Warm-Up...

Given that (-2 , 5) is a point on the graph of $y = f(x)$, determine the coordinates of this point once the following transformations are applied...

$a=3$ $b=1$
$h=0$ $k=0$

(1) $y = 3f(x)$

$(x,y) \rightarrow (x, 3y)$
$(-2,5) \rightarrow (-2, 15)$

$a=1$ $b=\frac{1}{3}$
$h=0$ $k=0$

(2) $y = f\left(-\frac{1}{3}x\right)$

$(x,y) \rightarrow (-3x, y)$
$(-2,5) \rightarrow (6, 5)$

$a=4$ $b=\frac{1}{2}$
$h=-5$ $k=-3$

(3) $y = 4f\left[\frac{1}{2}(x+5)\right]-3$

$(x,y) \rightarrow (2x-5, 4y-3)$
$(-2,5) \rightarrow (-9, 17)$

$a=2$ $b=2$
$h=3$ $k=5$

(4) $y-5 = -2f(-2x+6)$

$y = -2f(-2(x-3))+5$

$(x,y) \rightarrow \left(-\frac{1}{2}x+3, -2y+5\right)$
$(-2,5) \rightarrow (4, -5)$

Summary of Transformations...

Transformations of the graphs of functions	
$f(x)+c$	shift $f(x)$ up c units
$f(x)-c$	shift $f(x)$ down c units
$f(x+c)$	shift $f(x)$ left c units
$f(x-c)$	shift $f(x)$ right c units
$f(-x)$	reflect $f(x)$ about the y-axis
$-f(x)$	reflect $f(x)$ about the x-axis
$cf(x)$	When $0 < c < 1$ – vertical shrinking of $f(x)$ When $c > 1$ – vertical stretching of $f(x)$ **Multiply the y values by c**
$f(cx)$	When $0 < c < 1$ – horizontal stretching of $f(x)$ When $c > 1$ – horizontal shrinking of $f(x)$ **Divide the x values by c**

} vertical translation

} horizontal translation

horizontal reflection

vertical reflection

Questions from Homework

③ a) $y = f(x)$

$y = x^2$

$y = \frac{1}{4}x^2$

$a = \frac{1}{4}$

$(x, y) \longrightarrow (x, \frac{1}{4}y)$

(x, y)	$(x, \frac{1}{4}y)$
0,0	0,0
±1,1	±1, 1/4
±2,4	±2,1
±3,9	±3, 9/4
±4,16	±4,4

b) $y = f(x)$

$y = x^2$

$y = (\frac{1}{2}x)^2$

$y = \frac{1}{4}x^2$

$b = \frac{1}{2}$

$(x, y) \longrightarrow (2x, y)$

(x, y)	$(2x, y)$
0,0	0,0
±1,1	±2,1
±2,4	±4,4
±3,9	±6,9
±4,16	±8,16

Transformations:

$g(x) = -3f(4(x-4)) - 10$

$a = 3$
$b = 4$
$h = 4$
$k = -10$

2. The function $y = f(x)$ is transformed to the function $g(x) = -3f(4x - 16) - 10$. Copy and complete the following statements by filling in the blanks.

The function $f(x)$ is transformed to the function $g(x)$ by a horizontal stretch about the a by a factor of b. It is vertically stretched about the c by a factor of d. It is reflected in the e, and then translated f units to the right and g units down.

a) y-axis
b) 1/4
c) x-axis
d) 3
e) x-axis
f) 4
g) 10

Transformations:

$$y = f(x) \longrightarrow y = af\left(b(x-h)\right) + k$$

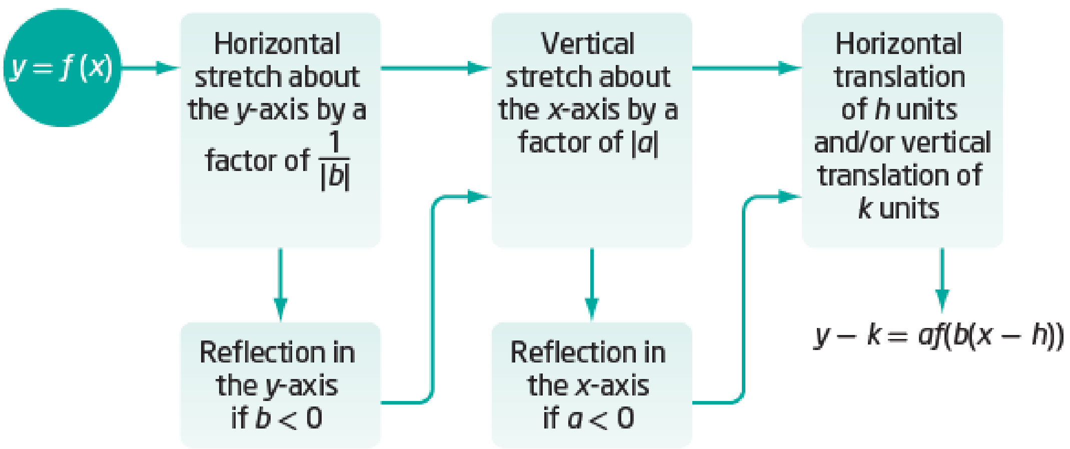
Mapping Rule: $(x, y) \rightarrow \left(\frac{1}{b}x + h, ay + k\right)$

Important note for sketching...

Transformations should be applied in following order:

1. Reflections
2. Stretches
3. Translations

Remember....RST



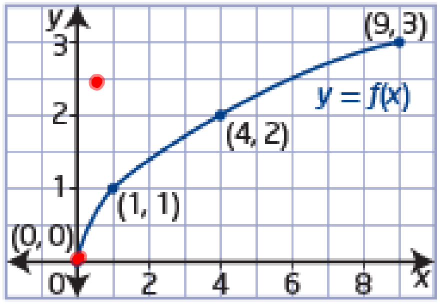
Example 1

Graph a Transformed Function

Describe the combination of transformations that must be applied to the function $y = f(x)$ to obtain the transformed function. Sketch the graph, showing each step of the transformation.

a) $y = 3f(2x)$

b) $y = f(3x + 6)$



a) $y = 3f(2x)$

$a = 3$

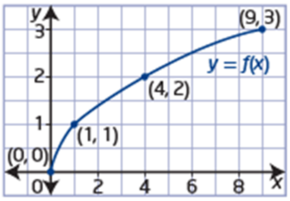
Vertical Stretch by a factor of 3

$b = 2$

Horizontal Stretch by a factor of $\frac{1}{2}$

$(x, y) \longrightarrow (\frac{1}{2}x, 3y)$

$(0,0)$	$(0, 0)$
$(1,1)$	$(\frac{1}{2}, 3)$
$(4,2)$	$(2, 6)$
$(9,3)$	$(\frac{9}{2}, 9)$

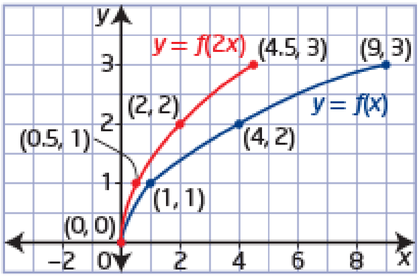


a) $y = 3f(2x)$

a) Compare the function to $y = af(b(x - h)) + k$. For $y = 3f(2x)$, $a = 3$, $b = 2$, $h = 0$, and $k = 0$.

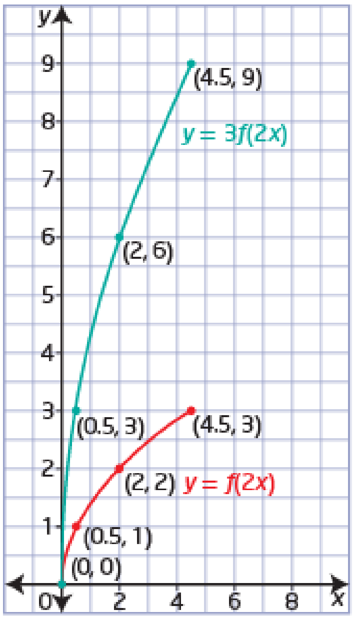
The graph of $y = f(x)$ is horizontally stretched about the y-axis by a factor of $\frac{1}{2}$ and then vertically stretched about the x-axis by a factor of 3.

- Apply the horizontal stretch by a factor of $\frac{1}{2}$ to obtain the graph of $y = f(2x)$.



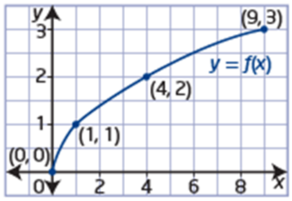
- Apply the vertical stretch by a factor of 3 to $y = f(2x)$ to obtain the graph of $y = 3f(2x)$.

