homework

Examine the function $f(x)=\frac{x^2}{1-x^2}$ with respect to…

- Intercepts ✓
- Symmetry ✓
- Asymptotes ✓
- Intervals of Increase or Decrease ✓
- Local Maximum and Minimum values ✓
- Concavity and Points of Inflection ✓
- Sketch the Curve

$f(x)=\frac{x^2}{1-x^2}$ | $f'(x)=\frac{2x}{(1-x^2)^2}$ | $f''(x)=\frac{2+6x^2}{(1-x^2)^3}=\frac{2(3x^2+1)}{(1-x^2)^3}$

① Intercepts:

x int ($y=0$): $x^2=0$, $x=0$, $(0,0)$

y int ($x=0$): $y=\frac{0}{1}=0$, $(0,0)$

② Symmetry:

$f(-x)=\frac{(-x)^2}{1-(-x)^2}=\frac{x^2}{1-x^2}$

$\therefore f(-x)=f(x)$ Even

③ Asymptotes:

HA: $y=-1$

VA: $1-x^2=0$, $1=x^2$, $\pm 1=x$

$\lim_{x\to 1^-}\frac{(+)}{(-)}=-\infty$ ($x=1.01$)

$\lim_{x\to -1^+}\frac{(+)}{(+)}=+\infty$ ($x\to -0.99$)

$\lim_{x\to 1^-}\frac{(+)}{(+)}=+\infty$

$\lim_{x\to 1^+}\frac{(+)}{(-)}=-\infty$

④ Intervals of Inc/Dec:

$f'(x)=\frac{2x}{(1-x^2)^2}$

min

− − + +

(-2) -1 -½ 0 ½ 1 (2)

Inc on $(0,\infty)$

Dec on $(-\infty,0)$

CV: $x=0,\pm 1$

⑤ Max/Mins:

$f(0)=0$ $(0,0)$ min

⑥ Concavity:

$f''(x)=\frac{2(3x^2+1)}{(1-x^2)^3}$

Always (+)

I.P. I.P.

− + −

(-2) -1 (0) 1 (2)

CU: on $(-1,1)$

CD: on $(-\infty,-1)\cup(1,\infty)$

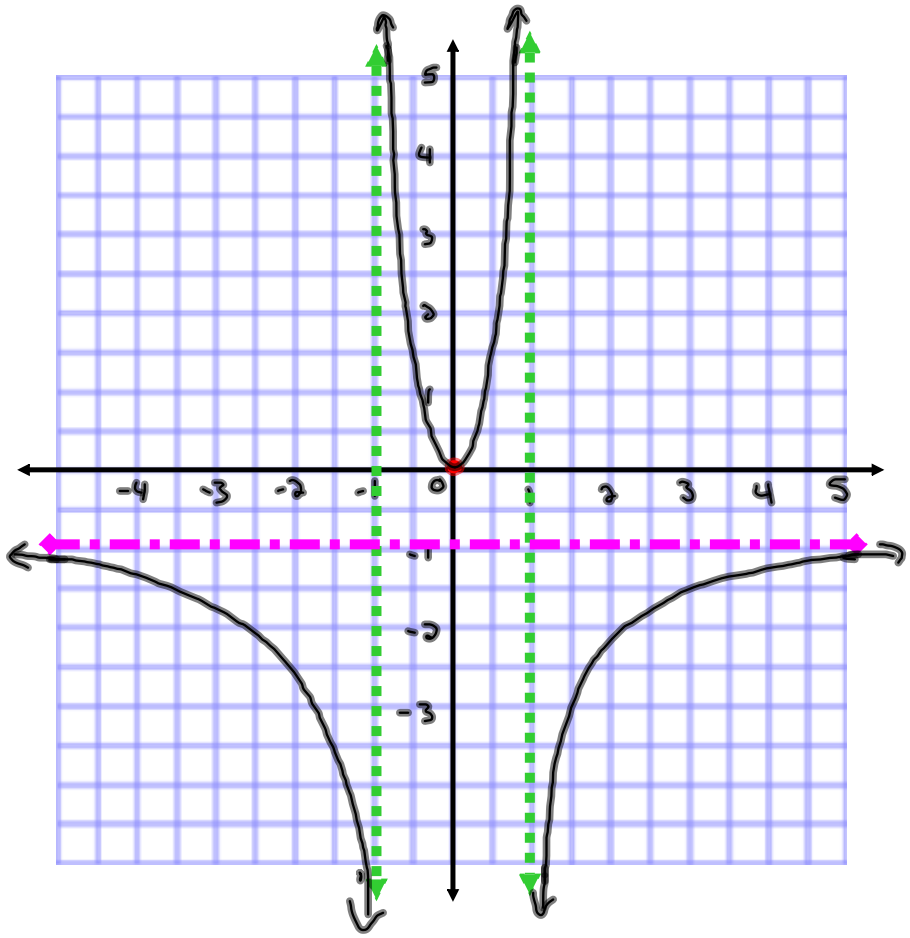
⑦ Inflection Points

CV: $x=\pm 1$

$f(-1)$ = undefined

$f(1)$ = undefined

No Inflection Points $x=\pm 1$ are V.A.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Examine the function $f(x) = 3x^5 - 5x^3$ with respect to…

