

Warm-Up

8. Copy and complete the table.

Translation	Transformed Function	Transformation of Points
vertical	$y = f(x) + 5$	$(x, y) \rightarrow (x, y + 5)$
H	$y = f(x + 7)$	$(x, y) \rightarrow (x - 7, y)$
H	$y = f(x - 3)$	$(x, y) \rightarrow (x + 3, y)$
V	$y = f(x) - 6$	$(x, y) \rightarrow (x, y - 6)$
horizontal and vertical	$y + 9 = f(x + 4)$	$(x, y) \rightarrow (x - 4, y - 9)$
horizontal and vertical	$y + 6 = f(x - 4)$	$(x, y) \rightarrow (x + 4, y - 6)$
H+V	$y - 3 = f(x + 2)$	$(x, y) \rightarrow (x - 2, y + 3)$
horizontal and vertical	$y = f(x - h) + k$	$(x, y) \rightarrow (x + h, y + k)$

Questions from Homework

① c) $y = f(x - \underline{17}) + 13$ or $y - 13 = f(x - 17)$

$h = 17$ $k = 13$

② Given $h = 2$ and $k = -5 \rightarrow y - k = f(x - h)$

a) $y = x^2$ b) $y = |x|$ c) $y = \frac{1}{x}$

$y + 5 = (x - 2)^2$ $y + 5 = |x - 2|$ $y + 5 = \frac{1}{x - 2}$

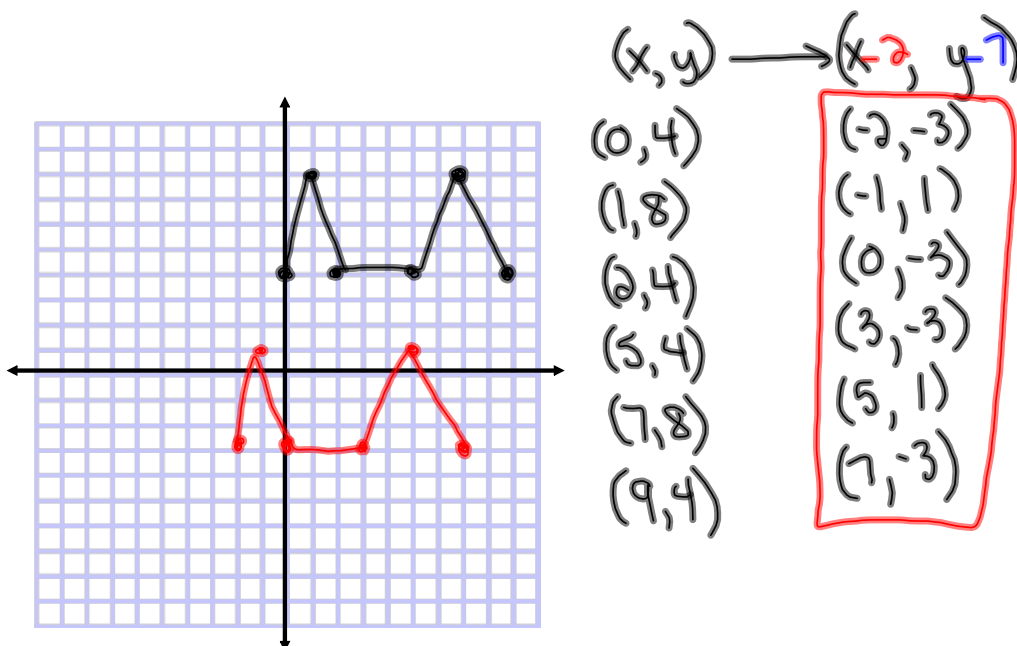
③ c) $y - 10 = f(x + 20)$ $h = -20$ $k = 10$

$(x, y) \rightarrow (x - 20, y + 10)$

④ a) $y + 7 = f(x + 2)$

$h = -2$

$k = -7$



Transformations:

New Functions From Old Functions

Translations

✓ Stretches

✓ Reflections

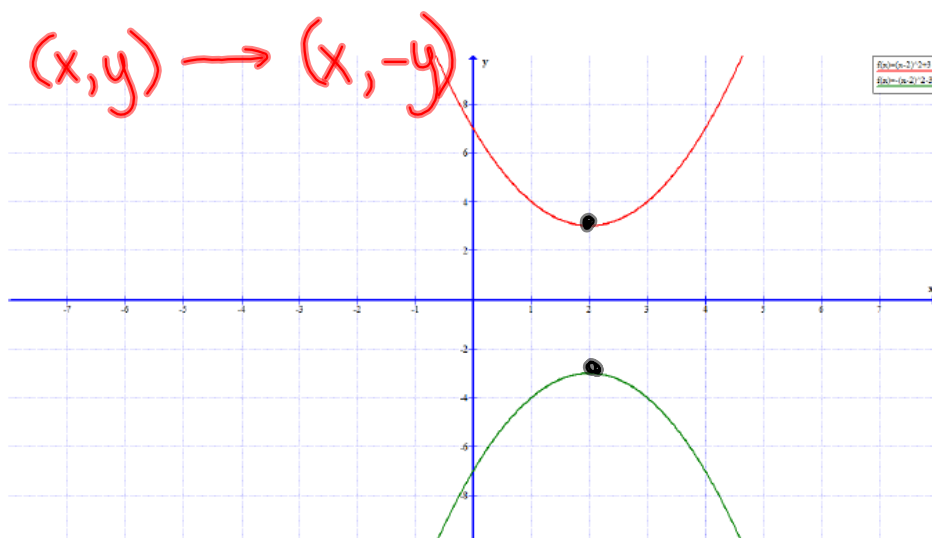
Reflections and Stretches

Focus on...

- ✓ developing an understanding of the effects of reflections on the graphs of functions and their related equations
- developing an understanding of the effects of vertical and horizontal stretches on the graphs of functions and their related equations

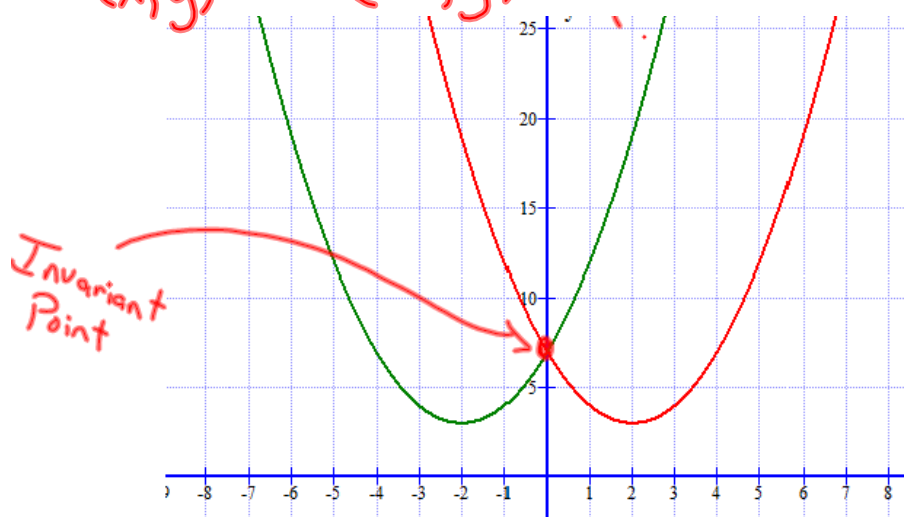
A **reflection** of a graph creates a mirror image in a line called the line of reflection. Reflections, like translations, do not change the shape of the graph. However, unlike translations, reflections may change the orientation of the graph.

- When the **output** of a function $y = f(x)$ is multiplied by -1 , the result, $y = -f(x)$, is a reflection of the graph in **the x-axis**.



