Questions from Homework

② c) $\left(\frac{1}{\underline{2}}\right)^{2x}$ and $\left(\frac{1}{\underline{4}}\right)^{x-1}$

$$(2^{-1})^{2x} \qquad (2^{-2})^{x-1}$$

$$2^{-2x} \qquad 2^{-2x+2}$$

③ a) $(\sqrt{16})^2$

$$(16^{1/2})^2$$

$$16 \longrightarrow \frac{\log 16}{\log 4} = 2 \Big\} \frac{\ln 16}{\ln 4} = 2$$

$$4^2$$

c) $\sqrt{16}(\sqrt[3]{64})^2$

$$(16^{1/2})(64^{1/3})^2$$

$$(\underline{16}^{1/2})(\underline{64}^{2/3})$$

$$((4^2)^{1/2})((4^3)^{2/3})$$

$$(4^1)(4^2)$$

$$4^3$$

$$4(4)^2$$

$$4(4^2)$$

$$4^3$$

④ d) $\underline{16}^{2k-3} = \underline{32}^{k+3}$

$$(2^4)^{2k-3} = (2^5)^{k+3}$$

$$\cancel{2}^{8k-12} = \cancel{2}^{5k+15}$$

$$8k-12 = 5k+15$$

$$3k = 27$$

$$k = 9$$

LS	RS
16^{15}	32^{12}
$(2^4)^{15}$	$(2^5)^{12}$
2^{60}	2^{60}

$k=9$ is a solution

Understanding Logarithms

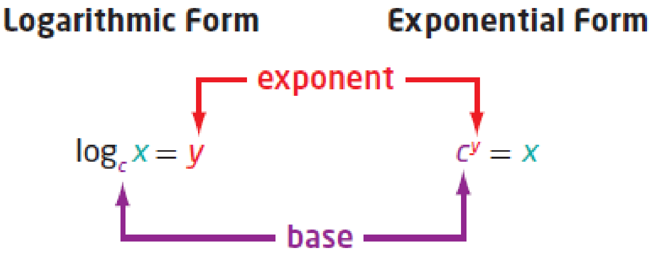
$(x,y) \rightarrow (y,x)$

reflection in the line $y=x$

Focus on...

- demonstrating that a logarithmic function is the inverse of an exponential function
- sketching the graph of $y = \log_c x$, $c > 0$, $c \neq 1$
- determining the characteristics of the graph of $y = \log_c x$, $c > 0$, $c \neq 1$
- explaining the relationship between logarithms and exponents
- expressing a logarithmic function as an exponential function and vice versa
- evaluating logarithms using a variety of methods

For the exponential function $y = c^x$, the inverse is $x = c^y$. This inverse is also a function and is called a **logarithmic function**. It is written as $y = \log_c x$, where c is a positive number other than 1.



Since our number system is based on powers of 10, **logarithms** with base 10 are widely used and are called **common logarithms**. When you write a common logarithm, you do not need to write the base. For example, log 3 means $\log_{10} 3$.

logarithmic function

- a function of the form $y = \log_c x$, where $c > 0$ and $c \neq 1$, that is the inverse of the exponential function $y = c^x$

logarithm

- an exponent
- in $x = c^y$, y is called the logarithm to base c of x

common logarithm

- a logarithm with base 10

Write each of the following in logarithmic form

a) $\overset{\text{ans.}}{32} = \underset{\text{base}}{2}^{\overset{\text{exp.}}{5}}$

$\log_2 32 = 5$

b) $2^{-5} = \dfrac{1}{32}$

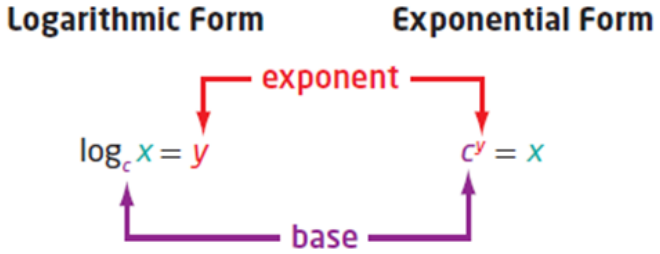
$\log_2\left(\frac{1}{32}\right) = -5$

c) $x = 10^y$

$\log_{10} x = y$

or $\log x = y$

common logarithm



Write each of the following in exponential form

a) $\log_{\underset{\text{base}}{4}} \overset{\text{ans}}{16} = \overset{\text{exp}}{2}$

$4^2 = 16$

b) $\log_2\left(\dfrac{1}{32}\right) = -5$

$2^{-5} = \dfrac{1}{32}$

c) $\log 65 = 1.8129$

$10^{1.8129} = 65$

Example 1

Evaluating a Logarithm

Evaluate.

a) $\log_7 49$ **b)** $\log_6 1$ **c)** $\log 0.001$ **d)** $\log_2 \sqrt{8}$

a) Let $x = \log_7 49$

$x = \log_7 49$

$7^x = 49$

$7^x = 7^2$

$x = 2$

$\log_7 49 = 2$

b) $x = \log_6 1$

$6^x = 1$

$6^x = 6^0$

$x = 0$

$\log_6 1 = 0$

c) $x = \log 0.001$

$10^x = 0.001$

$10^x = 10^{-3}$

$x = -3$

$\log 0.001 = -3$

d) $x = \log_2 \sqrt{8}$

$2^x = \sqrt{8}$

$2^x = \underline{\underline{8}}^{1/2}$

$2^x = (2^3)^{1/2}$

$2^x = 2^{3/2}$

$x = 3/2$

$\log_2 \sqrt{8} = \frac{3}{2}$

Example 2

Determine an Unknown in an Expression in Logarithmic Form

Determine the value of x.

a) $\log_5 x = -3$

b) $\log_x 36 = 2$

c) $\log_{64} x = \frac{2}{3}$

a) $\log_5 x = -3$

$5^{-3} = x$

$\left(\frac{1}{5}\right)^3 = x$

$\boxed{\frac{1}{125} = x}$

b) $\log_x 36 = 2$

$x^2 = 36$

$x = \pm 6$

Choose $+6$ because the base of a logarithm must be greater than 0

Example 3

Graph the Inverse of an Exponential Function

a) State the inverse of $f(x) = 3^x$.

b) Sketch the graph of the inverse. Identify the following characteristics of the inverse graph:

- the domain and range
- the x-intercept, if it exists
- the y-intercept, if it exists
- the equations of any asymptotes

Solution

a) The inverse of $y = f(x) = 3^x$ is or, expressed in logarithmic form, . Since the inverse is a function, it can be written in function notation as

How do you know that $y = \log_3 x$ is a function?

b) Set up tables of values for both the exponential function, $f(x)$, and its inverse, $f^{-1}(x)$. Plot the points and join them with a smooth curve.

$f(x) = 3^x$	
x	y
−3	
−2	
−1	
0	
1	
2	
3	

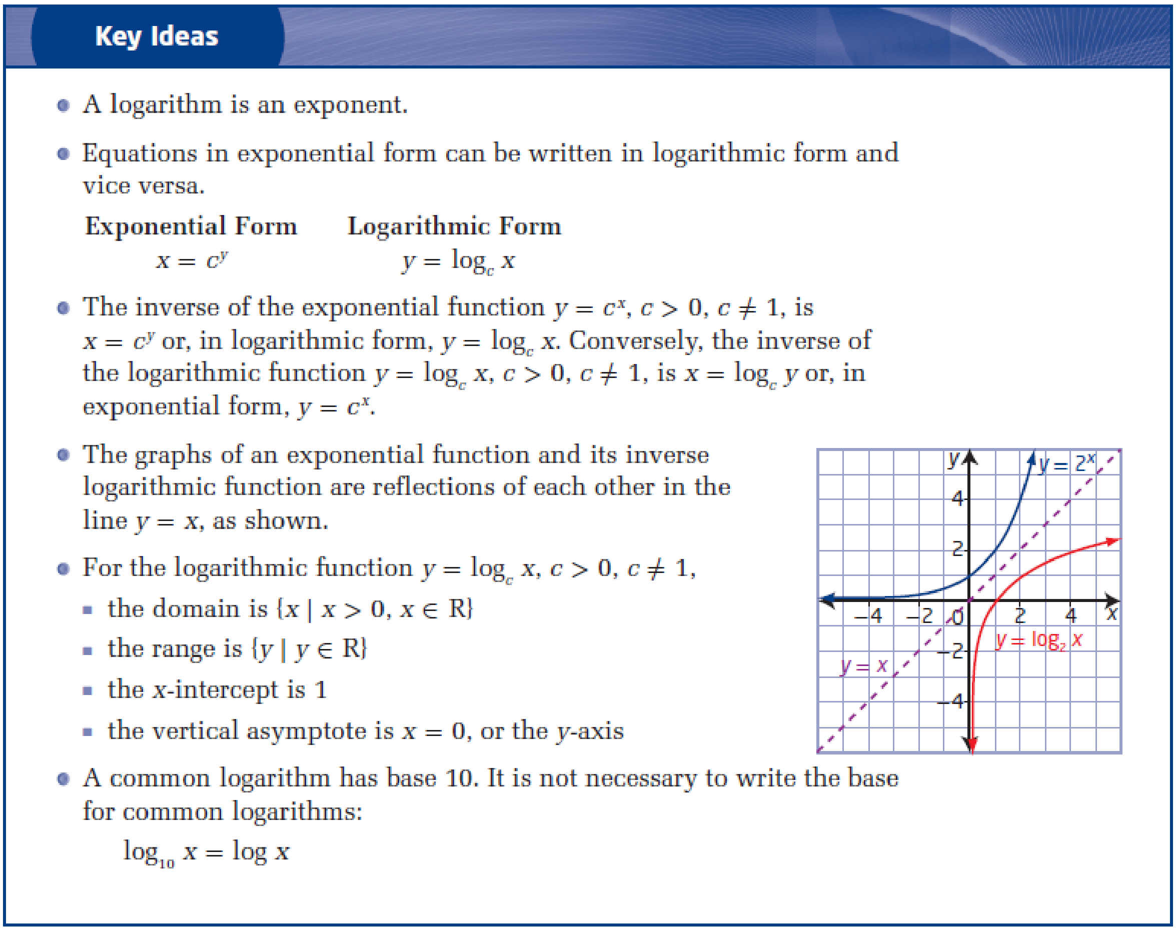
$f^{-1}(x) = \log_3 x$	
x	y

[redacted]

The graph of the inverse, $f^{-1}(x) = \log_3 x$, is a reflection of the graph of $f(x) = 3^x$ about the line $y = x$. For $f^{-1}(x) = \log_3 x$,

- the domain is and the range is
- the x-intercept is
- there is no y-intercept
- the vertical asymptote, the axis, has equation there is no asymptote

How do the characteristics of $f^{-1}(x) = \log_3 x$ compare to the characteristics of $f(x) = 3^x$?



Key Ideas

- A logarithm is an exponent.
- Equations in exponential form can be written in logarithmic form and vice versa.

Exponential Form	Logarithmic Form
$x = c^y$	$y = \log_c x$

- The inverse of the exponential function $y = c^x$, $c > 0$, $c \neq 1$, is $x = c^y$ or, in logarithmic form, $y = \log_c x$. Conversely, the inverse of the logarithmic function $y = \log_c x$, $c > 0$, $c \neq 1$, is $x = \log_c y$ or, in exponential form, $y = c^x$.
- The graphs of an exponential function and its inverse logarithmic function are reflections of each other in the line $y = x$, as shown.
- For the logarithmic function $y = \log_c x$, $c > 0$, $c \neq 1$,
  - the domain is $\{x \mid x > 0, x \in \mathrm{R}\}$
  - the range is $\{y \mid y \in \mathrm{R}\}$
  - the x-intercept is 1
  - the vertical asymptote is $x = 0$, or the y-axis
- A common logarithm has base 10. It is not necessary to write the base for common logarithms:

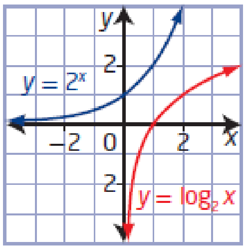
  $\log_{10} x = \log x$

Homework

#1-5, 8, 10, 12, 13, 17 on page 380

8.1 Understanding Logarithms, pages 380 to 382

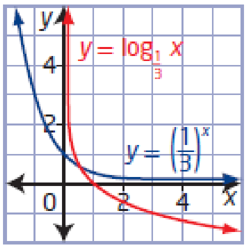
1. a) i)



ii) $y = \log_2 x$

iii) domain $\{x \mid x > 0, x \in \mathbb{R}\}$, range $\{y \mid y \in \mathbb{R}\}$, x-intercept 1, no y-intercept, vertical asymptote $x = 0$

b) i)



ii) $y = \log_{\frac{1}{3}} x$

iii) domain $\{x \mid x > 0, x \in \mathbb{R}\}$, range $\{y \mid y \in \mathbb{R}\}$, x-intercept 1, no y-intercept, vertical asymptote $x = 0$

2. a) $\log_{12} 144 = 2$ **b)** $\log_8 2 = \frac{1}{3}$

c) $\log_{10} 0.000\,01 = -5$ **d)** $\log_7 (y + 3) = 2x$

3. a) $5^2 = 25$ **b)** $8^{\frac{2}{3}} = 4$

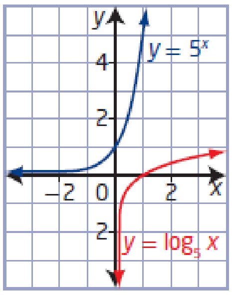
c) $10^6 = 1\,000\,000$ **d)** $11^y = x + 3$

4. a) 3 **b)** 0 **c)** $\frac{1}{3}$ **d)** −3

5. $a = 4$; $b = 5$

8. a) $y = \log_5 x$

b)



domain $\{x \mid x > 0, x \in \mathbb{R}\}$, range $\{y \mid y \in \mathbb{R}\}$, x-intercept 1, no y-intercept, vertical asymptote $x = 0$

10. They are reflections of each other in the line $y = x$.

11. a) They have the exact same shape.

b) One of them is increasing and the other is decreasing.

12. a) 216 **b)** 81 **c)** 64 **d)** 8

13. a) 7 **b)** 6

14. a) 0 **b)** 1

15. −1

16. 16

17. a) $t = \log_{1.1} N$ **b)** 145 days

18. The larger asteroid had a relative risk that was 1479 times as dangerous.

19. 1000 times as great

20. 5

21. $m = 14$, $n = 13$

22. $4n$

23. $y = 3^{2^x}$