Warm-Up...

Given that (-2 , 5) is a point on the graph of $y = f(x)$, determine the coordinates of this point once the following transformations are applied...

$a=3$ $b=1$
$h=0$ $k=0$

(1) $y = 3f(x)$

$(x, y) \rightarrow (x, 3y)$
$(-2, 5) \rightarrow (-2, 15)$

$a=1$ $b=\frac{1}{3}$
$h=0$ $k=0$

(2) $y = f\left(-\frac{1}{3}x\right)$

$(x, y) \rightarrow (-3x, y)$
$(-2, 5) \rightarrow (6, 5)$

$a=4$ $b=\frac{1}{2}$
$h=-5$ $k=-3$

(3) $y = 4f\left[\frac{1}{2}(x+5)\right]-3$

$(x, y) \rightarrow (2x-5, 4y-3)$
$(-2, 5) \rightarrow (-9, 17)$

$a=2$ $b=2$
$h=3$ $k=5$

(4) $y-5 = -2f(-2x+6)$

$y = -2f(-2(x-3))+5$

$(x, y) \rightarrow (\frac{-1}{2}x+3, -2y+5)$
$(-2, 5) \rightarrow (4, -5)$

Summary of Transformations...

Transformations of the graphs of functions		
$f(x)+c$	shift $f(x)$ up c units	vertical translation
$f(x)-c$	shift $f(x)$ down c units	
$f(x+c)$	shift $f(x)$ left c units	horizontal translation
$f(x-c)$	shift $f(x)$ right c units	
$f(-x)$	reflect $f(x)$ about the y-axis	horizontal reflection
$-f(x)$	reflect $f(x)$ about the x-axis	vertical reflection
$cf(x)$	When $0 < c < 1$ – vertical shrinking of $f(x)$ When $c > 1$ – vertical stretching of $f(x)$ **Multiply the y values by c**	
$f(cx)$	When $0 < c < 1$ – horizontal stretching of $f(x)$ When $c > 1$ – horizontal shrinking of $f(x)$ **Divide the x values by c**	

Questions from Homework

③ a) $y = f(x)$

$y = x^2$

$y = \frac{1}{4}x^2$

$a = \frac{1}{4}$

(x, y)	$\longrightarrow$	$(x, \frac{1}{4}y)$
$0, 0$		$0, 0$
$\pm 1, 1$		$\pm 1, \frac{1}{4}$
$\pm 2, 4$		$\pm 2, 1$
$\pm 3, 9$		$\pm 3, \frac{9}{4}$
$\pm 4, 16$		$\pm 4, 4$

b) $y = f(x)$

$y = x^2$

$y = \left(\frac{1}{2}x\right)^2$

$y = \frac{1}{4}x^2$

$b = \frac{1}{2}$

(x, y)	$\longrightarrow$	$(2x, y)$
$0, 0$		$0, 0$
$\pm 1, 1$		$\pm 2, 1$
$\pm 2, 4$		$\pm 4, 4$
$\pm 3, 9$		$\pm 6, 9$
$\pm 4, 16$		$\pm 8, 16$

Transformations:

$g(x) = -3f(4(x-4)) - 10$

$a = 3$
$b = 4$
$h = 4$
$k = -10$

2. The function $y = f(x)$ is transformed to the function $g(x) = -3f(4x - 16) - 10$. Copy and complete the following statements by filling in the blanks.

The function $f(x)$ is transformed to the function $g(x)$ by a horizontal stretch about the a by a factor of b. It is vertically stretched about the c by a factor of d. It is reflected in the e, and then translated f units to the right and g units down.