- When the **input** of a function $y = f(x)$ is multiplied by -1 , the result, $y = f(-x)$, is a reflection of the graph in **the y-axis**.

$$(x, y) \rightarrow (-x, y)$$



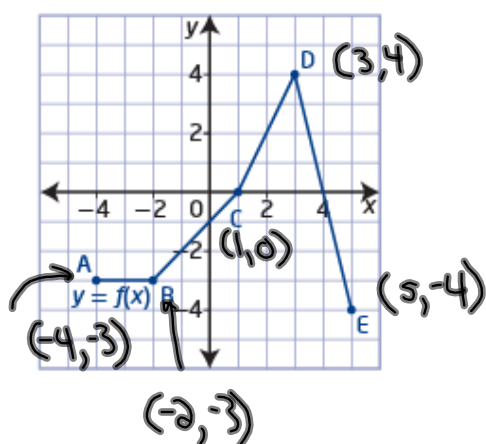
invariant point

- a point on a graph that remains unchanged after a transformation is applied to it
- any point on a curve that lies on the line of reflection is an invariant point

Example 1

Compare the Graphs of $y = f(x)$, $y = -f(x)$, and $y = f(-x)$

- a) Given the graph of $y = f(x)$, graph the functions $y = -f(x)$ and $y = f(-x)$.
- b) How are the graphs of $y = -f(x)$ and $y = f(-x)$ related to the graph of $y = f(x)$?

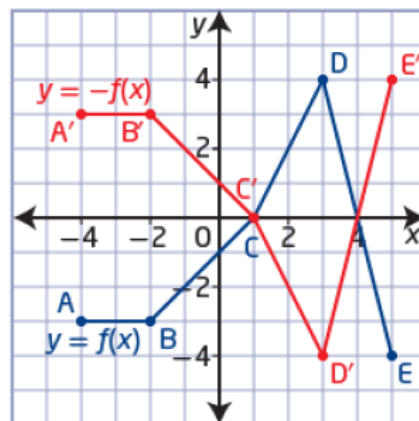
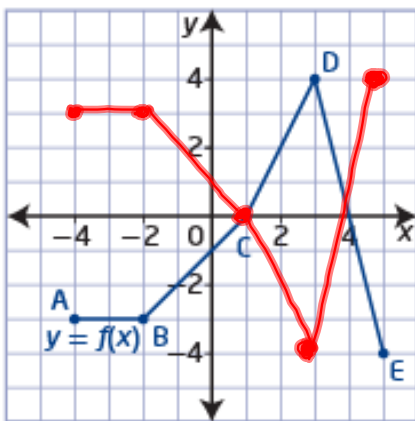


$$a) y = -f(x)$$

$$b) y = f(-x)$$

Remember...

- When the output of a function $y = f(x)$ is multiplied by -1 , the result, $y = -f(x)$, is a reflection of the graph in the x -axis.
- Sketch $y = -f(x)$ on the axis below

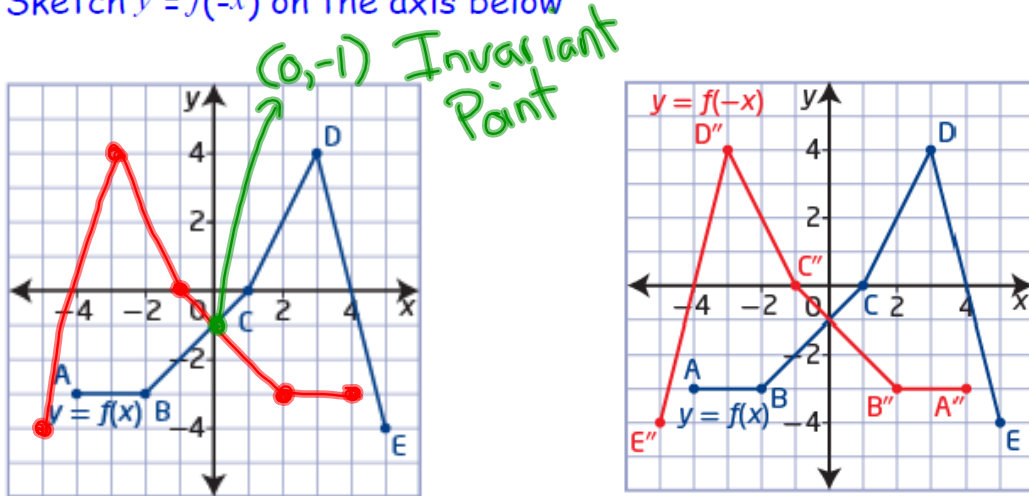


	$(x, y) \rightarrow (x, -y)$	
A	$(-4, -3)$	$(-4, 3)$
B	$(-2, -3)$	$(-2, 3)$
C	$(1, 0)$	$(1, 0)$
D	$(3, 4)$	$(3, -4)$
E	$(5, -4)$	$(5, 4)$

← Invariant point

Remember...