- Intercepts
- Symmetry
- Asymptotes
- Intervals of Increase or Decrease
- Local Maximum and Minimum values
- Concavity and Points of Inflection
- Sketch the Curve

$f(x) = 3x^5 - 5x^3$ | $f'(x) = 15x^4 - 15x^2$ | $f''(x) = 60x^3 - 30x$

$f(x) = x^3(3x^2 - 5)$ | $f'(x) = 15x^2(x^2 - 1)$ | $f''(x) = 30x(2x^2 - 1)$

① Intercepts:

x int ($y=0$): $x = 0, \pm\sqrt{5/3}$

$(0,0)$ $(1.29, 0)$ $(-1.29, 0)$

y int ($x=0$): $y = 0$

$(0,0)$

② Symmetry:

$f(-x) = 3(-x)^5 - 5(-x)^3$

$= -3x^5 + 5x^3$

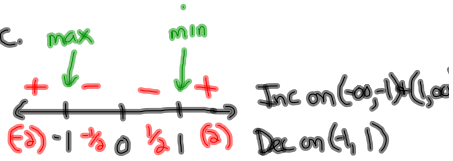
$\therefore f(-x) = -f(x)$ Odd

③ Asymptotes: None

④ Intervals of Inc/Dec.

$f'(x) = 15x^2(x^2 - 1)$

CV: $x = 0, \pm 1$



Inc on $(-\infty, -1) \cup (1, \infty)$

Dec on $(-1, 1)$

⑤ Max/Mins:

$f(-1) = 3(-1)^5 - 5(-1)^3$ $(-1, 2)$ max

$= -3 + 5$

$= 2$

$f(1) = 3(1)^5 - 5(1)^3$ $(1, -2)$ min

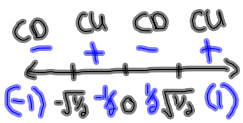
$= 3 - 5$

$= -2$

⑥ Concavity:

$f''(x) = 30x(2x^2 - 1)$

CV: $x = 0, \pm\sqrt{1/2}$

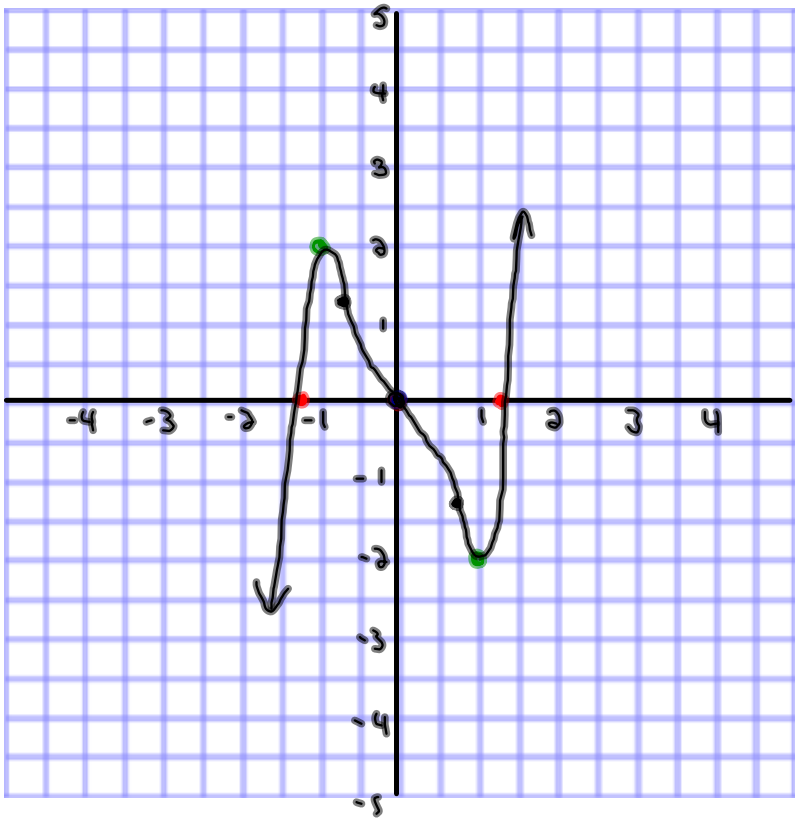


⑦ Inflection Points:

$f(-\sqrt{1/2}) \approx -0.53 + 1.767 \approx 1.238$ $(-0.707, 1.238)$

$f(0) = 0$ $(0,0)$

$f(\sqrt{1/2}) \approx 0.53 - 1.767 \approx -1.238$ $(0.707, -1.238)$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Examine the function $f(x) = \frac{x^2}{x-7}$ with respect to…

