

Combining Functions

- This is summarized in the following table:

Algebra of Functions Let f and g be functions with domains A and B . Then the functions $f + g$, $f - g$, fg , and f/g are defined as follows:

$$(f + g)(x) = f(x) + g(x) \quad \text{domain} = A \cap B$$

$$(f - g)(x) = f(x) - g(x) \quad \text{domain} = A \cap B$$

$$(fg)(x) = f(x)g(x) \quad \text{domain} = A \cap B$$

$$\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)} \quad \text{domain} = \{x \in A \cap B \mid g(x) \neq 0\}$$

Intersection of Overlap

Example

■ If $f(x) = \sqrt{x}$ and $g(x) = \sqrt{4-x^2}$, find the functions $f+g$, $f-g$, fg , and f/g .

**Also examine the domain of each of these new functions

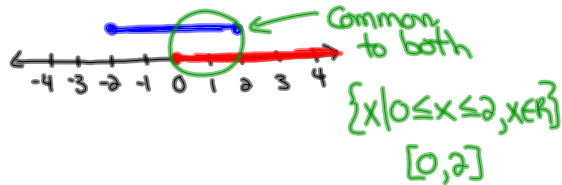
$$f(x) = \sqrt{x} \leftarrow \text{radicand} \geq 0 \quad \left| \quad g(x) = \sqrt{4-x^2} \leftarrow \text{radicand} \geq 0$$

$$x \geq 0 \quad \left| \quad \begin{array}{l} \text{factor} \\ 4-x^2 \geq 0 \\ (2-x)(2+x) \geq 0 \end{array} \quad \leftarrow \text{positive } y \text{ values}$$

$$D: \{x \mid x \geq 0, x \in \mathbb{R}\} \bullet$$



• $D: \{x \mid -2 \leq x \leq 2, x \in \mathbb{R}\}$



a) $f(x) + g(x)$
 $\sqrt{x} + \sqrt{4-x^2}$
 $\{x \mid 0 \leq x \leq 2, x \in \mathbb{R}\}$

b) $f(x) - g(x)$
 $\sqrt{x} - \sqrt{4-x^2}$
 $\{x \mid 0 \leq x \leq 2, x \in \mathbb{R}\}$

c) $f(x) \cdot g(x)$
 $\sqrt{x} \cdot \sqrt{4-x^2}$
 $\sqrt{x(4-x^2)}$
 $\sqrt{4x-x^3}$
 $\{x \mid 0 \leq x \leq 2, x \in \mathbb{R}\}$

Ex $\sqrt{5} \cdot \sqrt{3} = \sqrt{15}$
 $\sqrt{a} \cdot \sqrt{b} = \sqrt{ab}$

d) $\frac{f(x)}{g(x)}$
 $\frac{\sqrt{x}}{\sqrt{4-x^2}} \cdot \frac{(\sqrt{4-x^2})}{(\sqrt{4-x^2})}$

$\frac{\sqrt{4x-x^3}}{4-x^2}$ ← Non permissible values

$4-x^2 \neq 0$
 $4 \neq x^2$
 $\pm 2 \neq x$

$\{x \mid 0 \leq x < 2, x \neq \pm 2, x \in \mathbb{R}\}$

Compositions of Functions

(combining functions in a different way)

When the input in a function is another function, the result is called a **composite function**. If

$$f(x) = 3x + 2 \text{ and } g(x) = 4x - 5$$

then $f[g(x)]$ is a composite function. The statement $f[g(x)]$ is read "f of g of x" or "the composition of f with g." $f[g(x)]$ can also be written as

$$(f \circ g)(x) \text{ or } f \circ g(x) = f[g(x)]$$

The symbol between f and g is a small open circle. When replacing one function with another, be very careful to get the order correct because compositions of functions are not necessarily commutative (as you'll see).

Example 1

Evaluate a Composite Function

If $f(x) = 4x$, $g(x) = x + 6$, and $h(x) = x^2$, determine each value.