- When the input of a function $y = f(x)$ is multiplied by -1 , the result, $y = f(-x)$, is a reflection of the graph in the y -axis.
- Sketch $y = f(-x)$ on the axis below



	$(x, y) \rightarrow$	$(-x, y)$
A	$(-4, -3)$	$(4, -3)$
B	$(-2, -3)$	$(2, -3)$
C	$(1, 0)$	$(-1, 0)$
D	$(3, 4)$	$(-3, 4)$
E	$(5, 4)$	$(-5, 4)$

stretch + Compression

- a transformation in which the distance of each x -coordinate or y -coordinate from the line of reflection is multiplied by some scale factor
- scale factors between 0 and 1 result in the point moving closer to the line of reflection; scale factors greater than 1 result in the point moving farther away from the line of reflection

Vertical and Horizontal Stretches

A **stretch**, unlike a translation or a reflection, changes the shape of the graph. However, like translations, stretches do not change the orientation of the graph.

- When the output of a function $y = f(x)$ is multiplied by a non-zero constant a , the result, $y = af(x)$ or $\frac{y}{a} = f(x)$, is a vertical stretch of the graph about the x -axis by a factor of $|a|$. If $a < 0$, then the graph is also reflected in the x -axis.
- When the input of a function $y = f(x)$ is multiplied by a non-zero constant b , the result, $y = f(bx)$, is a horizontal stretch of the graph about the y -axis by a factor of $\frac{1}{|b|}$. If $b < 0$, then the graph is also reflected in the y -axis.

Vertical Stretch $(x, y) \rightarrow (x, ay)$

① $y = 2f(x)$ $a=2$
 $(x, y) \rightarrow (x, 2y)$

② $y = 3|x|$ $a=3$
 $(x, y) \rightarrow (x, 3y)$

③ $y = \frac{1}{3}f(x)$ $a=\frac{1}{3}$
 $(x, y) \rightarrow (x, \frac{1}{3}y)$

④ $y = -4x^2$ $a=4$
 $(x, y) \rightarrow (x, -4y)$
 reflection
 x axis

Horizontal Stretch: $(x, y) \rightarrow (\frac{1}{b}x, y)$

① $y = |2x|$ $b=2$
 $(x, y) \rightarrow (\frac{1}{2}x, y)$

② $y = f(\frac{1}{3}x)$ $b=\frac{1}{3}$
 $(x, y) \rightarrow (3x, y)$

Vertical Stretch or Compression...

- When the output of a function $y = f(x)$ is multiplied by a non-zero constant a , the result, $y = af(x)$ or $\frac{y}{a} = f(x)$, is a vertical stretch of the graph about the x -axis by a factor of $|a|$. If $a < 0$, then the graph is also reflected in the x -axis.

Example 2

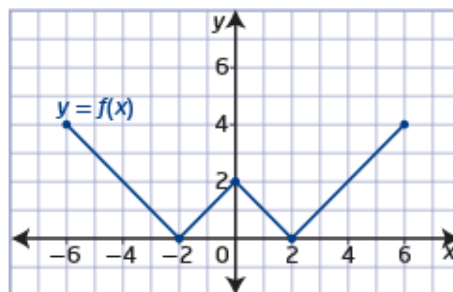
Graph $y = af(x)$

Given the graph of $y = f(x)$,

- transform the graph of $f(x)$ to sketch the graph of $g(x)$
- describe the transformation
- state any invariant points
- state the domain and range of the functions