- Intercepts ✓
- Symmetry ✓
- Asymptotes ✓
- Intervals of Increase or Decrease ✓
- Local Maximum and Minimum values ✓
- Concavity and Points of Inflection ✓
- Sketch the Curve

$f(x) = \frac{x^2}{x-7}$ | $f'(x) = \frac{x(x-14)}{(x-7)^2}$ | $f''(x) = \frac{98}{(x-7)^3}$

① Intercepts:

x int ($y=0$): $x^2 = 0$, $x = 0$, (0,0)

y int ($x=0$): $y = \frac{0}{-7} = 0$, (0,0)

② Symmetry

$f(-x) = \frac{(-x)^2}{(-x)-7} = \frac{x^2}{-x-7}$

None.

③ Asymptotes:

VA: $x=7$

$\lim_{x\to 7^-} \frac{(+)}{(-)} = -\infty$, $x = 6.99$

$\lim_{x\to 7^+} \frac{(+)}{(+)} = +\infty$, $x = 7.01$

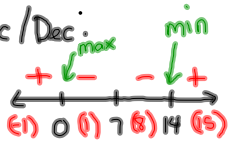
SA:

$$\begin{array}{r} x+7 \\ x-7 \overline{)\,x^2} \\ -(x^2-7x) \\ \hline 7x \\ -(7x-49) \\ \hline 49 \end{array}$$

$y = x+7$, $m = \frac{1}{1}$, $b = 7$

④ Intervals of Inc / Dec:

$f'(x) = \frac{x(x-14)}{(x-7)^2}$



Inc on $(-\infty,0) \cup (14,\infty)$

Dec on $(0,14)$

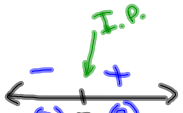
CV: 0, 7, 14

$f(0) = 0$ (0,0) max

$f(14) = 28$ (14,28) min

⑤ Concavity:

$f''(x) = \frac{98}{(x-7)^3}$



CU: on $(7,\infty)$

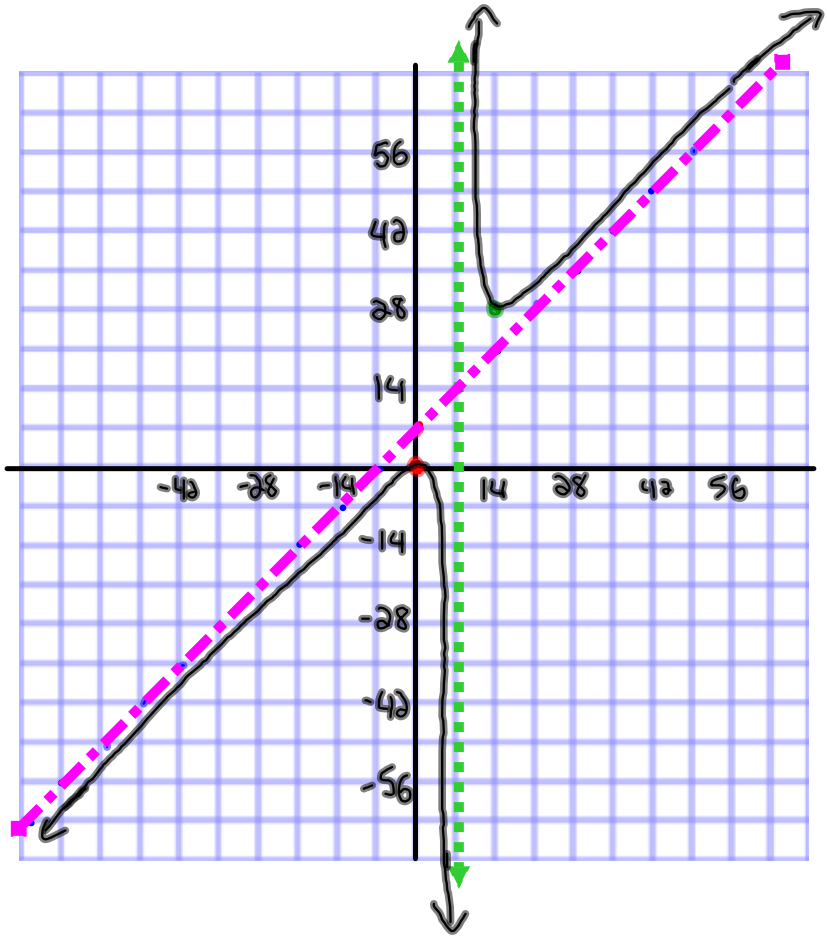
CD: on $(-\infty,7)$

CV: $x=7$

⑥ Inflection Point:

$f(7)$ = undefined

$x=7$ is V.A. No I.P.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