# Function Operations

To combine two functions, $f(x)$ and $g(x)$, add or subtract as follows:

*Sum of Functions*

$h(x) = f(x) + g(x)$

$h(x) = (f + g)(x)$

*Difference of Functions*

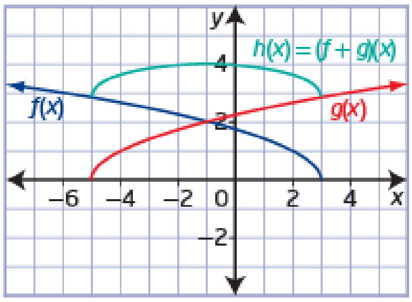
$h(x) = f(x) - g(x)$

$h(x) = (f - g)(x)$

## Key Ideas

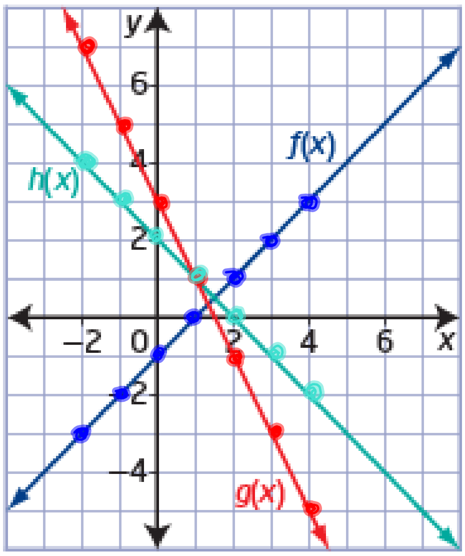
- You can add two functions, $f(x)$ and $g(x)$, to form the combined function $h(x) = (f + g)(x)$.
- You can subtract two functions, $f(x)$ and $g(x)$, to form the combined function $h(x) = (f - g)(x)$.
- The domain of the combined function formed by the sum or difference of two functions is the domain common to the individual functions. For example,

  Domain of $f(x)$: $\{x \mid x \leq 3, x \in \mathrm{R}\}$ or $(-\infty, 3]$
  Domain of $g(x)$: $\{x \mid x \geq -5, x \in \mathrm{R}\}$ or $[-5, \infty)$
  Domain of $h(x)$: $\{x \mid -5 \leq x \leq 3, x \in \mathrm{R}\}$ or $[-5, 3]$




- The range of a combined function can be determined using its graph.
- To sketch the graph of a sum or difference of two functions given their graphs, add or subtract the $y$-coordinates at each point.

**1.** Consider the graphs of the functions $f(x)$, $g(x)$, and $h(x)$.




**a)** Copy the table and use the graph of each function to complete the columns.

| x | f(x) | g(x) | h(x) |
|---|---|---|---|
| −2 | −3 | 7 | 4 |
| −1 | −2 | 5 | 3 |
| 0 | −1 | 3 | 2 |
| 1 | 0 | 1 | 1 |
| 2 | 1 | −1 | 0 |
| 3 | 2 | −3 | −1 |
| 4 | 3 | −5 | −2 |

**b)** What do you notice about the relationship between each value of $h(x)$ and the corresponding values of $f(x)$ and $g(x)$?

- We are adding $f(x)$ and $g(x)$ to get $h(x)$

$$h(x) = f(x) + g(x)$$
$$= (f+g)(x)$$

## Example 1

### Determine the Sum of Two Functions

add

Consider the functions $f(x) = 2x + 1$ and $g(x) = x^2$.

a) Determine the equation of the function $h(x) = (f + g)(x)$.

b) Sketch the graphs of $f(x)$, $g(x)$, and $h(x)$ on the same set of coordinate axes.

c) State the domain and range of $h(x)$.

d) Determine the values of $f(x)$, $g(x)$, and $h(x)$ when $x = 4$.

### Solution

a) Add $f(x)$ and $g(x)$ to determine the equation of the function $h(x) = (f + g)(x)$.