Would performing the stretches in reverse order change the final result?



b) $y = f(3x + 6)$

$y = 1\,f(3(x+2)) + 0$

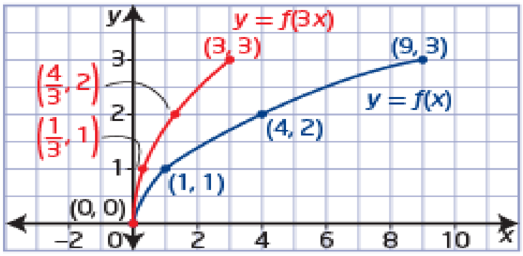


b) First, rewrite $y = f(3x + 6)$ in the form $y = af(b(x - h)) + k$. This makes it easier to identify specific transformations.

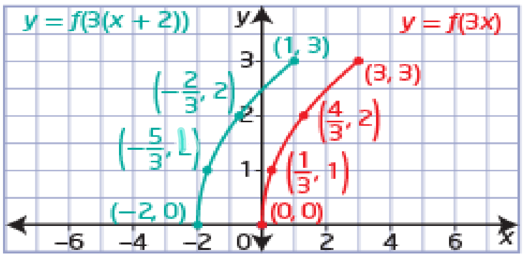
$y = f(3x + 6)$
$y = f(3(x + 2))$ Factor out the coefficient of x.
For $y = f(3(x + 2))$, $a = 1$, $b = 3$, $h = -2$, and $k = 0$.

The graph of $y = f(x)$ is horizontally stretched about the y-axis by a factor of $\frac{1}{3}$ and then horizontally translated 2 units to the left.

- Apply the horizontal stretch by a factor of $\frac{1}{3}$ to obtain the graph of $y = f(3x)$.



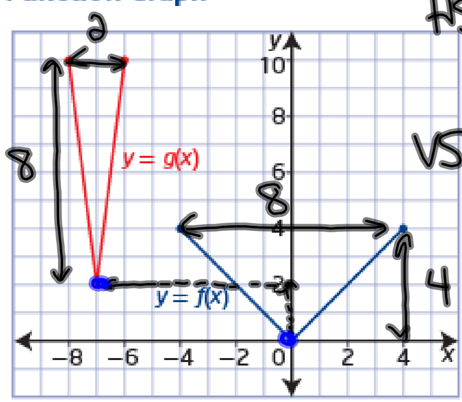
- Apply the horizontal translation of 2 units to the left to $y = f(3x)$ to obtain the graph of $y = f(3(x + 2))$.



Example 3

Write the Equation of a Transformed Function Graph

The graph of the function $y = g(x)$ represents a transformation of the graph of $y = f(x)$. Determine the equation of $g(x)$ in the form $y = af(b(x - h)) + k$. Explain your answer.



HS $= \frac{2}{8} = \frac{1}{4}$

VS $= \frac{8}{4} = 2$

Solution

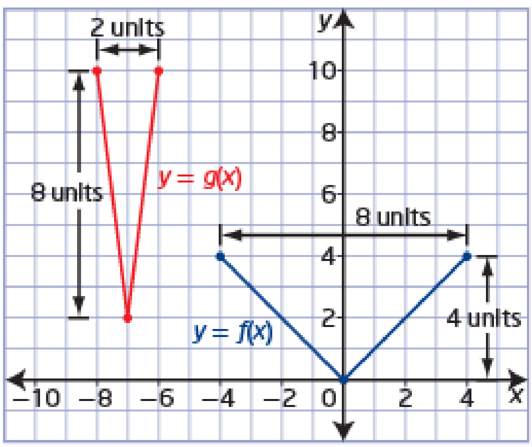
up 2 and Left 7

Locate key points on the graph of $f(x)$ and their image points on the graph of $g(x)$.

$a=2$ $b=4$ $h=-7$ $k=2$

$(-4, 4) \rightarrow (-8, 10)$
$(0, 0) \rightarrow (-7, 2)$
$(4, 4) \rightarrow (-6, 10)$

The equation of the transformed function is $g(x) = 2f(4(x + 7)) + 2$.



How could you use the mapping $(x, y) \rightarrow \left(\frac{1}{b}x + h, ay + k\right)$ to verify this equation?

