMPLE 4

(a) $\frac{d}{dx}(3x^4) = 3\frac{d}{dx}(x^4) = 3(4x^3) = 12x^3$

(b) $\frac{d}{dx}(-x) = \frac{d}{dx}[(-1)x] = (-1)\frac{d}{dx}(x) = -1(1) = -1$

Examples:

1. $f(x) = 4x^3$

$F'(x) = 12x^2$

2. $f(x) = \dfrac{8}{x^2}$

$f(x) = 8x^{-2}$

$f'(x) = -16x^{-3}$

$f'(x) = \dfrac{-16}{x^3}$

3. $f(x) = 5x^{\frac{6}{5}}$

$F'(x) = 6x^{1/5}$

4. $f(x) = (3x^2)^2$

$f(x) = 9x^4$

$F'(x) = 36x^3$

5. $f(x) = 12x$

$f'(x) = 12x^0$

$= 12$

Recall the derivative of a function is equal to the slope of a line that is tangent to the function.

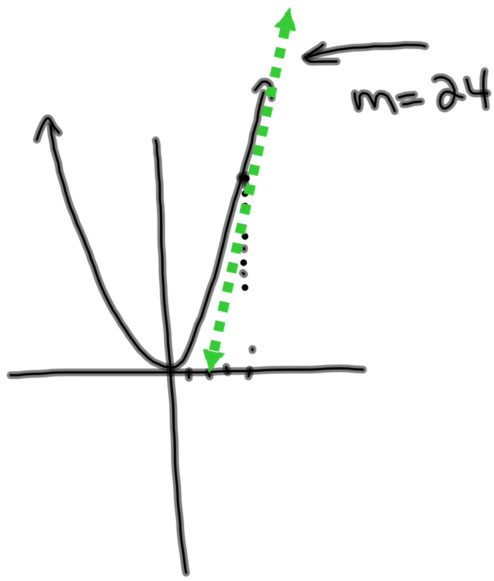
Find the slope of the tangent line to the function at the given "x" coordinate!

$f(x) = 3x^2$ at x = 4

$f'(x) = 6x$

$f'(4) = 6(4)$

$\boxed{f'(4) = 24}$



3 a) $f(x) = 2x^3$, $x = \boxed{\frac{1}{3}}$

$f'(x) = 6x^2$

$f'\left(\frac{1}{3}\right) = 6\left(\frac{1}{3}\right)^2$

$= 6\left(\frac{1}{9}\right)$

$= \frac{6}{9}$

$= \frac{2}{3}$

Homework

#4 To find an equation of a line:

1. Slope
2. Point
3. $y - y_1 = m(x - x_1)$

a) $f(x) = x^5$ $(2, 32)$

1. $f'(x) = 5x^4$
2. $f'(2) = 5(2)^4$
   $= 5(16)$
   $= 80$
3. $y - y_1 = m(x - x_1)$
   $y - 32 = 80(x - 2)$
   $y - 32 = 80x - 160$
   $-80x + y + 128 = 0$
   $\boxed{80x - y - 128 = 0}$