Chapter 4

Chapter 4 REVIEW

Assignment

Complete pgs. 258 - 260

Questions 3, 6ac, 7, 9ab, 10, 13, 14, 16, 18, 19

Solutions

3. Explain what must be true about the graph of the corresponding function for a quadratic equation to have no real roots.

For a quadratic equation to have no real roots, its corresponding graph must open upward and have a vertex above the x-axis or open downward and have a vertex below the x-axis.

6. Factor.

a) $4x^2-13x+9$

$(4x^2-4x)(-9x+9)$

$4x(x-1)-9(x-1)$

$(4x-9)(x-1)$

(Decomposition)

$-4 \times -9 = 36$

$-4 + -9 = -13$

c) $3(v+1)^2+10(v+1)+7$

Let $r = v+1$

$3r^2+10r+7$

$(3r^2+3r)(+7r+7)$

$3r(r+1)+7(r+1)$

$(r+1)(3r+7)$

$(v+1+1)(3(v+1)+7)$

$(v+2)(3v+3+7)$

$(v+2)(3v+10)$

(Decomposition)

$3 \times 7 = 21$

$3 + 7 = 10$

Solutions

7. Solve by factoring.

a) $0 = x^2 + 10x + 21$

$0 = (x+3)(x+7)$

$x+3=0$ or $x+7=0$

$x=-3$ $\quad$ $x=-7$

b) $\frac{1}{4}m^2 + 2m - 5 = 0$ $\quad \left(\div \frac{1}{4} \Rightarrow \times \frac{1}{4}\right)$

$\frac{1}{4}(m^2 + 8m - 20) = 0$

$\frac{1}{4}(m+10)(m-2) = 0$

$m+10=0$ or $m-2=0$

$m=-10$ $\quad$ $m=2$

c) $5p^2 + 13p - 6 = 0$ $\quad$ (Decomposition)

$5p^2 + 15p - 2p - 6 = 0$ $\quad$ $15 \times -2 = -30$

$5p(p+3) - 2(p+3) = 0$ $\quad$ $15 + -2 = 13$

$(p+3)(5p-2) = 0$

$p+3=0$ or $5p-2=0$

$p=-3$ $\quad$ $\frac{5p}{5} = \frac{2}{5}$

$p = \frac{2}{5}$

Solutions

d) $0 = 6z^2 - 21z + 9$
$0 = 3(2z^2 - 7z + 3)$ (Decomposition)
$0 = 3(2z^2 - 1z - 6z + 3)$ $-1 \times -6 = 6$
$0 = 3[(2z^2 - 1z)(-6z + 3)]$ $-1 + -6 = -7$
$0 = 3[z(2z-1) - 3(2z-1)]$
$0 = 3(2z-1)(z-3)$

9. Write a quadratic equation in standard form with the given roots.

a) 2 and 3

↳ $(x-2)(x-3) = 0$
$x^2 - 3x - 2x + 6 = 0$
$x^2 - 5x + 6 = 0$

b) -1 and -5

↳ $(x+1)(x+5) = 0$
$x^2 + 5x + 1x + 5 = 0$
$x^2 + 6x + 5 = 0$

Solutions

10. The path of a paper airplane can be modelled approximately by the function $h(t) = -\frac{1}{4}t^2 + t + 3$, where h is the height above the ground, in meters, and t is the time of flight, in seconds. Determine how long it takes for the paper airplane to hit the ground, $h(t) = 0$.

$$-\frac{1}{4}t^2 + t + 3 = 0$$

$$-\frac{1}{4}(t^2 - 4t - 12) = 0$$

$$-\frac{1}{4}(t+2)(t-6) = 0$$

$$t + 2 = 0 \text{ or } t - 6 = 0$$

$$t = -2 \qquad t = 6$$

$t = -2$ ↑ Extraneous Root

It takes 6 s for the paper airplane to hit the ground.

Solutions

13. Determine the value of K that makes each expression a perfect square trinomial.

a) $x^2 + 4x + K$

$K = \left(\frac{4}{2}\right)^2$

$K = \frac{16}{4}$

$K = 4$

b) $x^2 + 3x + K$

$K = \left(\frac{3}{2}\right)^2$

$K = \frac{9}{4}$

14. Solve. Express your answers as exact values.

a) $2x^2 - 98 = 0$

$\frac{2x^2}{2} = \frac{98}{2}$

$x^2 = 49$

$x = \pm\sqrt{49}$

$x = \pm 7$

b) $(x+3)^2 = 25$

$x + 3 = \pm\sqrt{25}$

$x = -3 \pm \sqrt{25}$

$x = -3 \pm 5$

$x = -3 - 5$ or $x = -3 + 5$

$x = -8$ $x = 2$

Solutions

c) $(x-5)^2 = 24$

$x - 5 = \pm\sqrt{24}$

$x = 5 \pm \sqrt{24}$ (we will stop here for now)