$(x, y) \rightarrow \left(\frac{1}{4}x - 7, 2y + 2\right)$

$(-4, 4) \longrightarrow (-8, 10)$

17. The graph of the function $y = 2x^2 + x + 1$ is stretched vertically about the x-axis by a factor of 2, stretched horizontally about the y-axis by a factor of $\frac{1}{3}$, and translated 2 units to the right and 4 units down. Write the equation of the transformed function. Complete the square.

$$y = 2x^2 + x + 1$$

$$y - 1 + \frac{1}{8} = 2\left(x^2 + \frac{1}{2}x + \frac{1}{16}\right)$$

$$y - \frac{7}{8} = 2\left(x + \frac{1}{4}\right)^2$$

$$y = 2\left(x + \frac{1}{4}\right)^2 + \frac{7}{8}$$

$$y = \cdot 2\, 2\left(3\left(x + \frac{1}{4} - 2\right)\right)^2 + \frac{7}{8} - \frac{4}{1}$$

$$\boxed{y = 4\left(3\left(x - \frac{7}{4}\right)\right)^2 - \frac{25}{8}}$$

VS by a factor of 2
$a = 2$

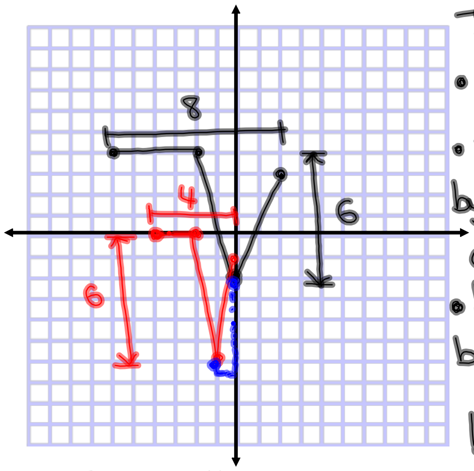
HS by a factor of $\frac{1}{3}$
$b = 3$

Translated 2 units right
$h = 2$

Translated 4 units down
$k = -4$

Questions from Homework

4 b)



Think RST
- No reflections
- vertical stretch by a factor of $\frac{6}{6}$ or 1
  $a=1$
- horizontal stretch by a factor of $\frac{4}{8}$ or $\frac{1}{2}$
  $b=2$
- translated down 4 units and left 1 unit
  $h=-1$ $k=-4$

$y=af(b(x-h))+k$

$y=1f(2(x+1))-4$

$y=f(2(x+1))-4$

6 Key point $\longrightarrow$ Image Point

$(x,y) \longrightarrow \left(\frac{1}{b}x+h, ay+k\right)$

a) $y+6=f(x-4)$

$y=f(x-4)-6$

$(x,y) \longrightarrow (x+4, y-6)$

$(-12,18) \longrightarrow \boxed{(-8,12)}$

$a=1$
$b=1$
$h=4$
$k=-6$

d) $y=-2f\left(-\frac{2}{3}x-6\right)+4$

$y=-2f\left(-\frac{2}{3}(x+9)\right)+4$

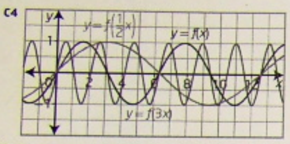
$(x,y) \longrightarrow \left(-\frac{3}{2}x-9, 2y+4\right)$

$(-12,18) \longrightarrow (9, -32)$

$a=2$
$b=\frac{2}{3}$
$h=-9$
$k=4$

Homework

finish #7-11 on page 40

C4 

C5 a) $t_n = 4n - 14$ b) $t_n = -4n + 14$

c) They are reflections of each other in the x-axis.

1.3 Combining Transformations, pages 38 to 43

1. a) $y = -f\left(\frac{1}{2}x\right)$ or $y = -\frac{1}{4}x^2$

b) $y = \frac{1}{4}f(-4x)$ or $y = 4x^2$

2. The function $f(x)$ is transformed to the function $g(x)$ by a horizontal stretch about the y-axis by a factor of $\frac{1}{4}$. It is vertically stretched about the x-axis by a factor of 3. It is reflected in the x-axis, and then translated 4 units right and 10 units down.

3.

