Arithmetic (common difference "d")	Geometric (Common Ratio "r")
$t_n = a + (n-1)d$	$t_n = ar^{n-1}$
$S_n = \frac{n}{2}(2a + (n-1)d)$	$S_n = \frac{a(r^n - 1)}{r-1}$
$S_n = \frac{n}{2}(a + t_n)$	* $\boxed{S_n = \frac{a}{1-r}}$
	$-1 < r < 1$

Ex 1.7

① b) $1 - \frac{2}{3} + \frac{4}{9} - \frac{8}{27} + \ldots$ Infinite

$a = 1$

$r = -\frac{2}{3}$

$$S_n = \frac{a}{1-r} = \frac{1}{1-\left(-\frac{2}{3}\right)} = \frac{1}{5/3} = (1)\left(\frac{3}{5}\right) = \boxed{\frac{3}{5}}$$

Infinite Series

② b) $\sum_{n=1}^{\infty} \left(\frac{-2}{5}\right)^n$

Generate Series:

$\left(\frac{-2}{5}\right)^1 = -\frac{2}{5}$

$\left(\frac{-2}{5}\right)^2 = \frac{4}{25}$

$\left(\frac{-2}{5}\right)^3 = -\frac{8}{125}$

$-\frac{2}{5} + \frac{4}{25} - \frac{8}{125} + \ldots$

$a = -\frac{2}{5}$

$r = -\frac{2}{5}$

$$S_n = \frac{-2/5}{1-\left(-2/5\right)} = \frac{-2}{5} \div \frac{7}{5} = \frac{-2}{\cancel{5}} \times \frac{\cancel{5}}{7} = \frac{-2}{7}$$

④ b) 1, 2, 2, 4, 8, 32, . . .

Recursive Rule
→ must use previous term(s)
t_{n-1} or t_{n-2}

you are multiplying the previous two terms to get the next term:

$t_n = (t_{n-1})(t_{n-2})$

③ 60, 57, 54.15, 51.4425, . . .

a) Geometric: $r = \frac{57}{60} = \frac{54.15}{57} = \frac{51.4425}{54.15} = 0.95$

b) $t_n = ar^{n-1}$

$t_n = 60(0.95)^{n-1}$

c) 60, 57, 54.15, 51.4425, ..., 22.64

$a = 60$
$r = 0.95$
$t_n = 22.64$
$n = ?$

$t_n = ar^{n-1}$

$\frac{22.64}{60} = \frac{\cancel{60}(0.95)^{n-1}}{\cancel{60}}$

$0.377\overline{3} = (0.95)^{n-1}$

$\frac{\log 0.377\overline{3}}{\log 0.95} = 19$

$(0.95)^{19} = (0.95)^{n-1}$

$19 = n - 1$

$20 = n$

① $\underline{80000}$, ___, ___, ___, $\underline{117128}$

$a = 80000$
$t_5 = 117128$
$n = 5$

Find r:

$t_n = ar^{n-1}$

$\frac{117128}{80000} = \frac{\cancel{80000}r^{5-1}}{\cancel{80000}}$

$1.4641 = r^4$

$1.1 = r$

AROI:

$AROI = (1.1 - 1) \times 100$
$= 10\%$

① $1 - \frac{2}{3} + \frac{4}{9} - \frac{8}{27} + \ldots$

$a = 1$

$r = \frac{-2}{3}$

$$S_n = \frac{a}{1-r}$$

$$= \frac{1}{1-\left(\frac{-2}{3}\right)}$$

$$= \frac{1}{5/3}$$

$$= 1 \times \frac{3}{5}$$

$$= \boxed{\frac{3}{5}}$$

c) $\frac{1}{4} - \frac{5}{16} + \frac{25}{64} - \frac{125}{256} + \ldots$

$a = \frac{1}{4}$

Divergent (r is too small)

$r = \frac{-5}{4}$

②b) $\sum_{n=1}^{\infty} \left(\frac{-2}{5}\right)^n = \frac{-2}{5} + \frac{4}{25} - \frac{8}{125} + \frac{16}{625} - \ldots$

$a = \frac{-2}{5}$

$r = \frac{4}{25} \div \frac{-2}{5}$

$= \frac{\cancel{4}^{2}}{\cancel{25}_{5}} \times \frac{\cancel{5}^{1}}{-\cancel{2}_{1}}$