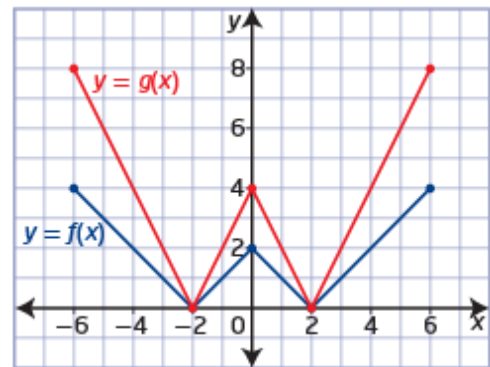
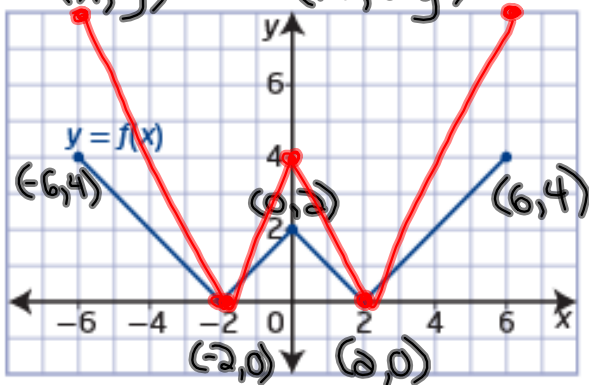
a) $g(x) = 2f(x)$

b) $g(x) = \frac{1}{2}f(x)$



a) $g(x) = 2f(x)$ Vertical Stretch $a=2$

$(x, y) \rightarrow (x, 2y)$



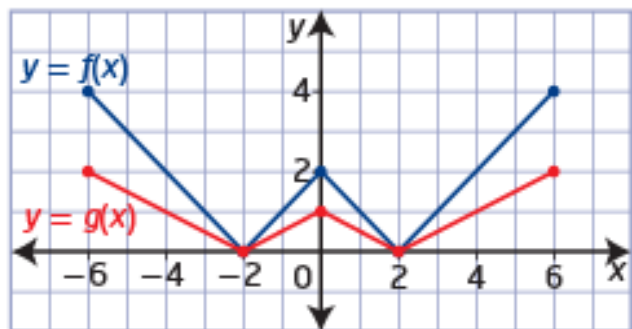
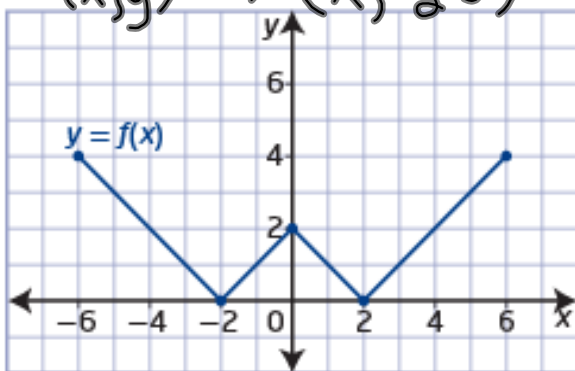
The invariant points are $(-2, 0)$ and $(2, 0)$.

For $f(x)$, the domain is
 $\{x \mid -6 \leq x \leq 6, x \in \mathbb{R}\}$, or $[-6, 6]$,
 and the range is
 $\{y \mid 0 \leq y \leq 4, y \in \mathbb{R}\}$, or $[0, 4]$.

For $g(x)$, the domain is $\{x \mid -6 \leq x \leq 6, x \in \mathbb{R}\}$, or $[-6, 6]$,
 and the range is $\{y \mid 0 \leq y \leq 8, y \in \mathbb{R}\}$, or $[0, 8]$.

$$b) g(x) = \frac{1}{2}f(x) \quad a = \frac{1}{2}$$

$$(x, y) \rightarrow (x, \frac{1}{2}y)$$



The invariant points are $(-2, 0)$ and $(2, 0)$.

For $f(x)$, the domain is $\{x \mid -6 \leq x \leq 6, x \in \mathbb{R}\}$, or $[-6, 6]$,
and the range is $\{y \mid 0 \leq y \leq 4, y \in \mathbb{R}\}$, or $[0, 4]$.