Function	Reflections	Vertical Stretch Factor	Horizontal Stretch Factor	Vertical Translation	Horizontal Translation
$y - 4 = f(x - 5)$	none	none	none	4	5
$y + 5 = 2f(3x)$	none	2	$\frac{1}{3}$	−5	none
$y = \frac{1}{2}f\left(\frac{1}{2}(x - 4)\right)$	none	$\frac{1}{2}$	2	none	4
$y + 2 = -3f(2(x + 2))$	x-axis	3	$\frac{1}{2}$	−2	−2

7. a) vertical stretch by a factor of 2 and translation of 3 units right and 4 units up; $(x, y) \rightarrow (x + 3, 2y + 4)$

b) horizontal stretch by a factor of $\frac{1}{3}$, reflection in the x-axis, and translation of 2 units down; $(x, y) \rightarrow \left(\frac{1}{3}x, -y - 2\right)$

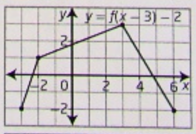
c) reflection in the y-axis, reflection in the x-axis, vertical stretch by a factor of $\frac{1}{4}$, and translation of 2 units left; $(x, y) \rightarrow \left(-x - 2, -\frac{1}{4}y\right)$

d) horizontal stretch by a factor of $\frac{1}{4}$, reflection in the x-axis, and translation of 2 units right and 3 units up; $(x, y) \rightarrow \left(\frac{1}{4}x + 2, -y + 3\right)$

e) reflection in the y-axis, horizontal stretch by a factor of $\frac{4}{3}$, reflection in the x-axis, and vertical stretch by a factor of $\frac{2}{3}$; $(x, y) \rightarrow \left(-\frac{4}{3}x, -\frac{2}{3}y\right)$

f) reflection in the y-axis, horizontal stretch by a factor of $\frac{1}{2}$, vertical stretch by a factor of $\frac{1}{3}$, and translation of 6 units right and 2 units up; $(x, y) \rightarrow \left(-\frac{1}{2}x + 6, \frac{1}{3}y + 2\right)$

8. a) $y + 5 = -3f(x + 4)$ b) $y - 2 = -\frac{3}{4}f(-3(x - 6))$

9. a) 

b)

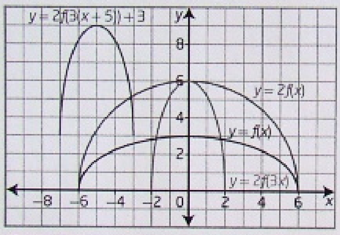
translated 4 units right and 10 units down.

3.

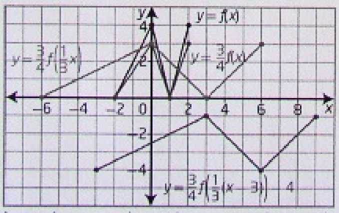
Function	Reflections	Vertical Stretch Factor	Horizontal Stretch Factor	Vertical Translation	Horizontal Translation
$y - 4 = f(x - 5)$	none	none	none	4	5
$y + 5 = 2f(3x)$	none	2	$\frac{1}{3}$	−5	none
$y = \frac{1}{2}f\left(\frac{1}{2}(x - 4)\right)$	none	$\frac{1}{2}$	2	none	4
$y + 2 = -3f(2(x + 2))$	x-axis	3	$\frac{1}{2}$	−2	−2

4. a) $y = f(-(x + 2)) - 2$ b) $y = f(2(x + 1)) - 4$

5. a)



b)

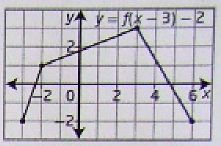


6. a) $(-8, 12)$ b) $(-4, 72)$ c) $(-6, -32)$
d) $(9, -32)$ e) $(-12, -9)$

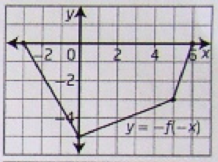
factor of $\frac{1}{2}$, vertical stretch by a factor of $\frac{1}{3}$, and translation of 6 units right and 2 units up;
$(x, y) \to \left(-\frac{1}{2}x + 6, \frac{1}{3}y + 2\right)$

8. a) $y + 5 = -3f(x + 4)$ b) $y - 2 = -\frac{3}{4}f(-3(x - 6$

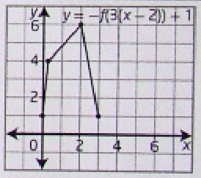
9. a)



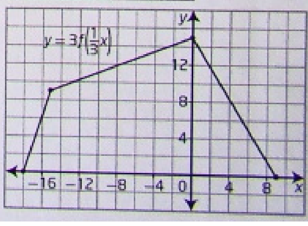
b)

