Warm-Up...

Given that (-2 , 5) is a point on the graph of $y = f(x)$, determine the coordinates of this point once the following transformations are applied...

$a=3 \quad b=1$
$h=0 \quad k=0$

(1) $y = 3f(x)$

$(x, y) \rightarrow (x, 3y)$
$(-2, 5) \rightarrow (-2, 15)$

$a=1 \quad b=\frac{1}{3}$
$h=0 \quad k=0$

(2) $y = f\left(-\frac{1}{3}x\right)$

$(x, y) \rightarrow (-3x, y)$
$(-2, 5) \rightarrow (6, 5)$

$a=4 \quad b=\frac{1}{2}$
$h=-5 \quad k=-3$

(3) $y = 4f\left[\frac{1}{2}(x+5)\right] - 3$

$(x, y) \rightarrow (2x-5, 4y-3)$
$(-2, 5) \rightarrow (-9, 17)$

$a=2 \quad b=2$
$h=3 \quad k=5$

(4) $y - 5 = -2f(-2x+6)$

$y = -2f(-2(x-3)) + 5$

$(x, y) \rightarrow (-\frac{1}{2}x+3, -2y+5)$
$(-2, 5) \rightarrow (4, -5)$

Summary of Transformations...

Transformations of the graphs of functions		
$f(x)+c$	shift $f(x)$ up c units	} vertical translation
$f(x)-c$	shift $f(x)$ down c units	
$f(x+c)$	shift $f(x)$ left c units	} horizontal translation
$f(x-c)$	shift $f(x)$ right c units	
$f(-x)$	reflect $f(x)$ about the y-axis	horizontal reflection
$-f(x)$	reflect $f(x)$ about the x-axis	vertical reflection
$cf(x)$	When $0 < c < 1$ – vertical shrinking of $f(x)$ When $c > 1$ – vertical stretching of $f(x)$ **Multiply the y values by c**	
$f(cx)$	When $0 < c < 1$ – horizontal stretching of $f(x)$ When $c > 1$ – horizontal shrinking of $f(x)$ **Divide the x values by c**	

Questions from Homework

③ a) $y = f(x)$

$y = x^2$

$y = \frac{1}{4}x^2$

$a = \frac{1}{4}$

(x, y)	$\longrightarrow$	$(x, \frac{1}{4}y)$
0,0		0,0
±1,1		±1,1/4
±2,4		±2,1
±3,9		±3,9/4
±4,16		±4,4

b) $y = f(x)$

$y = x^2$

$y = (\frac{1}{2}x)^2$

$y = \frac{1}{4}x^2$

$b = \frac{1}{2}$

(x, y)	$\longrightarrow$	$(2x, y)$
0,0		0,0
±1,1		±2,1
±2,4		±4,4
±3,9		±6,9
±4,16		±8,16

Transformations:

$g(x) = -3f(4(x-4)) - 10$

2. The function $y = f(x)$ is transformed to the function $g(x) = -3f(4x - 16) - 10$. Copy and complete the following statements by filling in the blanks.

The function $f(x)$ is transformed to the function $g(x)$ by a horizontal stretch about the a by a factor of b. It is vertically stretched about the c by a factor of d. It is reflected in the e, and then translated f units to the right and g units down.

$a = 3$
$b = 4$
$h = 4$
$k = -10$

a) y-axis
b) 1/4
c) x-axis
d) 3
e) x-axis
f) 4
g) 10

Transformations:

$$y = f(x) \longrightarrow y = af\left(b(x-h)\right)+k$$

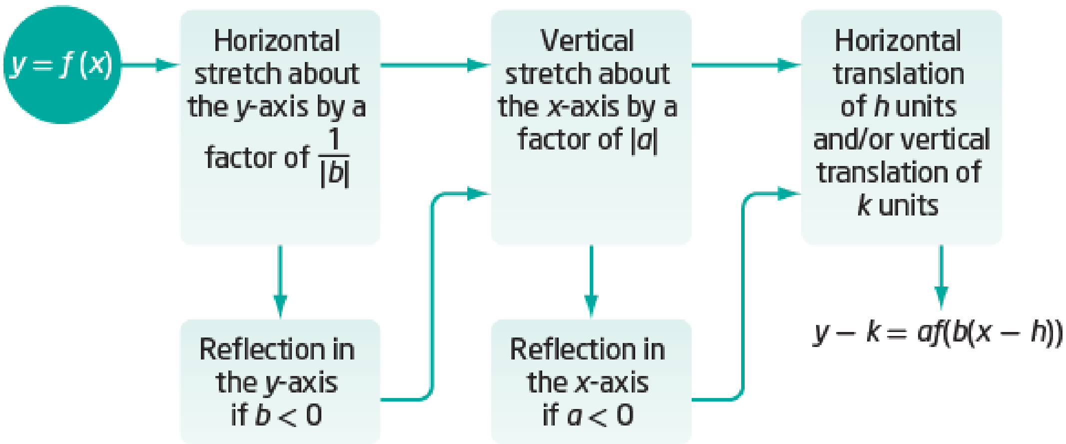
Mapping Rule: $$(x,y) \rightarrow \left(\frac{1}{b}x+h, ay+k\right)$$

Important note for sketching...