For $g(x)$, the domain is $\{x \mid -6 \leq x \leq 6, x \in \mathbb{R}\}$, or $[-6, 6]$,
and the range is $\{y \mid 0 \leq y \leq 2, y \in \mathbb{R}\}$, or $[0, 2]$.

Horizontal Stretch or Compression...

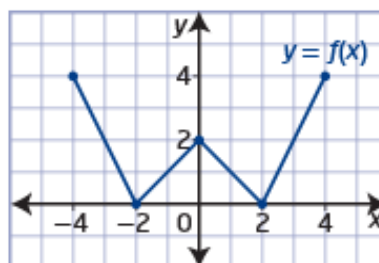
- When the input of a function $y = f(x)$ is multiplied by a non-zero constant b , the result, $y = f(bx)$, is a horizontal stretch of the graph about the y -axis by a factor of $\frac{1}{|b|}$. If $b < 0$, then the graph is also reflected in the y -axis.

Example 3

Graph $y = f(bx)$

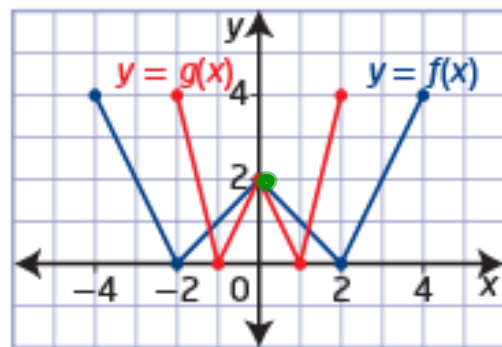
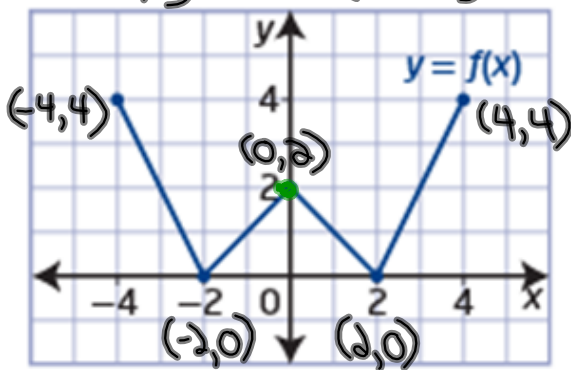
Given the graph of $y = f(x)$,

- transform the graph of $f(x)$ to sketch the graph of $g(x)$
- describe the transformation
- state any invariant points
- state the domain and range of the functions



- $g(x) = f(2x)$
- $g(x) = f\left(\frac{1}{2}x\right)$

a) $g(x) = f(2x)$ Horizontal Stretch: $b = 2$
 $(x, y) \rightarrow (\frac{1}{2}x, y)$



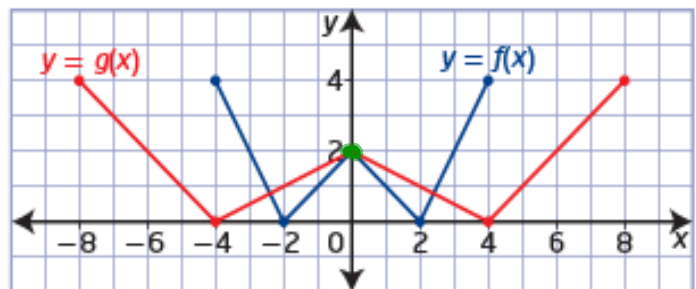
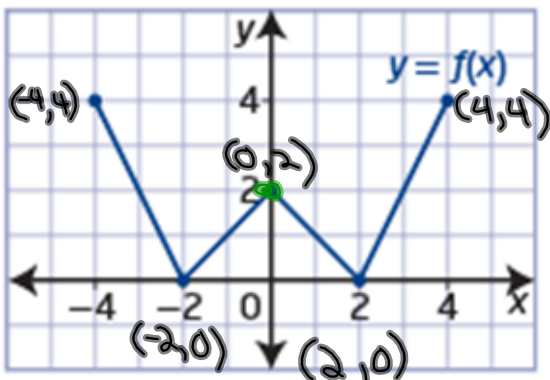
The invariant point is $(0, 2)$. \clubsuit

For $f(x)$, the domain is $\{x \mid -4 \leq x \leq 4, x \in \mathbb{R}\}$,
 or $[-4, 4]$, and the range is $\{y \mid 0 \leq y \leq 4, y \in \mathbb{R}\}$,
 or $[0, 4]$.