- a) $f(g(3))$
- b) $g(h(-2))$
- c) $h(h(2))$

Method 1: Determine the Value of the Inner Function and Then Substitute

$$a) f(g(3))$$

$$g(3) = (3) + 6 = 9$$

$$f(9) = 4(9) = \boxed{36}$$

$$b) g(h(-2))$$

$$h(-2) = (-2)^2 = 4$$

$$g(4) = 4 + 6 = \boxed{10}$$

$$c) h(h(2))$$

$$h(2) = (2)^2 = 4$$

$$h(4) = (4)^2 = \boxed{16}$$

Example 2

If $f(x) = 3x^2 + 2x + 1$ and $g(x) = 4x - 5$, find each of the following:

1. $f[g(x)]$

2. $g[f(x)]$

$$\begin{aligned}
 \textcircled{1} \quad f(4x-5) &= 3(4x-5)^2 + 2(4x-5) + 1 \\
 &= 3(16x^2 - 40x + 25) + 8x - 10 + 1 \\
 &= 48x^2 - 120x + 75 + 8x - 10 + 1 \\
 &= 48x^2 - 112x + 66 \\
 &= 2(24x^2 - 56x + 33)
 \end{aligned}$$

$$\begin{aligned}
 \textcircled{2} \quad g(3x^2 + 2x + 1) &= 4(3x^2 + 2x + 1) - 5 \\
 &= 12x^2 + 8x + 4 - 5 \\
 &= 12x^2 + 8x - 1
 \end{aligned}$$

Key Ideas

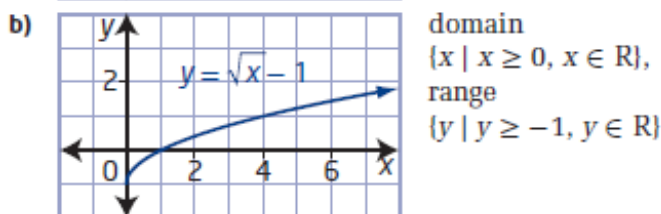
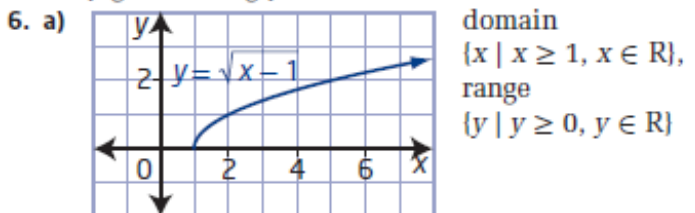
- Two functions, $f(x)$ and $g(x)$, can be combined using composition to produce two new functions, $f(g(x))$ and $g(f(x))$.
- To evaluate a composite function, $f(g(x))$, at a specific value, substitute the value into the equation for $g(x)$ and then substitute the result into $f(x)$ and evaluate, or determine the composite function first and then evaluate for the value of x .
- To determine the equation of a composite function, substitute the second function into the first as read from left to right. To compose $f(g(x))$, substitute the equation of $g(x)$ into the equation of $f(x)$.
- The domain of $f(g(x))$ is the set of all values of x in the domain of g for which $g(x)$ is in the domain of f . Restrictions on the inner function as well as the composite function must be considered.

Homework

#1-10 on page 507 (omit #6)

10.3 Composite Functions, pages 507 to 509

1. a) 3 b) 0 c) 2 d) -1
2. a) 2 b) 2 c) -4 d) -5
3. a) 10 b) -8 c) -2 d) 28
4. a) $f(g(a)) = 3a^2 + 1$ b) $g(f(a)) = 9a^2 + 24a + 15$
 c) $f(g(x)) = 3x^2 + 1$ d) $g(f(x)) = 9x^2 + 24x + 15$
 e) $f(f(x)) = 9x + 16$ f) $g(g(x)) = x^4 - 2x^2$
5. a) $f(g(x)) = x^4 + 2x^3 + 2x^2 + x$,
 $g(f(x)) = x^4 + 2x^3 + 2x^2 + x$
 b) $f(g(x)) = \sqrt{x^4 + 2}$, $g(f(x)) = x^2 + 2$
 c) $f(g(x)) = x^2$, $g(f(x)) = x^2$



7. a) $g(x) = 2x - 5$ b) $g(x) = 5x + 1$
8. Christine is right. Ron forgot to replace all x 's with the other function in the first step.
9. Yes. $k(j(x)) = j(k(x)) = x^6$; using the power law:
 $2(3) = 6$ and $3(2) = 6$.
10. No. $s(t(x)) = x^2 - 6x + 10$ and $t(s(x)) = x^2 - 2$.
11. a) $W(C(t)) = 3\sqrt{100 + 35t}$
 b) domain $\{t \mid t \geq 0, t \in \mathbb{R}\}$, range $\{W \mid W \geq 30, W \in \mathbb{W}\}$