Transformations should be applied in following order:

1. Reflections
2. Stretches
3. Translations

Remember....RST



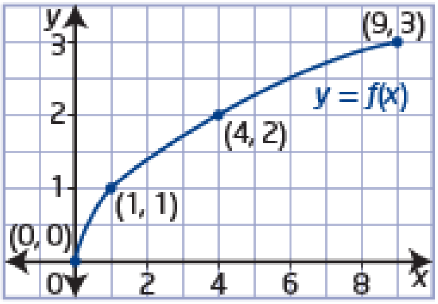
Example 1

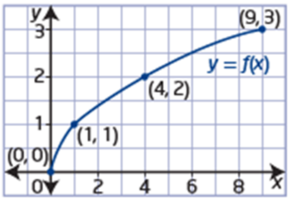
Graph a Transformed Function

Describe the combination of transformations that must be applied to the function $y = f(x)$ to obtain the transformed function. Sketch the graph, showing each step of the transformation.

a) $y = 3f(2x)$

b) $y = f(3x + 6)$





a) $y = 3f(2x)$

a) Compare the function to $y = af(b(x - h)) + k$. For $y = 3f(2x)$, $a = 3$, $b = 2$, $h = 0$, and $k = 0$.

The graph of $y = f(x)$ is horizontally stretched about the y-axis by a factor of $\frac{1}{2}$ and then vertically stretched about the x-axis by a factor of 3.

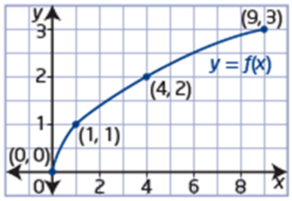
- Apply the horizontal stretch by a factor of $\frac{1}{2}$ to obtain the graph of $y = f(2x)$.

[redacted]

- Apply the vertical stretch by a factor of 3 to $y = f(2x)$ to obtain the graph of $y = 3f(2x)$.

Would performing the stretches in reverse order change the final result?

[redacted]



b) $y = f(3x + 6)$

b) First, rewrite $y = f(3x + 6)$ in the form $y = af(b(x - h)) + k$. This makes it easier to identify specific transformations.

$y = f(3x + 6)$
$y =$ Factor out the coefficient of x.
For $y =$, $a =$, $b =$, $h =$, and $k =$

The graph of $y = f(x)$ is horizontally stretched about the y-axis by a factor of and then horizontally translated units to the left.

- Apply the horizontal stretch by a factor of to obtain the graph of $y =$

[redacted]

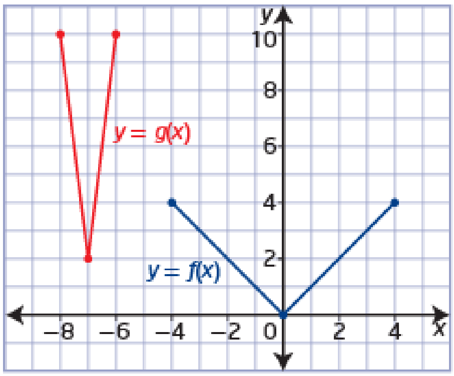
- Apply the horizontal translation of units to the left to $y =$ to obtain the graph of

[redacted]

Example 3

Write the Equation of a Transformed Function Graph

The graph of the function $y = g(x)$ represents a transformation of the graph of $y = f(x)$. Determine the equation of $g(x)$ in the form $y = af(b(x - h)) + k$. Explain your answer.



Solution

Locate key points on the graph of $f(x)$ and their image points on the graph of $g(x)$.

$(-4, 4) \rightarrow (-8, 10)$
$(0, 0) \rightarrow (-7, 2)$
$(4, 4) \rightarrow (-6, 10)$

The equation of the transformed function is [redacted]

[redacted]

How could you use the mapping $(x, y) \rightarrow \left(\frac{1}{b}x + h, ay + k\right)$ to verify this equation?

17. The graph of the function $y = 2x^2 + x + 1$ is stretched vertically about the x-axis by a factor of 2, stretched horizontally about the y-axis by a factor of $\frac{1}{3}$, and translated 2 units to the right and 4 units down. Write the equation of the transformed function.

is stretched vertically about the x-axis by a factor of 2, stretched horizontally about the y-axis by a factor of $\frac{1}{3}$, and translated 2 units to the right and 4 units down. Write the equation of the transformed function.

Homework