Warm up

As it aged, a maple tree produced sap according to the pattern shown in the table below.

Year	2001	2002	2003	2004
Sap (Litres)	$t_1 = 60.000$	t ₂ = 57.000	t ₃ = 54.150	t ₄ = 51.4425

a) Does the data follow an arithmetic of geometric pattern?

$$\Gamma = \frac{1}{4} = \frac{57}{60} = 0.95$$

b) Write down a formula for t_n ?

$$t_n = \alpha r^{n-1}$$
 $t_n = 60(0.95)^{n-1}$

c) Assuming the pattern continues, how long will it take for the sap production to be approximately 17.5L?

$$t_{n} = 17.5$$
 $t_{n} = ar^{n-1}$ In 35 years 0.96 $17.5 = 60(0.95)^{n-1}$ the tree will produce 17.51 $0.3916 = (0.95)^{n-1}$ $(0.95)^{n} = (0.95)^{n-1}$ $34 = n-1$ $35 = n$

Questions from homework

$$= \frac{4}{n^{3}}$$

$$= (1)^{3} + (3)^{3} + (4)^{3}$$

$$= 1 + 4 + 9 + 16$$

$$= 30$$

$$00 \quad 3+6+12+34+48$$

$$0 = 3$$

$$0 = 3$$

$$0 = 3$$

$$0 = 3$$

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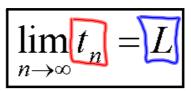
$$0 = 3$$

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Limit (of a sequence - t_n)

A finite number L that the value of t_n gets closer and closer to, or "approaches," as n becomes very large, or "approaches infinity." The value of t_n can be made as close as you like to L by using a sufficiently large value for n.

The notation for a limit is



Converging Sequence -> Has a Limit

A sequence in which the terms approach a limit

For example,
$$\frac{1}{4}$$
, $\frac{2}{5}$, $\frac{3}{6}$, $\frac{4}{7}$,... converges to 1

$$t_n = \frac{n}{n+3}$$

What happens if "n" is a very large number?

$$t_{100} = \frac{100}{103} = 0.97$$

$$t_{000} = \frac{1000}{1003} = 0.997$$

$$\lim_{n\to\infty} \frac{n}{n+3} = \boxed{1}$$

Diverging Sequence -> Has no Limit

A sequence in which the terms do not approach a limit

For example, 1, 2, 3, 4,... diverges. (no limit exists)

The above sequence was generated using the following general term.

$$t_{n} = n$$

What happens if "n" is a very large number?

Decide whether each sequence *converges* or *diverges* then state the limit.

2, 4, 8, 16, 32,... diverges

$$a = 3$$
 $t_n = (3)(3)^{n-1}$
 $t_n = (3)^n$
 $t_n = (3)(4)^{n-1}$
 $t_n = (3)(4)^{n-1}$

Infinite Sequences

Suppose we have a sequence defined by
$$t_n = \frac{n}{2n+1}$$
 $n \in \mathbb{N}$

Generate the first 4 terms of the sequence

$$\frac{1}{3}, \frac{2}{5}, \frac{3}{7}, \frac{4}{9}, \dots$$

You may notice that as "n" increases " t_n " approaches $\frac{1}{2}$

Symbolically this is written
$$\lim_{n \to \infty} \frac{n}{2n+1} = \frac{1}{2}$$

and is read "The limit as n approaches infinity of n over (2n + 1) is $\frac{1}{2}$."

Algebraically we solve by dividing the numerator and the denominator by the highest power of n.

$$\lim_{N \to \infty} \frac{1}{N} = \frac{1}{N}$$

Diverging
$$t_n = \underline{n+5}$$

Find the limit if it exists

Diverging

$$t_n = \frac{n+5}{1}$$
 $t_n = \frac{3n+1}{4n-2}$
 $t_n = \frac{3n+1}{4n-2}$

$$t_n = \frac{3n+1}{4n-2}$$

$$\lim_{n \to \infty} \frac{3+0}{4-0} = \frac{3}{4}$$

if the degree of the numerator and denominator are the same, then your limit will be the quotient of the leading coefficients.

$$\lim_{n\to\infty} \frac{n^3-3}{4+3n^3} = \frac{3}{3}$$

if the degree of the numerator is larger than the degree of the denominator, then your limit will not exist.

$$\lim_{n\to\infty} \frac{n^2+3n}{n-3} = DNE$$

if the degree of the denominator is larger than the degree of the numerator, then your limit will always equal 0.

$$\lim_{n\to\infty}\frac{1}{3n^5-2}=0$$

Homework

(a)
$$c_1 = \frac{5}{1} + \frac{1}{1}$$

 $= \frac{5n+1}{n}$
 $\frac{5n+1}{n} = 5$

3
$$t_{n} = \frac{1}{2}$$
 $t_{n} = \frac{1}{2}$
 $t_{n} = \frac$

$$\lim_{N \to \infty} \frac{1 + 0}{1 + 0} = 0$$

$$\lim_{N \to \infty} \frac{1 + 0}{1 - 0} = 0$$

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