a) y-axis
b) 1/4
c) x-axis
d) 3
e) x-axis
f) 4
g) 10

Transformations:

$$y = f(x) \longrightarrow y = af\left(b(x-h)\right) + k$$

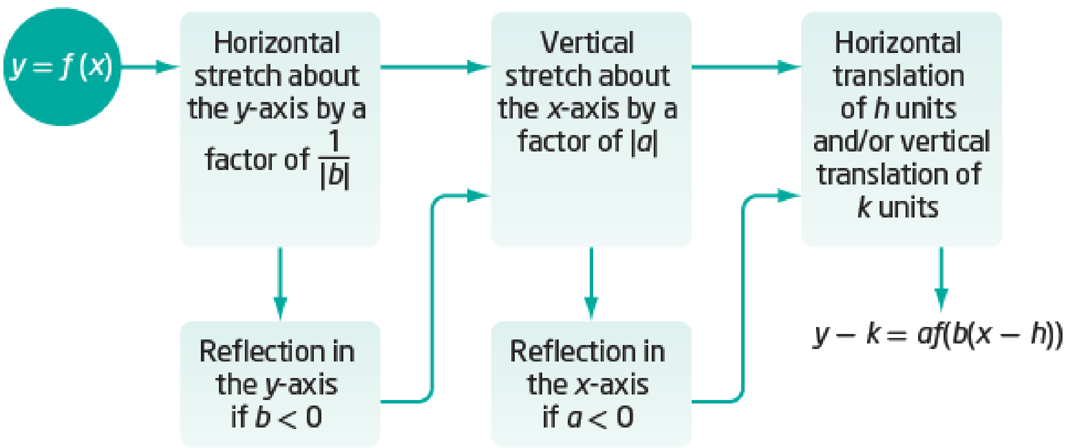
Mapping Rule: $$(x, y) \rightarrow \left(\frac{1}{b}x + h, ay + k\right)$$

Important note for sketching...

Transformations should be applied in following order:

1. Reflections
2. Stretches
3. Translations

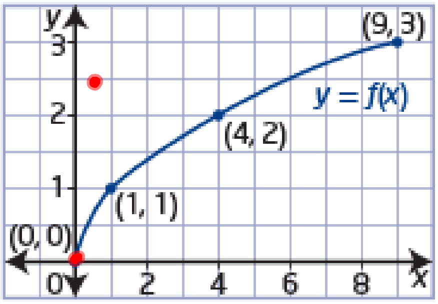
Remember....RST



Example 1

Graph a Transformed Function

Describe the combination of transformations that must be applied to the function $y = f(x)$ to obtain the transformed function. Sketch the graph, showing each step of the transformation.



a) $y = 3f(2x)$

b) $y = f(3x + 6)$

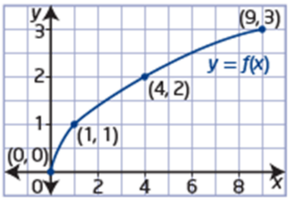
a) $y = 3f(2x)$

$a = 3$
Vertical Stretch by a factor of 3

$b = 2$
Horizontal Stretch by a factor of $\frac{1}{2}$

$(x, y) \longrightarrow \left(\frac{1}{2}x, 3y\right)$

$(0, 0)$	$(0, 0)$
$(1, 1)$	$(\frac{1}{2}, 3)$
$(4, 2)$	$(2, 6)$
$(9, 3)$	$(\frac{9}{2}, 9)$

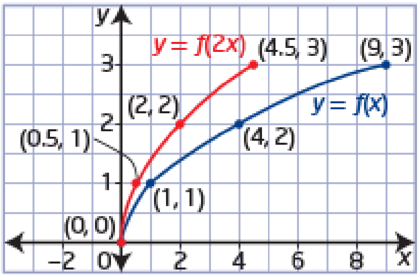


a) $y = 3f(2x)$

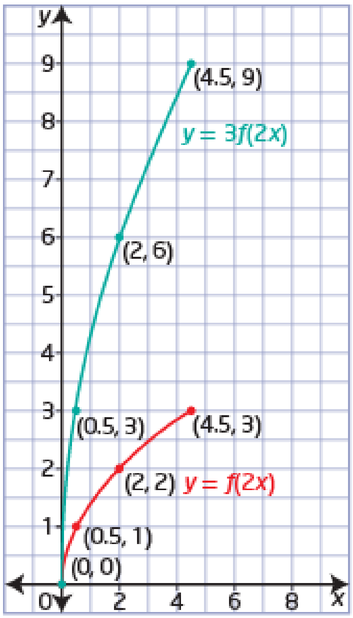
a) Compare the function to $y = af(b(x - h)) + k$. For $y = 3f(2x)$, $a = 3$, $b = 2$, $h = 0$, and $k = 0$.

