Logarithms

exponential form

ans $\longrightarrow$ $x = b^y$ $\leftarrow$ exponent

base

Say "the base ***b*** to the exponent ***y*** is ***x***."

logarithmic form

exponent $\longrightarrow$ $y = \log_b x$ $\leftarrow$ ans

base

Say "***y*** is the exponent to which you raise base ***b*** to get the answer ***x***."

Example 1

Write each of the following in logarithmic form

a) $32 = 2^5$ b) $2^{-5} = \frac{1}{32}$ c) $x = 10^y$

Solutions

Compare $\overset{\text{ans}}{x} = \underset{\text{base}}{b}^{\overset{\text{exp}}{y}} \longleftrightarrow \overset{\text{exp}}{y} = \log_{\underset{\text{base}}{b}} \overset{\text{ans}}{x}$

a) $32 = 2^5 \longrightarrow 5 = \log_2 32$

b) $2^{-5} = \frac{1}{32} \longrightarrow -5 = \log_2\left(\frac{1}{32}\right)$

c) $x = 10^y \longrightarrow y = \log_{10} x$

$y = \log x$ (common logarithm)

When writing logarithms or evaluating expressions involving logarithms, you will find it useful to bear in mind the equivalent exponential form.

$$x = b^y \longleftrightarrow y = \log_b x$$

Example 2

Evaluate each of the following.

a) $\log_{10} 100$ b) $\log_2 64$ c) $\log_5 \sqrt{5}$

Solutions

a) $\log_{10} 100 \longleftarrow$ Think: to what exponent is the base 10 raised to obtain 100?

2

$10^y = 100$

$10^y = 10^2$

$y = 2$

b) $\log_2 64$

6

$2^y = 64$

$2^y = 2^6 \longrightarrow \frac{\log 64}{\log 2} = 6$

$y = 6$

c) $\log_5 \sqrt{5}$

$\frac{1}{2}$

$5^y = \sqrt{5}$

$5^y = 5^{1/2}$

$y = 1/2$

Skills with logarithms are needed to solve equations involving logarithms. When solving these equations, you must remember the meanings of the exponential form and the logarithmic form.

$$x = b^y \longleftrightarrow y = \log_b x$$

Example 3

$$\log_3 m = 4$$

$$3^4 = m$$

$$81 = m$$

$$x = b^y \longleftrightarrow y = \log_b x$$

Example 4

$$\log_8 4 = y$$

$$8^y = 4$$

$$(2^3)^y = 2^2$$

$$\cancel{2}^{3y} = \cancel{2}^2$$

$$3y = 2$$

$$y = \frac{2}{3}$$

$$x = b^y \longleftrightarrow y = \log_b x$$

Example 5

a) $\log_x 49 = 2$

$(x^2)^{1/2} = (49)^{1/2}$

$x = 7$

b) $\log_x 4 = \dfrac{2}{3}$

$(x^{2/3})^{3/2} = (4)^{3/2}$

$x = 8$

c) $\log_x 81 = 4$

$(x^4)^{1/4} = (81)^{1/4}$

$x = 3$

When solving some logarithmic equations, or simplifying logarithmic expressions, you will use the following property.

$$b^{\log_b m} = m$$

Example 4

a) $2^{\log_2 4} = 4$

b) $7^{\log_7 2401} = 2401$

Homework