$h(x) = (f + g)(x)$
$h(x) = f(x) + g(x)$
$h(x) = 2x + 1 + x^2$
$h(x) = x^2 + 2x + 1$

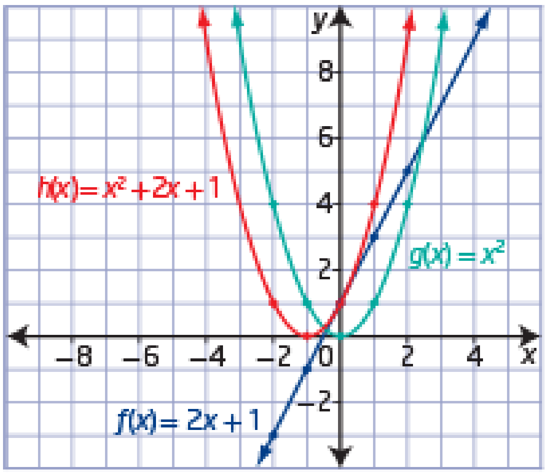
Is the function $(f + g)(x)$ the same as $(g + f)(x)$? Will this always be true?

yes → not the same if subtracting

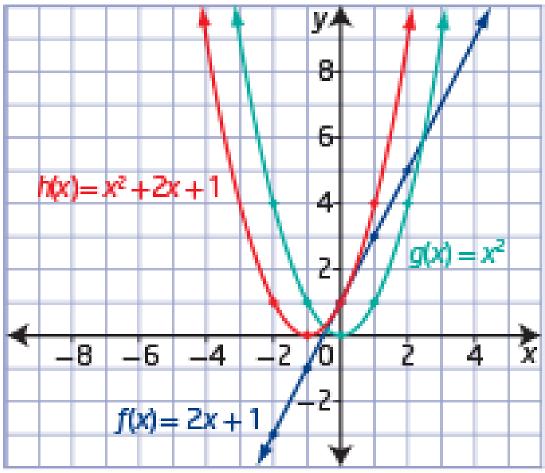
b) Method 1: Use Paper and Pencil

| $x$ | $f(x) = 2x + 1$ | $g(x) = x^2$ | $h(x) = x^2 + 2x + 1$ |
|---|---|---|---|
| −2 | −3 | 4 | 1 |
| −1 | −1 | 1 | 0 |
| 0 | 1 | 0 | 1 |
| 1 | 3 | 1 | 4 |
| 2 | 5 | 4 | 9 |

How could you use the values in the columns for $f(x)$ and $g(x)$ to determine the values in the column for $h(x)$?




How are the $y$-coordinates of points on the graph of $h(x)$ related to those on the graphs of $f(x)$ and $g(x)$?




How are the y-coordinates of points on the graph of $h(x)$ related to those on the graph of $f(x)$ and $g(x)$?

**c)** The function $f(x) = 2x + 1$ has domain $\{x \mid x \in \mathbb{R}\}$. or $(-\infty, \infty)$
The function $g(x) = x^2$ has domain $\{x \mid x \in \mathbb{R}\}$.
The function $h(x) = (f + g)(x)$ has domain $\{x \mid x \in \mathbb{R}\}$, which consists of all values that are in both the domain of $f(x)$ and the domain of $g(x)$.
The range of $h(x)$ is $\{y \mid y \geq 0, y \in \mathbb{R}\}$. or $[0, \infty)$

**d)** Substitute $x = 4$ into $f(x)$, $g(x)$, and $h(x)$.

| | | |
|---|---|---|
| $f(x) = 2x + 1$ | $g(x) = x^2$ | $h(x) = x^2 + 2x + 1$ |
| $f(4) = 2(4) + 1$ | $g(4) = 4^2$ | $h(4) = 4^2 + 2(4) + 1$ |
| $f(4) = 8 + 1$ | $g(4) = 16$ | $h(4) = 16 + 8 + 1$ |
| $f(4) = 9$ | | $h(4) = 25$ |

## Example 2

### Determine the Difference of Two Functions

subtract

Consider the functions $f(x) = \sqrt{x - 1}$ and $g(x) = x - 2$.