The graph of $y = f(x)$ is horizontally stretched about the y-axis by a factor of $\frac{1}{2}$ and then vertically stretched about the x-axis by a factor of 3.

- Apply the horizontal stretch by a factor of $\frac{1}{2}$ to obtain the graph of $y = f(2x)$.



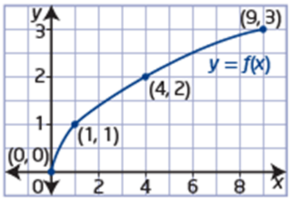
- Apply the vertical stretch by a factor of 3 to $y = f(2x)$ to obtain the graph of $y = 3f(2x)$.



Would performing the stretches in reverse order change the final result?

b) $y = f(3x + 6)$

$y = 1f(3(x+2)) + 0$

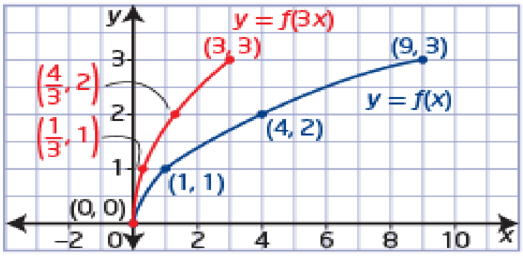


b) First, rewrite $y = f(3x + 6)$ in the form $y = af(b(x - h)) + k$. This makes it easier to identify specific transformations.

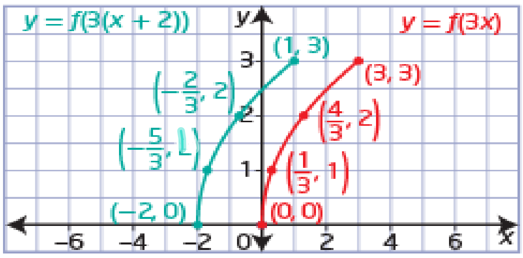
$y = f(3x + 6)$
$y = f(3(x + 2))$ Factor out the coefficient of x.
For $y = f(3(x + 2))$, $a = 1$, $b = 3$, $h = -2$, and $k = 0$.

The graph of $y = f(x)$ is horizontally stretched about the y-axis by a factor of $\frac{1}{3}$ and then horizontally translated 2 units to the left.

- Apply the horizontal stretch by a factor of $\frac{1}{3}$ to obtain the graph of $y = f(3x)$.



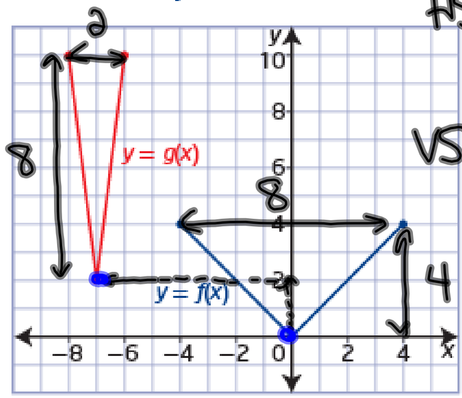
- Apply the horizontal translation of 2 units to the left to $y = f(3x)$ to obtain the graph of $y = f(3(x + 2))$.



Example 3

Write the Equation of a Transformed Function Graph

The graph of the function $y = g(x)$ represents a transformation of the graph of $y = f(x)$. Determine the equation of $g(x)$ in the form $y = af(b(x - h)) + k$. Explain your answer.



