Convert the following complex numbers from ***rectangular*** to **polar form**

$(5\sqrt{3} - 5i)$

$a = 5\sqrt{3}$, $b = -5$ } Quad 4

① $r = \sqrt{(5\sqrt{3})^2 + (-5)^2}$

$r = \sqrt{75 + 25}$

$r = 10$

② $\alpha = \tan^{-1}\left(\frac{5}{5\sqrt{3}}\right)$

$\alpha = 30°$

③ Quad 4

$\theta = 360 - \alpha$

$\theta = 360° - 30°$

$\theta = 330°$

④ $10\text{cis}\,330°$

$(1 - i\sqrt{3})$

$a = 1$, $b = -\sqrt{3}$ } Quad 4

① $r = \sqrt{(1)^2 + (\sqrt{3})^2}$

$r = \sqrt{1 + 3}$

$r = 2$

② $\alpha = \tan^{-1}\left(\frac{\sqrt{3}}{1}\right)$

$\alpha = 60°$

③ Quad 4

$\theta = 360 - \alpha$

$\theta = 360 - 60$

$\theta = 300°$

④ $2\text{cis}\,300°$

Multiply your answers

$(10\text{cis}\,330°)(2\text{cis}\,300°)$

$20\text{cis}\,630°$

$20\text{cis}\,270°$ Polar

$0 - 20i$ Rectangular

Divide your answers

$\frac{10\text{cis}\,330°}{2\text{cis}\,300°}$

$5\text{cis}\,30°$ Polar

$4.33 + 2.5i$ Rec.

Questions from Homework

1c) $\dfrac{(4-4i\sqrt{3})(2\sqrt{3}+2i)(1+i)}{(5-5i)(-\sqrt{3}+i)}$

$a=4$, $b=-4\sqrt{3}$; $r=\sqrt{16+48}$, $r=\sqrt{64}$, $r=8$; $\alpha=\tan^{-1}\left(\frac{4\sqrt{3}}{4}\right)$, $\alpha=60^\circ$; Quad 4, $\theta=300^\circ$

$a=2\sqrt{3}$, $b=2$; $r=\sqrt{12+4}$, $r=4$; $\alpha=\tan^{-1}\left(\frac{2}{2\sqrt{3}}\right)$, $\alpha=30^\circ$; Quad 1, $\theta=30^\circ$

$a=1$, $b=1$; $r=\sqrt{1+1}$, $r=\sqrt{2}$; $\alpha=\tan^{-1}\left(\frac{1}{1}\right)$, $\alpha=45^\circ$; Quad 1, $\theta=45^\circ$

$a=5$, $b=-5$; $r=\sqrt{25+25}$, $r=\sqrt{50}$, $r=5\sqrt{2}$; $\alpha=\tan^{-1}\left(\frac{5}{5}\right)$, $\alpha=45^\circ$; Quad 4, $\theta=315^\circ$

$a=-\sqrt{3}$, $b=1$; $r=\sqrt{3+1}$, $r=2$; $\alpha=\tan^{-1}\left(\frac{1}{\sqrt{3}}\right)$, $\alpha=30^\circ$; Quad 2, $\theta=150^\circ$

$$\frac{(8\,\mathrm{cis}\,300^\circ)(4\,\mathrm{cis}\,30^\circ)(\sqrt{2}\,\mathrm{cis}\,45^\circ)}{(5\sqrt{2}\,\mathrm{cis}\,315^\circ)(2\,\mathrm{cis}\,150^\circ)}$$

$$\frac{32\sqrt{2}\,\mathrm{cis}\,375^\circ}{10\sqrt{2}\,\mathrm{cis}\,465^\circ}$$

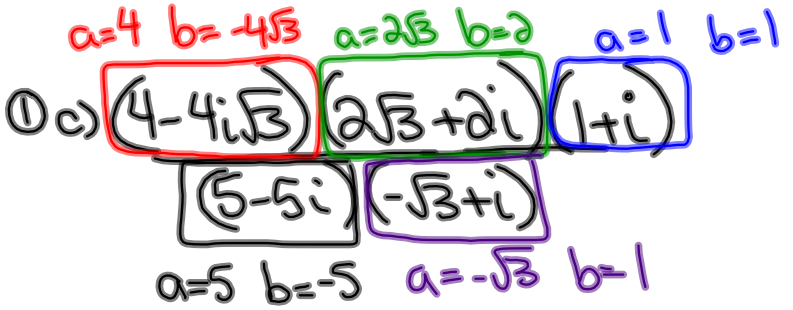
$$3.2\,\mathrm{cis}(-90^\circ)$$

$$\boxed{3.2\,\mathrm{cis}\,270^\circ}$$ Polar

$$3.2\cos 270^\circ + 3.2i\sin 270^\circ$$

$$3.2(0) + 3.2i(-1)$$

$$\boxed{-3.2i}$$ Rectangular



$$\frac{(8\text{cis}300°)(4\text{cis}30°)(\sqrt{2}\text{cis}45°)}{(5\sqrt{2}\text{cis}315°)(2\text{cis}150°)}$$