$= \frac{-2}{5}$

$$S_n = \frac{-2/5}{1-(-2/5)}$$

$$= \frac{\frac{-2}{5}}{7/5}$$

$$= \frac{-2}{\cancel{5}} \times \frac{\cancel{5}}{7}$$

$$= \boxed{\frac{-2}{7}}$$

Series & Sequence Review

② $\underline{\underline{12000}}$, ___, ___, ___, ___, $\underline{\underline{91125}}$

$a = 12000$

$t_6 = 91125$

$n = 6$

$r = ?$

$$t_n = ar^{n-1}$$

$$\frac{91125}{12000} = \frac{12000r^{6-1}}{12000}$$

$$7.59375 = r^5$$

$$1.5 = r$$

* Annual rate of Increase:

$1.5 - 1 = 0.5 \times 100 = 50\%$

④ $t_1 = 3$

$t_n = (t_{n-1})^2$

$$t_2 = (t_{2-1})^2$$
$$= (t_1)^2$$
$$= (3)^2$$
$$= 9$$

$t_3 = 81$

$t_4 = 6561$

$t_5 = 43\,046\,721$

4 $t_1 = 3 \qquad t_n = (t_{n-1})^2$

"recursive rules"

previous term

$t_2 = (t_{2-1})^2$
$= (t_1)^2$
$= (3)^2$
$= 9$

$t_3 = (t_2)^2$
$= (9)^2$
$= 81$

5 c) $4+8+12+\ldots+400$

$a=4$

$d=4$

$t_n=400$

$S_n=?$

Solve for "n"

$t_n=a+(n-1)d$

$400=4+(n-1)(4)$

$400=4+4n-4$

$400=4n$

$100=n$

$S_n=\frac{n}{2}(a+t_n)$

$S_{100}=\frac{100}{2}(4+400)$

$S_{100}=50(404)$

$S_{100}=20\,200$

5 a) $\sum_{n=1}^{5} 2n+1$

$=3+5+7+9+11$

$=35$

⑥ $t_5 = 16$ | $t_8 = 25$ $\qquad t_n = a+(n-1)d$

$t_5 = a+(5-1)d$ | $t_8 = a+(8-1)d$

$t_5 = a+4d$ | $t_8 = a+7d$

$\boxed{a+4d=16 \quad a+7d=25}$

$a+7d=25$
$-\ a+4d=16$
$3d=9$
$\boxed{d=3}$

$a+4d=16$
$a+4(3)=16$
$a+12=16$
$\boxed{a=4}$

$t_n = a+(n-1)d$
$t_n = 4+(n-1)3$
$t_n = 4+3n-3$
$\boxed{t_n = 3n+1}$

7 $t_5 = 48$ | $t_8 = 384$ $t_n = ar^{n-1}$

$t_5 = ar^{5-1}$ | $t_8 = ar^{8-1}$

$t_5 = ar^4$ | $t_8 = ar^7$

$\boxed{ar^4 = 48 \quad ar^7 = 384}$

$$\frac{ar^7 = 384}{ar^4 = 48}$$

$r^3 = 8$

$\boxed{r = 2}$

$ar^4 = 48$

$a(2)^4 = 48$

$16a = 48$

$\boxed{a = 3}$

$t_n = ar^{n-1}$

$\boxed{t_n = (3)(2)^{n-1}}$

8 $\frac{5}{8}, \frac{5}{2}, 10, \ldots, \underline{\underline{640}}$.

$a = \frac{5}{8}$

$r = 4$

$t_n = 640$

$t_n = ar^{n-1}$

$640 = \frac{5}{8}(4)^{n-1}$

$* \frac{\log 1024}{\log 4} = 5$

$1024 = 4^{n-1}$

$\cancel{4}^{5} = \cancel{4}^{n-1}$

$5 = n-1$

$\boxed{6 = n}$

S+S Review 2

① $t_{12} = 15$

$t_n = a + (n-1)d$

$t_{12} = a + (12-1)d$

$t_{12} = a + 11d$

$\boxed{a + 11d = 15}$

$S_{15} = 105$

$S_n = \frac{n}{2}(2a + (n-1)d)$

$S_{15} = \frac{15}{2}(2a + (15-1)d)$

$S_{15} = 7.5(2a + 14d)$

$S_{15} = 15a + 105d$

$\boxed{15a + 105d = 105}$

$$\begin{array}{r} a + 11d = 15 \\ (-)\ a + 7d = 7 \\ \hline 4d = 8 \end{array}$$

$\boxed{d = 2}$

$a + 7d = 7$

$a + 7(2) = 7$

$a + 14 = 7$

$\boxed{a = -7}$

$-7, -5, -3$