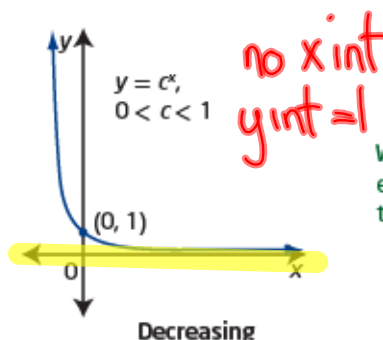
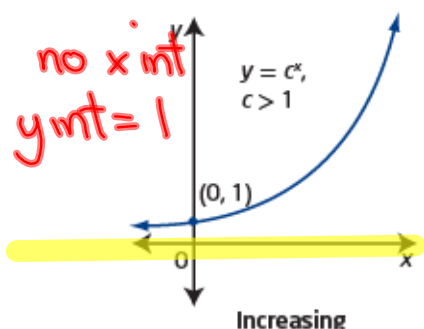


Exponential Functions

The graph of an **exponential function**, such as $y = c^x$, is increasing for $c > 1$, decreasing for $0 < c < 1$, and neither increasing nor decreasing for $c = 1$. From the graph, you can determine characteristics such as domain and range, any intercepts, and any **asymptotes**.

exponential function

- a function of the form $y = c^x$, where c is a constant ($c > 0$) and x is a variable



Why is the definition of an exponential function restricted to positive values of c ?

Did You Know?

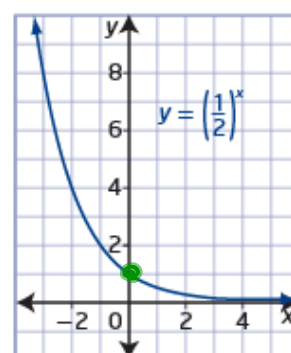
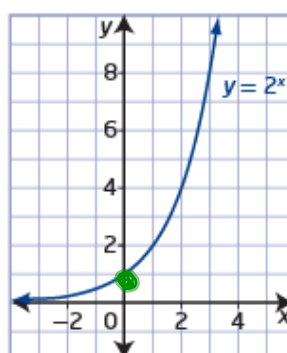
Any letter can be used to represent the base in an exponential function. Some other common forms are $y = a^x$ and $y = b^x$. In this chapter, you will use the letter c . This is to avoid any confusion with the transformation parameters, a , b , h , and k , that you will apply in Section 7.2.

* Any base raised to the power of 0 is equal to 1

① $5^0 = 1$ ③ $18^0 = 1$ ⑤ $(-18)^0 = 1$
 ② $7^0 = 1$ ④ $-18^0 = -1$ ⑥ $(\frac{1}{2})^0 = 1$

Key Ideas

- An exponential function of the form $y = c^x$, $c > 0$,
 - is increasing for $c > 1$
 - is decreasing for $0 < c < 1$
 - is neither increasing nor decreasing for $c = 1$
 - has a domain of $\{x \mid x \in \mathbb{R}\}$
 - has a range of $\{y \mid y > 0, y \in \mathbb{R}\}$
 - has a y-intercept of 1
 - has no x-intercept
 - has a horizontal asymptote at $y = 0$



Example 1

Analyse the Graph of an Exponential Function

Graph each exponential function. Then identify the following:

- the domain and range
- the x -intercept and y -intercept, if they exist
- whether the graph represents an increasing or a decreasing function
- the equation of the horizontal asymptote

a) $y = 4^x$

b) $f(x) = \left(\frac{1}{2}\right)^x$

Solution

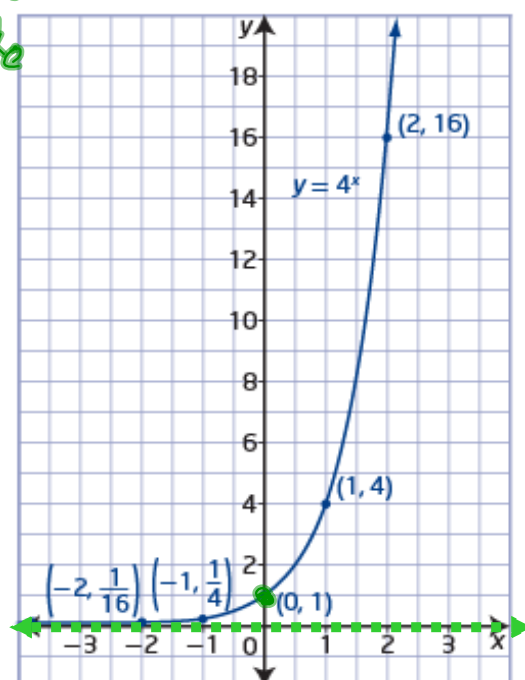
a) Method 1: Use Paper and Pencil

Use a table of values to graph the function.

