Warm Up

Differentiate each of the following:

1. $f(x) = 6^{x^3} + \ln\left(\tan^{-1} 2x^4\right)$

$f'(x) = 6^{x^3}(\ln 6)(3x^2) + \left(\frac{1}{\tan^{-1} 2x^4}\right)\left(\frac{8x^3}{1+(2x^4)^2}\right)$

2. $y = (8x-1)^{\sqrt{x}}$

$\ln y = \ln (8x-1)^{\sqrt{x}}$

$\ln y = \left[x^{1/2}\right]\left[\ln(8x-1)\right]$

$\cancel{y}\ \frac{y'}{\cancel{y}} = \left[x^{1/2}\left(\frac{8}{8x-1}\right) + \frac{1}{2}x^{-1/2}\left(\ln(8x-1)\right)\right]\left[(8x-1)^{\sqrt{x}}\right]$

Derivative Rules

Exponential Functions

$$d\left(b^u\right)=b^u \bullet (\ln b)\bullet du \text{ , where b} \in R$$

$$d\left(e^u\right)=e^u \bullet du \text{ , base is Euler's number}$$

Logarithmic Functions

$$d(\log_b u)=\frac{1}{u\ln b}\bullet du \text{ , where } b \in R$$

$$d(\ln u)=\frac{1}{u}\bullet du \text{ , base is Euler's number}$$

Inverse Trigonometric Functions

$$\frac{d\left(\sin^{-1} u\right)}{du}=\frac{1}{\sqrt{1-u^2}}du \qquad \frac{d\left(\csc^{-1} u\right)}{du}=\frac{-1}{u\sqrt{u^2-1}}du$$

$$\frac{d\left(\cos^{-1} u\right)}{du}=\frac{-1}{\sqrt{1-u^2}}du \qquad \frac{d\left(\sec^{-1} u\right)}{du}=\frac{1}{u\sqrt{u^2-1}}du$$

$$\frac{d\left(\tan^{-1} u\right)}{du}=\frac{1}{1+u^2}du \qquad \frac{d\left(\cot^{-1} u\right)}{du}=\frac{-1}{u^2+1}du$$

Derivatives of Transcendental Functions

- Inverse Trigonometric Functions
- Exponential Functions
- Logarithmic Functions

Practice Test

Solutions

May 06

1 a) $f'(x) = \frac{1}{x^3 \ln 5}(3x^2) + e^{\sin 5x}(\cos 5x)(5)$

b) $y' = \frac{-1}{\sqrt{1-(2/x)^2}}(x^{-1/2}) - \frac{1}{\ln x^2}\left(\frac{1}{x^2}\right)(2x)$

c) $h'(t) = \dfrac{5^{\tan t}\ln 5(\sec^2 t)\ln(3t^4+5) - 5^{\tan t}\left(\frac{1}{3t^4+5}(12t^3)\right)}{[\ln(3t^4+5)]^2}$

d) $\ln y = x\ln(5 \cdot 2x^2)$

$(y)\frac{1}{y}y' = \left[\ln(5-2x^2) + x\left(\frac{1}{5-2x^2}\right)(-4x)\right]y$

$y' = \left(\ln(5-2x^2) - \frac{4x^2}{5-2}\right)(5-2x^2)^x$

e) $y' = \frac{1}{1+[\ln^3(x^5-1)]^2}\left[3[\ln(x^5-1)]^2\left(\frac{1}{x^5-1}\right)(5x^4)\right]$

f) $g'(x) = 4^{5x}\ln 4(5)e^{\sin^{-1}\sqrt{x}} + 4^{5x}e^{\sin^{-1}\sqrt{x}}\left(\frac{1}{\sqrt{1-x}}\cdot\frac{1}{2}x^{-1/2}\right)$