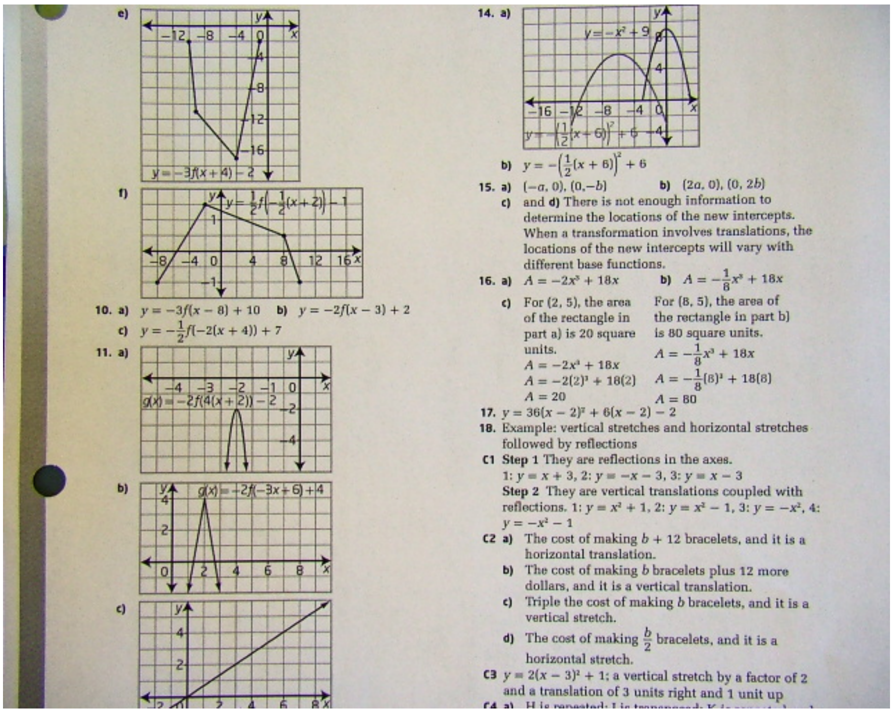


c)



d)





e) $y = -3f(x + 4) - 2$

f) $y = \frac{1}{2}f\left(-\frac{1}{2}(x + 2)\right) - 1$

10. a) $y = -3f(x - 8) + 10$ **b)** $y = -2f(x - 3) + 2$

c) $y = -\frac{1}{2}f(-2(x + 4)) + 7$

11. a) $g(x) = -2f(4(x + 2)) - 2$

b) $g(x) = -2f(-3x + 6) + 4$

c)

14. a) $y = -x^2 + 9$; $y = -\left(\frac{1}{2}(x + 6)\right)^2 + 6$

b) $y = -\left(\frac{1}{2}(x + 6)\right)^2 + 6$

15. a) $(-a, 0), (0, -b)$ **b)** $(2a, 0), (0, 2b)$

c) and **d)** There is not enough information to determine the locations of the new intercepts. When a transformation involves translations, the locations of the new intercepts will vary with different base functions.

16. a) $A = -2x^3 + 18x$ **b)** $A = -\frac{1}{8}x^3 + 18x$

c) For (2, 5), the area of the rectangle in part a) is 20 square units.
$A = -2x^3 + 18x$
$A = -2(2)^3 + 18(2)$
$A = 20$

For (8, 5), the area of the rectangle in part b) is 80 square units.
$A = -\frac{1}{8}x^3 + 18x$
$A = -\frac{1}{8}(8)^3 + 18(8)$
$A = 80$

17. $y = 36(x - 2)^2 + 6(x - 2) - 2$

18. Example: vertical stretches and horizontal stretches followed by reflections

C1 Step 1 They are reflections in the axes.
1: $y = x + 3$, 2: $y = -x - 3$, 3: $y = x - 3$
Step 2 They are vertical translations coupled with reflections. 1: $y = x^2 + 1$, 2: $y = x^2 - 1$, 3: $y = -x^2$, 4: $y = -x^2 - 1$

C2 a) The cost of making $b + 12$ bracelets, and it is a horizontal translation.

b) The cost of making b bracelets plus 12 more dollars, and it is a vertical translation.

c) Triple the cost of making b bracelets, and it is a vertical stretch.

d) The cost of making $\frac{b}{2}$ bracelets, and it is a horizontal stretch.

C3 $y = 2(x - 3)^2 + 1$; a vertical stretch by a factor of 2 and a translation of 3 units right and 1 unit up