Select integral values of x that make it easy to calculate the corresponding values of y for $y = 4^x$.

x	y
-2	$\frac{1}{16}$
-1	$\frac{1}{4}$
0	1
1	4
2	16

$y = 4^{-2}$ ← take reciprocal of base to make exponent positive
 $y = \left(\frac{1}{4}\right)^2 = \frac{(1)^2}{(4)^2} = \frac{1}{16}$



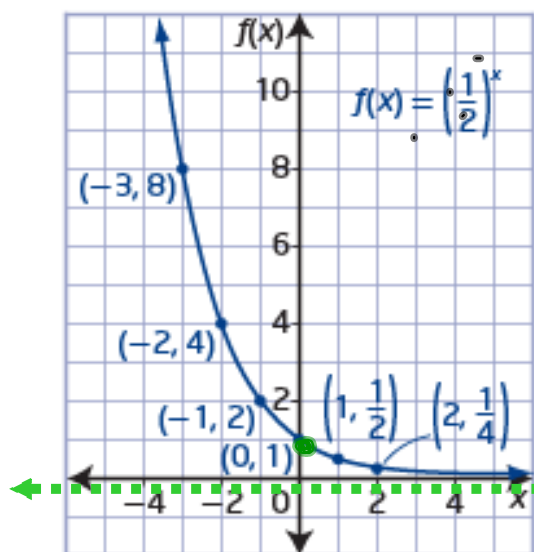
- Domain: $\{x | x \in \mathbb{R}\}$
- Range: $\{y | y > 0, y \in \mathbb{R}\}$
- x int: None
- y int: $y = 1$ or $(0, 1)$
- The function is increasing
- Horizontal Asymptote: $y = 0$

b) Method 1: Use Paper and Pencil

Use a table of values to graph the function.

Select integral values of x that make it easy to calculate the corresponding values of y for $f(x) = \left(\frac{1}{2}\right)^x$.

x	$f(x)$
-3	8
-2	4
-1	2
0	1
1	$\frac{1}{2}$
2	$\frac{1}{4}$

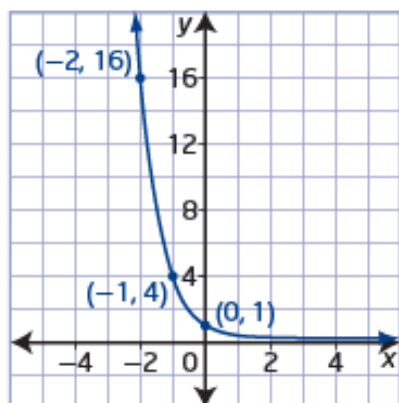


- Domain: $\{x | x \in \mathbb{R}\}$
- Range: $\{y | y > 0, y \in \mathbb{R}\}$
- x int: None
- y int: $y=1$ or $(0,1)$
- The function is *decreasing*
- Horizontal Asymptote: $y=0$

Example 2

Write the Exponential Function Given Its Graph

What function of the form $y = c^x$ can be used to describe the graph shown?



It is a decreasing $0 < c < 1$

Solution

Look for a pattern in the ordered pairs from the graph.

x	y
-2	16
-1	4
0	1

$$\frac{4}{16} = \frac{1}{4}$$

As the value of x increases by 1 unit, the value of y decreases by a factor of $\frac{1}{4}$. Therefore, for this function, $c = \frac{1}{4}$.

Choose a point other than $(0, 1)$ to substitute into the function $y = \left(\frac{1}{4}\right)^x$ to verify that the function is correct. Try the point $(-2, 16)$.

Check:

Left Side

$$y = 16$$

Right Side

$$\begin{aligned} &\left(\frac{1}{4}\right)^x \\ &\left(\frac{1}{4}\right)^{-2} \\ &(4)^2 \\ &16 \end{aligned}$$

Why should you not use the point $(0, 1)$ to verify that the function is correct?

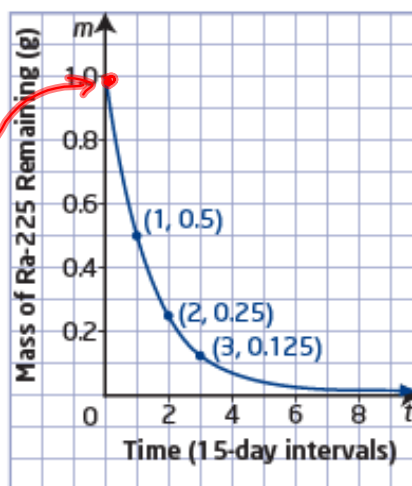
The point $(0, 1)$ is common to all exponential functions of the form $y = c^x$.

Example 3

Application of an Exponential Function

A radioactive sample of radium (Ra-225) has a **half-life** of 15 days. The mass, m , in grams, of Ra-225 remaining over time, t , in 15-day intervals, can be modelled using the exponential graph shown.