d) $(x-1)^2 = \frac{5}{9}$

$x - 1 = \pm\sqrt{\frac{5}{9}}$

$x = 1 \pm \sqrt{\frac{5}{9}}$

$x = 1 \pm \frac{\sqrt{5}}{3}$

Solutions

16. In a simulation, the path of a new aircraft after it has achieved weightlessness can be modelled approximately by $h(t) = -5t^2 + 200t + 9750$, where h is the altitude of the aircraft, in meters, and t is the time, in seconds, and weightlessness is achieved. How long does the aircraft take to return to the ground, $h(t) = 0$? Express your answer to the nearest tenth of a second.

$$-5t^2 + 200t + 9750 = 0$$
$$t^2 - 40t - 1950 = 0$$
$$t^2 - 40t = 1950$$
$$t^2 - 40t + 400 = 1950 + 400$$
$$(t-20)^2 = 2350$$
$$t - 20 = \pm\sqrt{2350}$$
$$t = 20 \pm \sqrt{2350}$$

$t = 20 - \sqrt{2350}$ or $t = 20 + \sqrt{2350}$

$t = -28.5$ $t = 68.5$

↑ Extraneous Root

The aircraft takes approximately 68.5s to return to the ground.

Solutions

18. Use the discriminant to determine the nature of the roots for each quadratic equation. Do not solve the equation.

a) $2x^2 + 11x + 5 = 0$ $a=2, b=11, c=5$

$D = b^2 - 4ac$
$= (11)^2 - 4(2)(5)$
$= 121 - 40$
$= 81$

Since $D > 0$, there are 2 real roots.

b) $4x^2 - 4x + 1 = 0$ $a=4, b=-4, c=1$

$D = b^2 - 4ac$
$= (-4)^2 - 4(4)(1)$
$= 16 - 16$
$= 0$

Since $D = 0$, there is one real root.

c) $3p^2 + 6p + 24 = 0$ $a=3, b=6, c=24$

$D = b^2 - 4ac$
$= (6)^2 - 4(3)(24)$
$= 36 - 288$
$= -252$

Since $D < 0$, there are no real roots.

Solutions

d) $4x^2+4x-7=0$ $a=4, b=4, c=-7$

$D = b^2-4ac$
$= (4)^2-4(4)(-7)$
$= 16+112$
$= 128$

Since $D>0$, there are 2 real roots.

19. Use the quadratic formula to determine the roots for each quadratic equation. Express your answers as exact values.

a) $-3x^2-2x+5=0$ $a=-3, b=-2, c=5$

$$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$$
$$= \frac{2 \pm \sqrt{(-2)^2-4(-3)(5)}}{2(-3)}$$
$$= \frac{2 \pm \sqrt{4+60}}{-6}$$
$$= \frac{2 \pm \sqrt{64}}{-6}$$
$$= \frac{2 \pm 8}{-6}$$

$x = \frac{2-8}{-6}$ or $x = \frac{2+8}{-6}$

$x = \frac{-6}{-6}$ $x = \frac{10}{-6}$

$x = 1$ $x = -\frac{5}{3}$

Solutions

b) $5x^2+7x+1=0$ $a=5, b=7, c=1$

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

$$x=\frac{-7\pm\sqrt{(7)^2-4(5)(1)}}{2(5)}$$

$$x=\frac{-7\pm\sqrt{49-20}}{10}$$

$$x=\frac{-7\pm\sqrt{29}}{10}$$

c) $3x^2-4x-1=0$ $a=3, b=-4, c=-1$

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

$$x=\frac{4\pm\sqrt{(-4)^2-4(3)(-1)}}{2(3)}$$

$$x=\frac{4\pm\sqrt{16+12}}{6}$$

$$x=\frac{4\pm\sqrt{28}}{6}$$

d) $25x^2+90x+81=0$ $a=25, b=90, c=81$

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

$$x=\frac{-90\pm\sqrt{(90)^2-4(25)(81)}}{2(25)}$$

$$x=\frac{-90\pm\sqrt{8100-8100}}{50}$$

$$x=\frac{-90\pm\sqrt{0}}{50}$$

$$x=\frac{-90}{50}$$

$$x=-\frac{9}{5}$$