$$\frac{32\sqrt{2}\text{cis}375°}{10\sqrt{2}\text{cis}465°}$$

$$3.2\text{cis}(-90°)$$

$$\boxed{3.2\text{cis}270°}$$ Polar form

$$3.2\cos 270° + 3.2i\sin 270°$$

$$3.2(0) + 3.2i(-1)$$

$$\boxed{0-3.2i}$$ Rectangular form

You may have noticed a shortcut when multiplying and dividing complex numbers in Polar form.

For Multiplication
Multiply the "r" values and add the arguments.

For Division
Divide the "r" values and subtract the arguments.

De Moivre's Theorem

$$(rcis\theta)^n = r^n cisn\theta$$

Evaluate

$$(2\cos\frac{\pi}{6} + 2i\sin\frac{\pi}{6})^5$$

$(2cis\frac{\pi}{6})^5$

$2^5 cis(5\cdot\frac{\pi}{6})$

$32cis\frac{5\pi}{6}$

In Degrees:

$\frac{\pi}{6}\cdot\frac{180}{\pi} = 30°$

$(2cis30°)^5$

$2^5 cis(5\cdot30)$

$32cis150°$

Remember $(rcis\theta)^n = r^n cisn\theta$

$(3-4i)^5$

$a = 3$
$b = -4$ } Quad 4

$r = \sqrt{(3)^2+(-4)^2}$
$r = \sqrt{9+16}$
$r = 5$

$\alpha = \tan^{-1}\left(\frac{4}{3}\right)$
$\alpha = 53°$

Quad 4
$\theta = 360 - \alpha$
$\theta = 360 - 53°$
$\theta = 307°$

$5cis307°$

$(5cis307°)^5$

$5^5 cis(5 \cdot 307°)$

$3125 cis 1535°$

$3125 cis 95°$ Polar

$-312.4 + 3113.1i$ Rectangular

Homework

⑧ Ex $\dfrac{(4-3i)^5(1-i)^3}{(5\sqrt{3}+5i)^3}$ $(r\,\text{cis}\,\theta)^n = r^n\,\text{cis}\,n\theta$

$a=4\ \ b=-3$ (Quad 4)

$r=\sqrt{16+9}$ $\alpha=\tan^{-1}\left(\frac{3}{4}\right)$ $\theta=360-36.9$ $5\,\text{cis}\,323.1°$

$r=\sqrt{25}$ $\alpha=36.9°$ $\theta=323.1°$

$r=5$

$a=1\ \ b=-1$ (Quad 4)

$r=\sqrt{1+1}$ $\alpha=\tan^{-1}\left(\frac{1}{1}\right)$ $\theta=360-45$ $\sqrt{2}\,\text{cis}\,315°$

$r=\sqrt{2}$ $\alpha=45°$ $\theta=315°$

$a=5\sqrt{3}\ \ b=5$ (Quad 1)

$r=\sqrt{(5\sqrt{3})^2+(5)^2}$ $\alpha=\tan^{-1}\left(\frac{5}{5\sqrt{3}}\right)$ $\theta=30°$ $10\,\text{cis}\,30°$

$r=\sqrt{75+25}$ $\alpha=30°$

$r=10$

$$\frac{(5\,\text{cis}\,323.1)^5(\sqrt{2}\,\text{cis}\,315)^3}{(10\,\text{cis}\,30)^3}$$

$$\frac{[5^5\,\text{cis}(5\cdot323.1)][\sqrt{2}^3\,\text{cis}(3\cdot315)]}{10^3\,\text{cis}(3\cdot30)}$$

$$\frac{(3125\,\text{cis}\,1615.5)(2\sqrt{2}\,\text{cis}\,945°)}{100\,\text{cis}\,60°}$$

$$\frac{6250\sqrt{2}\,\text{cis}\,2560.5°}{100\,\text{cis}\,60°}$$

$$62.5\sqrt{2}\,\text{cis}\,2500.5°$$

$$62.5\sqrt{2}\,\text{cis}\,340.5°$$