2. $\ln y = 3\ln(x^2-2x) + \ln 8x^5 - \frac{5}{2}\ln(5-x^2) - (x^5+2)$

$(y)\frac{1}{y}y' = \left[3\left(\frac{2x-2}{x^2-2x}\right) + \frac{40x^4}{8x^5} - \frac{5}{2}\left(\frac{(-2x)}{5-x^2}\right) - 5x^4\right]y$

$y' = \left(\frac{3(2x-2)}{x^2-2x} + \frac{5}{x} + \frac{5x}{5-x^2} - 5x^4\right)\left[\frac{(x^2-2x)^3(8x^5)}{(5-x^2)^{5/2}(e^{x^5+2})}\right]$

3. $e^{3x-y^5}\left(3-5y^4\frac{dy}{dx}\right) = 5^{xy^3}\ln 5\left(y^3 + x(3y^2\frac{dy}{dx})\right)$

$3e^{3x-y^5} - 5y^4e^{3x-y^5}\frac{dy}{dx} = 5^{xy^3}\ln 5\, y^3 + 3xy^2 5^{xy^3}\ln 5\frac{dy}{dx}$

$\dfrac{3e^{3x-y^5} - 5^{xy^3}\ln 5\, y^3}{3xy^2 5^{xy^3}\ln 5 + 5y^4e^{3x-y^5}} = \frac{dy}{dx}$

4. $\frac{dz}{dy} = \frac{1}{\sqrt{1-(y-3)^2}}(1) - 3y^2$ ~~when x = 0~~

$\frac{dy}{dx} = 3e^x + 2x$

$\frac{dz}{dx} = \frac{dz}{dy}\cdot\frac{dy}{dx} = \frac{dz}{dx}$

at $x = 0$ . . . $y = 3+0$, $y = 3$

$\frac{dz}{dy} = \frac{1}{\sqrt{1-0^2}} - 27$

$= -26$

$\frac{dy}{dx} = 3e^0 + 2(0)$

$= 3$

$\boxed{\frac{dz}{dx} = -78}$

5. $f'(x) = 2xe^{2x} + x^2e^{2x}(2)$

$f'(x) = 2xe^{2x}(1+x)$

Critical Values: $x = -1$, $x = 0$

	$2x$	e^{2x}	$1+x$	f'	f
$(-\infty,-1)$	−	+	−	+	Inc
$(-1,0)$	−	+	+	−	Dec
$(0,\infty)$	+	+	+	+	Inc

LOCAL MAX: $\left(-1, \frac{1}{e^2}\right)$

LOCAL MIN: $(0, 0)$

From text

2 a) $y = x^{x^2}$

$\ln y = \ln x^{x^2}$

$\ln y = x^2 \ln x$

$\frac{y'}{y} = x^2\left(\frac{1}{x}\right) + 2x\ln x$

$\frac{y'}{y} = x + 2x\ln x$

$y' = (x + 2x\ln x)x^{x^2}$

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10 $f(x) = xe^x$

a) $f'(x) = xe^x + 1e^x$

$f'(x) = e^x(x+1)$

CV:

$x+1=0$

$x=-1$

Decreasing on $(-\infty,-1)$

Increasing on $(-1,\infty)$

$f(-1) = (-1)e^{(-1)}$

$= -1\left(\frac{1}{e}\right) = \frac{-1}{e}$

abs min: $(-1, -1/e)$

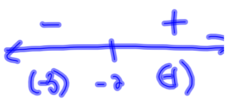
b) $f'(x) = e^x(x+1)$

$f''(x) = e^x(1) + e^x(x+1)$

$= e^x + e^x(x+1)$

$= e^x[1 + (x+1)]$

$= e^x(x+2)$



CV:

$x+2=0$

$x=-2$

CD: on $(-\infty,-2)$

CU: on $(-2,\infty)$

c) $f(-2) = (-2)e^{(-2)}$

$= -2\left(\frac{1}{e^2}\right)$

Inflection Point

$\left(-2, \frac{-2}{e^2}\right)$

① c) $h(t) = \dfrac{5^{\tan t}}{\ln(3t^4+5)}$

$$h'(t) = \frac{\ln(3t^4+5)\,5^{\tan t} \cdot \ln 5 \cdot \sec^2 t - 5^{\tan t}\left[\frac{12t^3}{3t^4+5}\right]}{\left[\ln(3t^4+5)\right]^2}$$