For $g(x)$, the domain is $\{x \mid -2 \leq x \leq 2, x \in \mathbb{R}\}$,
 or $[-2, 2]$, and the range is $\{y \mid 0 \leq y \leq 4, y \in \mathbb{R}\}$,
 or $[0, 4]$.

$$b) \quad g(x) = f\left(\frac{1}{2}x\right) \quad b = \frac{1}{2}$$

$$(x, y) \rightarrow (2x, y)$$

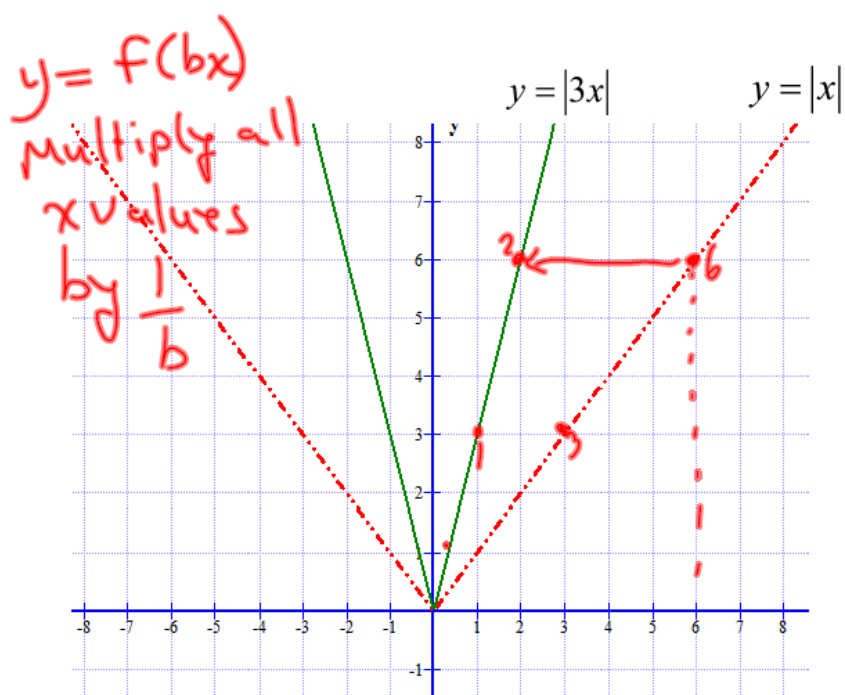


The invariant point is $(0, 2)$. ✨

For $f(x)$, the domain is $\{x \mid -4 \leq x \leq 4, x \in \mathbb{R}\}$, or $[-4, 4]$,
and the range is $\{y \mid 0 \leq y \leq 4, y \in \mathbb{R}\}$, or $[0, 4]$.

For $g(x)$, the domain is $\{x \mid -8 \leq x \leq 8, x \in \mathbb{R}\}$, or $[-8, 8]$,
and the range is $\{y \mid 0 \leq y \leq 4, y \in \mathbb{R}\}$, or $[0, 4]$.

Horizontal Stretch or Compression...



Putting it all together

$$y = -3f(-2x) + 7$$

$$a=3 \quad b=2 \quad k=7 \quad h=0$$

$$(x, y) \longrightarrow \left(-\frac{1}{2}x, 3y+7\right)$$

Reflection in both x & y axis

Stretch vertically by a factor of 3
 " horizontally " " " " $\frac{1}{2}$

Shift up 7

Homework

Working Example 1-3 on page 10
Questions 3-5 on page 15

Determine the Equation of a Translated Function:

