

Warm Up

Review of laws of logarithms...

Express the following as a single logarithm:

$$\frac{2}{3} \ln b^6 - \frac{1}{2} \left[\ln b^4 + 8 \ln b + 6 \ln \sqrt[3]{b} \right]$$

Questions from Homework

① b $y = \ln\left(\frac{x}{\sqrt{x^2+1}}\right)$ $u = \frac{x}{(x^2+1)^{1/2}}$ $du \Rightarrow$ quotient/chain

$$y' = \frac{(x^2+1)^{1/2}}{x} \left[\frac{(x^2+1)^{1/2}(1) - x\left(\frac{1}{2}\right)(x^2+1)^{-1/2}(2x)}{x^2+1} \right]$$

$$y' = \frac{(x^2+1)^{1/2}}{x} \left[\frac{(x^2+1)^{1/2} - x^2(x^2+1)^{-1/2}}{x^2+1} \right]$$

$$y' = \frac{(x^2+1)^{1/2}}{x} \left[\frac{(x^2+1)^{-1/2} [x^2+1 - x^2]}{x^2+1} \right]$$

$$y' = \frac{1}{x(x^2+1)}$$

② $y = \ln(\ln x)$ $u = \ln x$
 $du = \frac{1}{x}$

$$y' = \frac{1}{\ln x} \cdot \frac{1}{x} = \frac{1}{x \ln x}$$

Questions from Homework

③ d) $g(x) = \frac{1 + \log_3 x}{x}$

$g'(x) = \frac{x(\frac{1}{x \ln 3} \cdot 1) - (1 + \log_3 x)(1)}{x^2}$

$g'(x) = \frac{\frac{1}{\ln 3} - 1 - \log_3 x}{x^2 \ln 3}$

$g'(x) = \frac{1 - \ln 3 - \ln 3(\log_3 x)}{x^2 \ln 3}$

$g'(x) = \frac{1 - \ln 3 - \ln 3(\frac{\ln x}{\ln 3})}{x^2 \ln 3}$

$g'(x) = \frac{1 - \ln 3 - \ln x}{x^2 \ln 3}$

Change of base formula.
 $\log_b M = \frac{\log M}{\log b} = \frac{\log M}{\log N} = \frac{\ln M}{\ln N}$
 base b ← base e

⑦ $f(x) = x \ln x$

i) $f'(x) = x(\frac{1}{x}) + (1) \ln x$

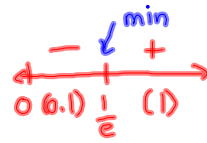
$f'(x) = 1 + \ln x$

$1 + \ln x = 0$

$\ln x = -1$

$e^{-1} = x$

CV: $\frac{1}{e} = x$



$f(\frac{1}{e}) = (\frac{1}{e}) \ln(\frac{1}{e})$

$= \frac{1}{e}(-1)$

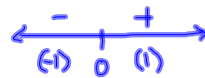
$= -\frac{1}{e}$

$(\frac{1}{e}, -\frac{1}{e})$ Min

ii) $f'(x) = 1 + \ln x$

$f''(x) = 0 + \frac{1}{x}$

$f''(x) = \frac{1}{x}$



CU on $(0, \infty)$

CO on $(-\infty, 0)$

CV: $x = 0$

⑧ $f(x) = x(\ln x)^2$

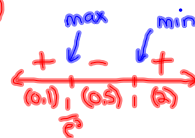
$f'(x) = x(2)(\ln x)(\frac{1}{x}) + (1)(\ln x)^2$

$f'(x) = 2 \ln x + (\ln x)^2$

$f'(x) = \ln x [2 + \ln x]$

$0 = \ln x [2 + \ln x]$

CV: $\ln x = 0$ | $2 + \ln x = 0$
 $e^0 = x$ | $\ln x = -2$
 $1 = x$ | $e^{-2} = x$
 $\frac{1}{e^2} = x$



$f(\frac{1}{e}) = \frac{4}{e^2}$ max

$f(1) = 0$ min

Logarithmic Differentiation

A differentiation process that requires taking the logarithm of both sides before differentiating.

This process will be used in TWO circumstances:

I. Simplifying messy products and quotients

What would it involve to differentiate the following?

$$y = \frac{(x^2 - 1)^5 \sqrt{2x + 9} (5x^3 + 2)^8}{(10x - 1) \sqrt{5 - x^7}}$$

- Quotient rule, multiple product rules and chain rules...

This would be possible but it would be easier to differentiate a group of terms added and subtracted rather than multiplied and divided

Laws of logarithms will do exactly that...turn this mess into a addition and subtraction of terms.

$$y = \frac{(x^2 - 1)^5 \sqrt{2x + 9} (5x^3 + 2)^8}{(10x - 1) \sqrt{5 - x^7}}$$

$$\ln y = \ln \left[\frac{(x^2 - 1)^5 (2x + 9)^{1/2} (5x^3 + 2)^8}{(10x - 1) (5 - x^7)^{1/2}} \right]$$

$$\ln y = \ln(x^2 - 1)^5 + \ln(2x + 9)^{1/2} + \ln(5x^3 + 2)^8 - \ln(10x - 1) - \ln(5 - x^7)^{1/2}$$

$$\ln y = 5 \ln(x^2 - 1) + \frac{1}{2} \ln(2x + 9) + 8 \ln(5x^3 + 2) - \ln(10x - 1) - \frac{1}{2} \ln(5 - x^7)$$

$$\frac{y'}{y} = \left[5 \left(\frac{2x}{x^2 - 1} \right) + \frac{1}{2} \left(\frac{2}{2x + 9} \right) + 8 \left(\frac{15x^2}{5x^3 + 2} \right) - \left(\frac{10}{10x - 1} \right) - \frac{1}{2} \left(\frac{-7x^6}{5 - x^7} \right) \right] y'$$

$$y' = \left[\frac{10x}{x^2 - 1} + \frac{1}{2x + 9} + \frac{120x^2}{5x^3 + 2} - \frac{10}{10x - 1} + \frac{7x^6}{2(5 - x^7)} \right] \left[\frac{(x^2 - 1)^5 \sqrt{2x + 9} (5x^3 + 2)^8}{(10x - 1) \sqrt{5 - x^7}} \right]$$

Steps in Logarithmic Differentiation

1. Take logarithms of both sides of an equation.
2. Differentiate implicitly with respect to x .
3. Solve the resulting equation for y'

Use Logarithmic Differentiation to Differentiate the following:

$$y = \frac{e^x \sqrt{x^2 + 1}}{(x^2 + 2)^3}$$

$$\ln y = \ln \left[\frac{e^x (x^2 + 1)^{1/2}}{(x^2 + 2)^3} \right]$$

$$\ln y = \underline{\ln e^x} + \ln(x^2 + 1)^{1/2} - \ln(x^2 + 2)^3$$

$$\ln y = x + \frac{1}{2} \ln(x^2 + 1) - 3 \ln(x^2 + 2)$$

$$y \cdot \frac{y'}{y} = \left[1 + \frac{1}{2} \left(\frac{2x}{x^2 + 1} \right) - 3 \left(\frac{2x}{x^2 + 2} \right) \right] y$$

$$y' = \left[1 + \frac{x}{x^2 + 1} - \frac{6x}{x^2 + 2} \right] \left[\frac{e^x \sqrt{x^2 + 1}}{(x^2 + 2)^3} \right]$$

II. Base and exponent both variables

Have a look at this example:

$$y = x^{x^5}$$

- Does not fit either the power rule or the rules for an exponential function

...What can be done to help this crazy situation??

Of Course...take the logarithm of both sides!!

$$\begin{aligned}
 y &= x^{x^5} \\
 \ln y &= \ln x^{x^5} \\
 \ln y &= x^5 \ln x \\
 \frac{y'}{y} &= x^5 \left(\frac{1}{x} \right) + 5x^4 \ln x \\
 y \cdot \frac{y'}{y} &= [x^4 + 5x^4 \ln x] y \\
 y' &= [x^4 + 5x^4 \ln x] x^{x^5}
 \end{aligned}$$

Example:

Differentiate:

$$y = (\ln x^5)^{\cos x}$$

Practice Questions...

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#1 b, d, e

#2 b, c, e

#3