① e) $y = \tan^{-1}(\ln^3(x^5-1))$

$$y' = \left[\frac{1}{1+(\ln^3(x^5-1))^2}\right]\left[3[\ln^2(x^5-1)]\left[\frac{5x^4}{x^5-1}\right]\right]$$

④ $z = \sin^{-1}(y-3) - y^3 \qquad y = 3e^x + x^2$

$$z = \sin^{-1}(3e^x + x^2 - 3) - (3e^x + x^2)^3$$

$$z' = \left[\frac{1}{\sqrt{1-(3e^x+x^2-3)^2}}\right]\left[3e^x + 2x\right] - 3(3e^x + x^2)^2(3e^x + 2x)$$

$$z' = \frac{3e^x + 2x}{\sqrt{1-(3e^x+x^2-3)^2}} - 3(3e^x+x^2)^2(3e^x+2x)$$

$$z'(0) = \frac{3e^0 + 2(0)}{\sqrt{1-(3e^0+(0)^2-3)^2}} - 3(3e^0+(0)^2)^2(3e^0+2(0))$$

$$z'(0) = 3 - 3(3)^2(3)$$

$$z'(0) = 3 - 81$$

$$\boxed{z'(0) = -78}$$

④ Find $\frac{dz}{dx}$ at $x=0$ Since $\frac{dz}{dx} = \frac{dz}{dy} \cdot \frac{dy}{dx}$

$z = \sin^{-1}(y-3) - y^3$ | $y = 3e^x + x^2$

$\frac{dz}{dy} = \frac{1}{\sqrt{1-(y-3)^2}} \cdot (1) - 3y^2$ | $\frac{dy}{dx} = 3e^x + 2x$

when $\underline{\underline{x=0}} \rightarrow y = 3e^{(0)} + (0)^2 = 3+0 = \underline{\underline{3}}$

$\frac{dz}{dy} = \frac{1}{\sqrt{1-(3-3)^2}} - 3(3)^2$ | $\frac{dy}{dx} = 3e^0 + 2(0) = 3$

$\frac{dz}{dy} = 1 - 27 = -26$

Since $\frac{dz}{dx} = \frac{dz}{dy} \cdot \frac{dy}{dx}$

$\frac{dz}{dx} = (-26)(3) = \boxed{-78}$

⑤ $f(x) = (x^2)(e^{2x})$

$f'(x) = x^2(e^{2x})(2) + e^{2x}(2x)$

$f'(x) = 2x^2e^{2x} + 2xe^{2x}$

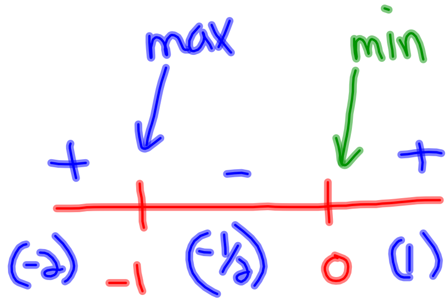
$f'(x) = 2xe^{2x}[x+1]$

CV:

$0 = 2xe^{2x}[x+1]$

$2xe^{2x} = 0$ | $x+1=0$

$x = 0$ | $x = -1$



max $(x=-1)$

$f(-1) = (-1)^2e^{2(-1)} = 1e^{-2} = \frac{1}{e^2}$ $\quad\left(-1, \frac{1}{e^2}\right)$

min $(x=0)$

$f(0) = (0)^2e^{2(0)} = 0e^0 = 0(1) = 0$ $\quad(0,0)$

Attachments

Review of Transcendentals.doc

logs & arcfunctions test 2006.doc