**a)** Determine the equation of the function $h(x) = (f - g)(x)$.

**b)** Sketch the graphs of $f(x)$, $g(x)$, and $h(x)$ on the same set of coordinate axes.

**c)** State the domain of $h(x)$.

**d)** Use the graph to approximate the range of $h(x)$.

## Solution

**a)** Subtract $g(x)$ from $f(x)$ to determine the equation of the function $h(x) = (f - g)(x)$.

$h(x) = (f - g)(x)$

$h(x) = f(x) - g(x)$

$h(x) = \sqrt{x - 1} - (x - 2)$

$h(x) = \sqrt{x - 1} - x + 2$

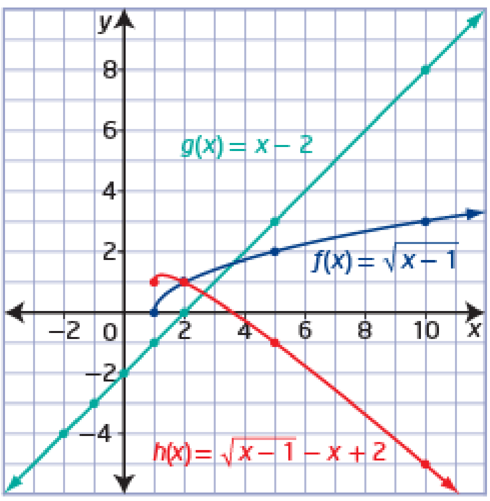
**b) Method 1: Use Paper and Pencil**

For the function $f(x) = \sqrt{x - 1}$, the value of the radicand must be greater than or equal to zero: $x - 1 \geq 0$ or $x \geq 1$.

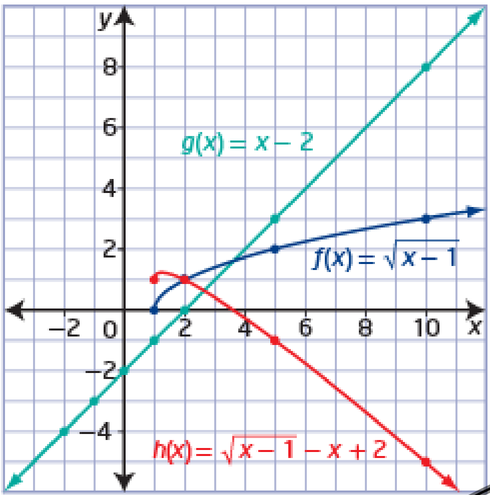
| $x$ | $f(x) = \sqrt{x - 1}$ | $g(x) = x - 2$ | $h(x) = \sqrt{x - 1} - x + 2$ |
|---|---|---|---|
| −2 | undefined | −4 | undefined |
| −1 | undefined | −3 | undefined |
| 0 | undefined | −2 | undefined |
| 1 | 0 | −1 | 1 |
| 2 | 1 | 0 | 1 |
| 5 | 2 | 3 | −1 |
| 10 | 3 | 8 | −5 |

Why is the function $h(x)$ undefined when $x < 1$?

How could you use the values in the columns for $f(x)$ and $g(x)$ to determine the values in the column for $h(x)$?



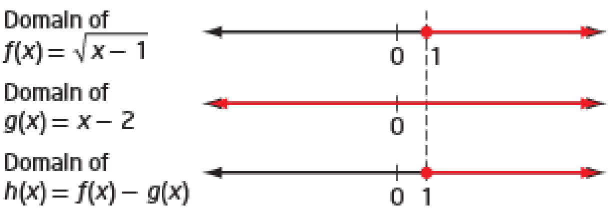

How could you use the $y$-coordinates of points on the graphs of $f(x)$ and $g(x)$ to create the graph of $h(x)$?




How could you use the y-coordinates of points on the graphs of $f(x)$ and $g(x)$ to create the graph of $h(x)$?

radicand ≥ 0
$x-1 \geq 0$
$x \geq 1$

**c)** The function $f(x) = \sqrt{x-1}$ has domain $\{x \mid x \geq 1, x \in R\}$.
The function $g(x) = x - 2$ has domain $\{x \mid x \in R\}$.
The function $h(x) = (f - g)(x)$ has domain $\{x \mid x \geq 1, x \in R\}$, which consists of all values that are in both the domain of $f(x)$ and the domain of $g(x)$.




What values of $x$ belong to the domains of both $f(x)$ and $g(x)$?

**d)** From the graph, the range of $h(x)$ appears to be approximately $\{y \mid y \leq 1.2, y \in R\}$.

How can you use a graphing calculator to verify the range?