$HS = \frac{2}{8} = \frac{1}{4}$

$VS = \frac{8}{4} = 2$

Solution

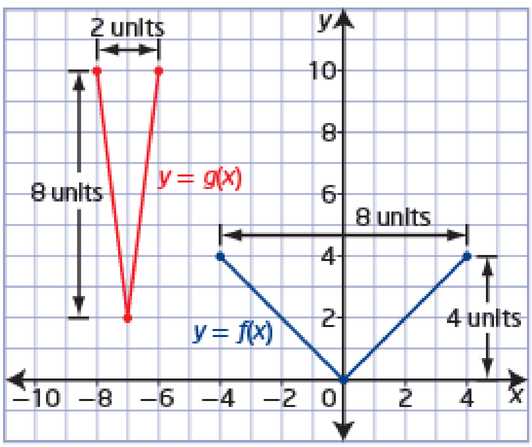
up 2 and Left 7

Locate key points on the graph of $f(x)$ and their image points on the graph of $g(x)$.

$a = 2$ $b = 4$ $h = -7$ $k = 2$

$(-4, 4) \rightarrow (-8, 10)$
$(0, 0) \rightarrow (-7, 2)$
$(4, 4) \rightarrow (-6, 10)$

The equation of the transformed function is $g(x) = 2f(4(x + 7)) + 2$.



How could you use the mapping $(x, y) \rightarrow \left(\frac{1}{b}x + h, ay + k\right)$ to verify this equation?

$(x, y) \rightarrow \left(\frac{1}{4}x - 7, 2y + 2\right)$

$(-4, 4) \longrightarrow (-8, 10)$

17. The graph of the function $y = 2x^2 + x + 1$ is stretched vertically about the x-axis by a factor of 2, stretched horizontally about the y-axis by a factor of $\frac{1}{3}$, and translated 2 units to the right and 4 units down. Write the equation of the transformed function.

is stretched vertically about the x-axis by a factor of 2, stretched horizontally about the y-axis by a factor of $\frac{1}{3}$, and translated 2 units to the right and 4 units down. Write the equation of the transformed function.

Homework

finish #3-6 on page 39

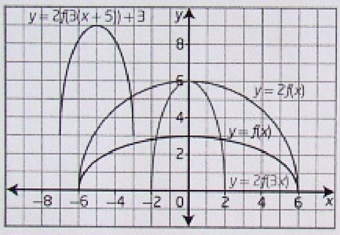
translated 4 units right and 10 units down.

3.

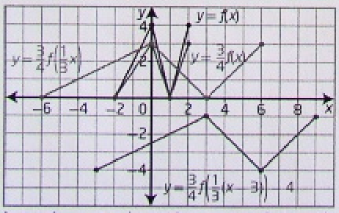
Function	Reflections	Vertical Stretch Factor	Horizontal Stretch Factor	Vertical Translation	Horizontal Translation
$y - 4 = f(x - 5)$	none	none	none	4	5
$y + 5 = 2f(3x)$	none	2	$\frac{1}{3}$	−5	none
$y = \frac{1}{2}f\left(\frac{1}{2}(x - 4)\right)$	none	$\frac{1}{2}$	2	none	4
$y + 2 = -3f(2(x + 2))$	x-axis	3	$\frac{1}{2}$	−2	−2

4. a) $y = f(-(x + 2)) - 2$ **b)** $y = f(2(x + 1)) - 4$

5. a)



b)

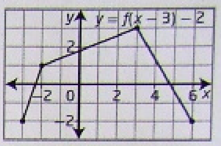


6. a) $(-8, 12)$ **b)** $(-4, 72)$ **c)** $(-6, -32)$
d) $(9, -32)$ **e)** $(-12, -9)$

factor of $\frac{1}{2}$, vertical stretch by a factor of $\frac{1}{3}$, and translation of 6 units right and 2 units up; $(x, y) \rightarrow \left(-\frac{1}{2}x + 6, \frac{1}{3}y + 2\right)$

8. a) $y + 5 = -3f(x + 4)$ **b)** $y - 2 = -\frac{3}{4}f(-3(x - 6$

9. a)



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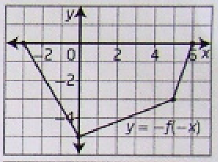
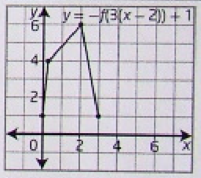


c)



d)