Combining Functions

Example

- If $f(x)=\sqrt{x}$ and $g(x)=\sqrt{4-x^2}$, find the functions $f+g$, $f-g$, fg, and f/g.

**Also examine the domain of each of these new functions

$f(x)=\sqrt{x}$ ← radicand ≥ 0

$x \geq 0$

D: $\{x \mid x\geq 0, x\in R\}$

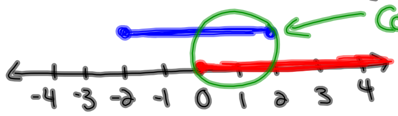
$g(x)=\sqrt{4-x^2}$ ← radicand ≥ 0

factor

$4-x^2 \geq 0$ ← positive y values

$(2-x)(2+x)\geq 0$

D: $\{x \mid -2\leq x\leq 2, x\in R\}$



← Common to both

$\{x \mid 0\leq x\leq 2, x\in R\}$ $[0,2]$

a) $f(x)+g(x)$

$\sqrt{x}+\sqrt{4-x^2}$

$\{x \mid 0\leq x\leq 2, x\in R\}$

b) $f(x)-g(x)$

$\sqrt{x}-\sqrt{4-x^2}$

$\{x \mid 0\leq x\leq 2, x\in R\}$

c) $f(x)\cdot g(x)$

$\sqrt{x}\cdot\sqrt{4-x^2}$

$\sqrt{x(4-x^2)}$

$\sqrt{4x-x^3}$

$\{x \mid 0\leq x\leq 2, x\in R\}$

Ex

$\sqrt{2}\cdot\sqrt{3}=\sqrt{6}$

$\sqrt{a}\cdot\sqrt{b}=\sqrt{ab}$

d) $\dfrac{f(x)}{g(x)}$

$\dfrac{\sqrt{x}}{\sqrt{4-x^2}}\cdot\dfrac{(\sqrt{4-x^2})}{(\sqrt{4-x^2})}$

$\dfrac{\sqrt{4x-x^3}}{4-x^2}$ ← Non permissable values

$4-x^2\neq 0$

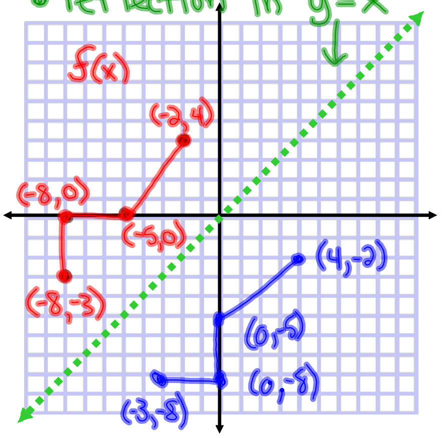
$4\neq x^2$

$\pm 2\neq x$

$\{x \mid 0\leq x<2, x\in R\}$

Inverse Relations

- $(x,y) \to (y,x)$
- reflection in $y=x$



a) Determine the Inverse

b) Is the Inverse a function?

No becaus f(x) does not pass the horizontal line test

a) Determine the Inverse of $f(x)=3\sqrt{x-5}+8$

① $y=3\sqrt{x-5}+8$

② $x=3\sqrt{y-5}+8$

③ $x-8=3\sqrt{y-5}$

$\frac{1}{3}(x-8)=\sqrt{y-5}$

$\frac{1}{9}(x-8)^2=y-5$

$\frac{1}{9}(x-8)^2+5=y$

④ $\boxed{f^{-1}(x)=\frac{1}{9}(x-8)^2+5}$

b) State the domain and range of $f(x)$ **and** $f^{-1}(x)$

$f(x)=3\sqrt{x-5}+8$	$f^{-1}(x)=\frac{1}{9}(x-8)^2+5$
D: $x\in[5,\infty)$	D: $x\in[8,\infty)$
R: $y\in[8,\infty)$	R: $y\in[5,\infty)$

Composite Functions

Given the three functions....

$f(x) = 1 - x$ $\quad g(x) = \sqrt{x+1}$ $\quad h(x) = x^2 + 5$

Evaluate each of the following:

1. $(f \circ g)(3) = f(g(3))$
2. $(g \circ h)(0) = g(h(0))$
3. $(g \circ g \circ f)(-7)$

$g(g(f(-7)))$

① $g(3) = \sqrt{3+1} = \sqrt{4} = 2$

$f(2) = 1 - 2 = \boxed{-1}$

② $h(0) = (0)^2 + 5 = 5$

$g(5) = \sqrt{5+1} = \sqrt{6}$

③ $f(-7) = 1 - (-7) = 8$

$g(8) = \sqrt{8+1} = \sqrt{9} = 3$

$g(3) = \sqrt{3+1} = \sqrt{4} = 2$

Homework

Chapter 1 Review

#1-6 on page 35 Workbook

Chapter 10 Review

#1-10 on page 356 (omit #4, 8, 9) Workbook

Unit Test:

- Function notation
- combinations:
- compositions:
- catalogue of essential functions
- transformations
  ↳ Reflections, Stretches, Translations
  ↳ $y = af(b(x-h))+k$

• vertical translation Shift up/down

• vertical stretch by a factor of a
• if $a<0$ reflect in x-axis

• horizontal stretch by a factor of $\frac{1}{|b|}$
• if $b<0$ reflect in y-axis

• horizontal translation shift left/right

- Mapping:
  $(x,y) \longrightarrow \left(\frac{1}{b}x+h, ay+k\right)$

⇒ Inverse Functions

- Switch "x" & "y" (Domain & Range)
- Sketch Inverses from a given graph (Reflects in line $y=x$)
- one-one function (Horizontal line)
- Switch to inverse algebraically

i.e. $f(x) = x + 7$

$x = y + 7$

$x - 7 = y$

$f^{-1}(x) = x - 7$