# Homework

finish #1-11 on page 483-484

## 10.1 Sums and Differences of Functions, pages 483 to 487

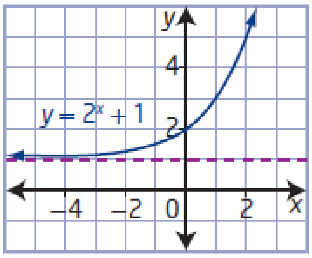
**1.** **a)** $h(x) = |x - 3| + 4$ **b)** $h(x) = 2x - 3$
**c)** $h(x) = 2x^2 + 3x + 2$ **d)** $h(x) = x^2 + 5x + 4$

**2.** **a)** $h(x) = 5x + 2$ **b)** $h(x) = -3x^2 - 4x + 9$
**c)** $h(x) = -x^2 - 3x + 12$ **d)** $h(x) = \cos x - 4$

**3.** **a)** $h(x) = x^2 - 6x + 1$; $h(2) = -7$
**b)** $m(x) = -x^2 - 6x + 1$; $m(1) = -6$
**c)** $p(x) = x^2 + 6x - 1$; $p(1) = 6$

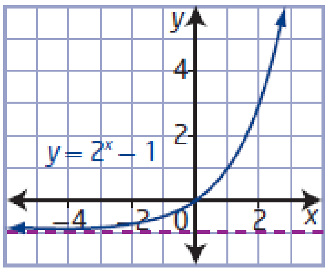
**4.** **a)** $y = 3x^2 + 2 + \sqrt{x + 4}$; domain $\{x \mid x \geq -4, x \in \mathrm{R}\}$
**b)** $y = 4x - 2 - \sqrt{x + 4}$; domain $\{x \mid x \geq -4, x \in \mathrm{R}\}$
**c)** $y = \sqrt{x + 4} - 4x + 2$; domain $\{x \mid x \geq -4, x \in \mathrm{R}\}$
**d)** $y = 3x^2 + 4x$; domain $\{x \mid x \in \mathrm{R}\}$

**5.** **a)**



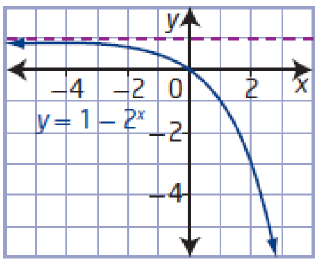

domain $\{x \mid x \in \mathrm{R}\}$, range $\{y \mid y > 1, y \in \mathrm{R}\}$

**b)**




domain $\{x \mid x \in \mathrm{R}\}$, range $\{y \mid y > -1, y \in \mathrm{R}\}$

**c)**

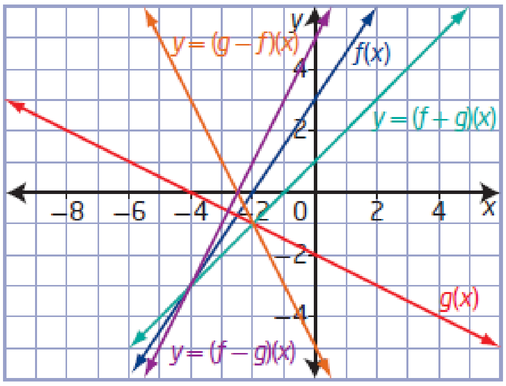



domain $\{x \mid x \in \mathrm{R}\}$, range $\{y \mid y < 1, y \in \mathrm{R}\}$

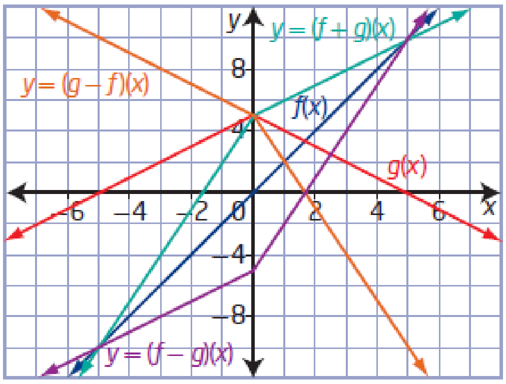
**6.** **a)** 8 **b)** 6 **c)** 7
**d)** not in the domain

**7.** **a)** B **b)** C **c)** A

**8. a)**




**b)**




**9. a)** $y = 3x^2 + 11x + 1$ **b)** $y = 3x^2 - 3x + 3$

**c)** $y = 3x^2 + 3x + 1$ **d)** $y = 3x^2 - 11x + 3$

**10. a)** $g(x) = x^2$ **b)** $g(x) = \sqrt{x + 7}$

**c)** $g(x) = -3x + 1$ **d)** $g(x) = 3x^2 - x - 4$

**11. a)** $g(x) = x^2 - 1$ **b)** $g(x) = -\sqrt{x - 4}$

**c)** $g(x) = 8x - 9$ **d)** $g(x) = 2x^2 - 11x - 6$