- What is the initial mass of Ra-225 in the sample? What value does the mass of Ra-225 remaining approach as time passes? 1g
- What are the domain and range of this function?
- Write the exponential decay model that relates the mass of Ra-225 remaining to time, in 15-day intervals.
- Estimate how many days it would take for Ra-225 to decay to $\frac{1}{30}$ of its original mass.



b) Domain: $\{t \mid t \geq 0, t \in \mathbb{R}\}$

Range $\{m \mid 0 < m \leq 1, m \in \mathbb{R}\}$ d)

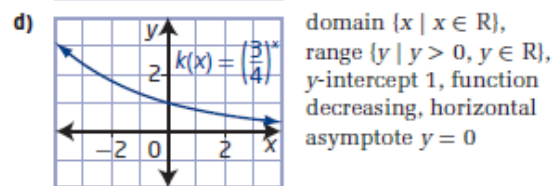
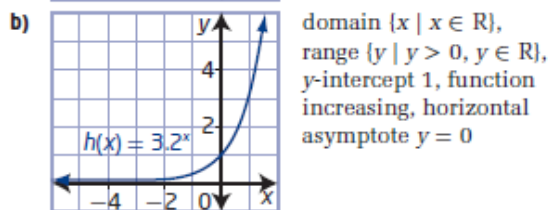
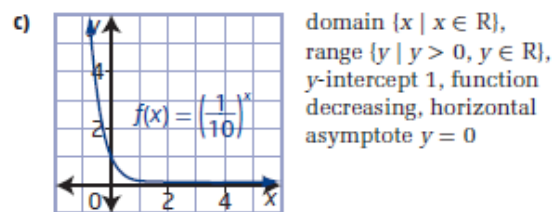
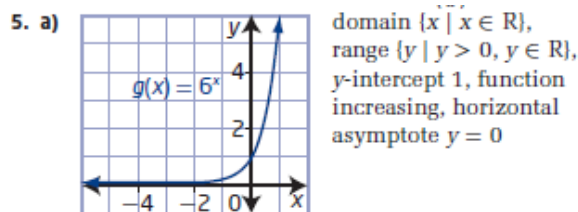
c) $m = \left(\frac{1}{2}\right)^t$

Homework

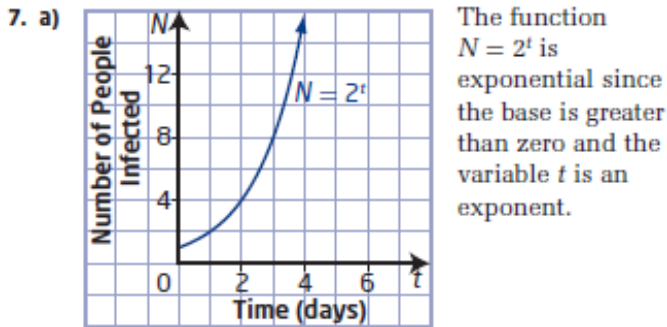
#1-8 on page 343

7.1 Characteristics of Exponential Functions, pages 342 to 345

1. a) No, the variable is not the exponent.
 b) Yes, the base is greater than 0 and the variable is the exponent.
 c) No, the variable is not the exponent.
 d) Yes, the base is greater than 0 and the variable is the exponent.
2. a) $f(x) = 4^x$ b) $g(x) = \left(\frac{1}{4}\right)^x$
 c) $x = 0$, which is the y-intercept
3. a) B b) C c) A
4. a) $f(x) = 3^x$ b) $f(x) = \left(\frac{1}{5}\right)^x$

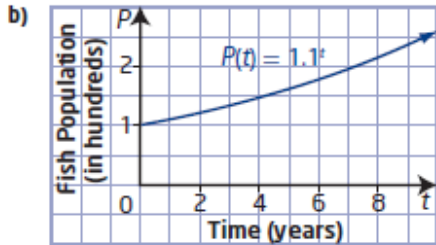


- 6. a) $c > 1$; number of bacteria increases over time
- b) $0 < c < 1$; amount of actinium-225 decreases over time
- c) $0 < c < 1$; amount of light decreases with depth
- d) $c > 1$; number of insects increases over time



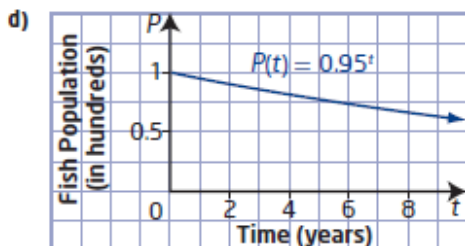
- b) i) 1 person ii) 2 people
- iii) 16 people iv) 1024 people

- 8. a) If the population increases by 10% each year, the population becomes 110% of the previous year's population. So, the growth rate is 110% or 1.1 written as a decimal.



domain $\{t \mid t \geq 0, t \in \mathbb{R}\}$ and range $\{P \mid P \geq 100, P \in \mathbb{R}\}$

- c) The base of the exponent would become $100\% - 5\%$ or 95%, written as 0.95 in decimal form.



domain $\{t \mid t \geq 0, t \in \mathbb{R}\}$ and range $\{P \mid 0 < P \leq 100, P \in \mathbb{R}\}$