# Warm Up

Differentiate:

$\sec^{-1}\left(\left(\tan\sqrt{x}\right)^2\right)$

(1) $f(x) = e^{\log_{10}\sqrt{x}} - 3^{\cos^{-1}(\ln x^5)} + Arc\sec\left(\tan^2\sqrt{x}\right)$

$$f'(x) = e^{\log\sqrt{x}}\left(\frac{1}{\sqrt{x}\ln 10}\right)\left(\frac{1}{2}x^{-1/2}\right) - 3^{\cos^{-1}(\ln x^5)}(\ln 3)\left(\frac{-1}{\sqrt{1-(\ln x^5)^2}}\right)\left(\frac{5x^4}{x^5}\right)$$

$$+\left(\frac{1}{\tan^2\sqrt{x}\sqrt{(\tan^2\sqrt{x})^2-1}}\right)(2)(\tan\sqrt{x})(\sec^2\sqrt{x})\left(\frac{1}{2}x^{-1/2}\right)$$

Problems with homework?

# Antiderivatives

Up to this point in our study of calculus, we have been concerned primarily with the problem:

**Given a function, find its derivative.**

Many important applications of calculus involve the inverse problem:

**Given the derivative, find the original function.**

"The basic problem of differentiation is: given the path of a moving point, to calculate its velocity, or given a curve, to calculate its slope. The basic problem of integration is the inverse: given the velocity of a moving point at every instant, to calculate its path, or given the slope of a curve at each of its points, to calculate the curve."

For example, suppose we are given the following derivatives: $f'(x) = 2$, $g'(x) = 3x^2$ $h'(t) = 4t$

Our goal is to determine f(x), g(x), and h(x) that have the respective derivatives given above. If we make some educated guesses, what would these functions be????

$f(x) = 2x + 2$ ? $g(x) = x^3$ $h(t) = 2t^2$

This operation of determining the original function from its derivative is the inverse operation of differentiation and we call it antidifferentiation.

Definition: A function F is called an antiderivative of f on an interval I if $F'(x) = f(x)$ for all x in I.

"F(x) is an antiderivative of f(x)"

It should be emphasized that if F(x) is an antiderivative of f(x), then F(x) + C (C is any constant) is also an antiderivative of f(x).

# Antiderivatives

Differentiate

$$f(x) \longrightarrow f'(x)$$

Antidifferentiate

$$f(x) \longleftarrow f'(x)$$

**Definition** A function $F$ is called an **antiderivative** of $f$ on an interval $I$ if $F'(x) = f(x)$ for all $x$ in $I$.

Example:

$$F(x) = \frac{x^2}{2} \quad \rightleftarrows \quad F'(x) = x$$

What about $F(x) = \dfrac{x^2}{2} + 2$ ?

**1 Theorem** If $F$ is an antiderivative of $f$ on an interval $I$, then the most general antiderivative of $f$ on $I$ is

$$F(x) + C$$

where $C$ is an arbitrary constant.

General antiderivatives are considered a family of curves...

Here is an example of a family of general antiderivatives:

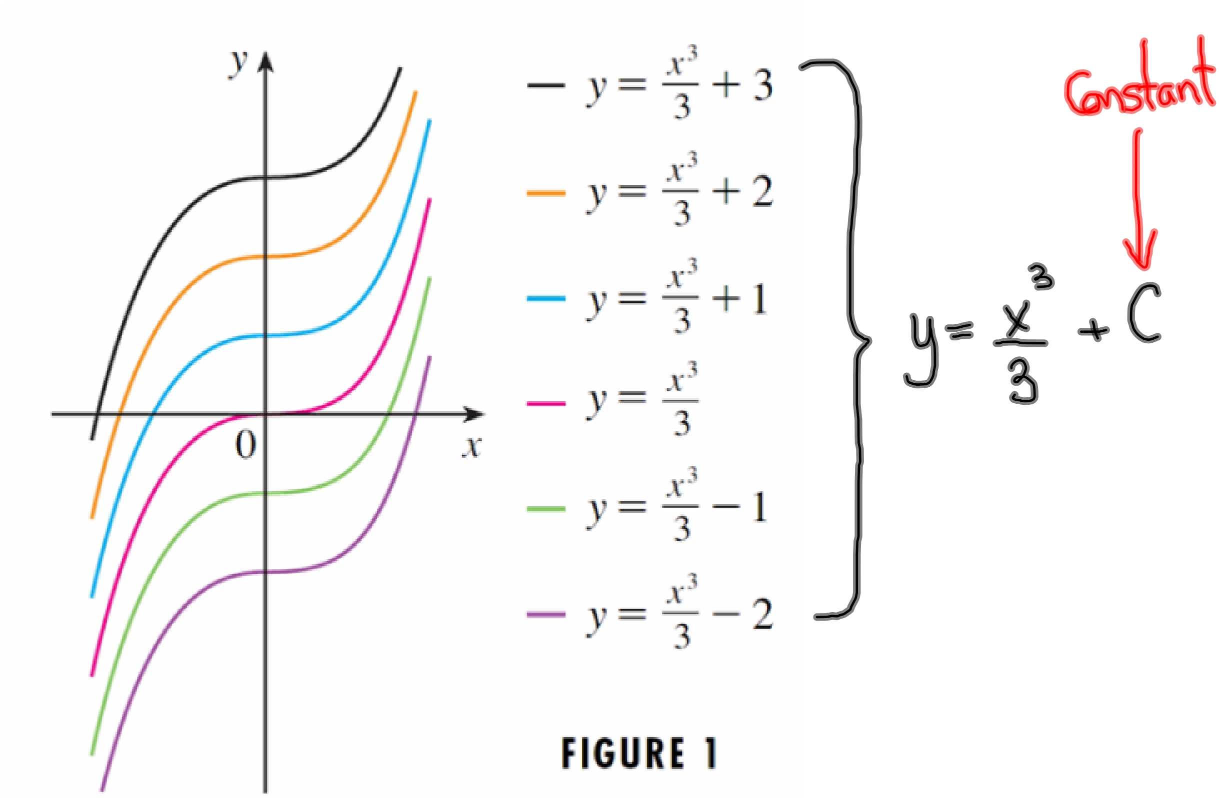


**FIGURE 1**

Notice the slopes of the tangents on each curve at the same *x*-coordinate.

# Antidifferentiation Rules...

Constants:

Determine the antiderivative of any constant

$f'(x) = 6$ $\quad$ $f'(x) = -3$ $\quad$ $f(x) = \pi$

$f(x) = 6x + C$ $\quad$ $f(x) = -3x + C$ $\quad$ $F(x) = \pi x + C$

# Rule:

$$f(x) = k \quad \Rightarrow \quad F(x) = kx + C$$

Power Law:

How will we put the power rule in reverse?

ie. If $f'(x) = 6x^2$ what is $f(x)$ ? $= \frac{6}{2+1}x^{2+1} + C = \boxed{2x^3 + C}$

Flip your brain into reverse...what will be the rule used to antidifferentiate power rules?

# Rule:

$$f(x) = kx^n \quad \Rightarrow \quad F(x) = \frac{k}{n+1}x^{n+1} + C$$

* Add one to the exponent and divide by this NEW exponent *

Determine the general antiderivative of each of the following...

(1) $f(x) = x^5 - 2x^4 - 3x^3 + \frac{2}{x^2} + 5x^{-3} + 5$

$f(x) = x^5 - 2x^4 - 3x^3 + 2x^{-2} + 5x^{-3} + 5$

$F(x) = \frac{1}{6}x^6 - \frac{2}{5}x^5 - \frac{3}{4}x^4 - 2x^{-1} - \frac{5}{2}x^{-2} + 5x + C$

$$F(x) = \frac{1}{6}x^6 - \frac{2}{5}x^5 - \frac{3}{4}x^4 - \frac{2}{x} - \frac{5}{2x^2} + 5x + C$$

(2) $f(x) = 3\sqrt{x} - \frac{2}{5x^4} + \sqrt[5]{x^7} - \frac{6\sqrt{x}}{x^2} + e^2$

$f(x) = 3x^{1/2} - \frac{2}{5}x^{-4} + x^{7/5} - 6x^{-3/2} + e^2$ ← const.

$F(x) = 2x^{3/2} + \frac{2}{15}x^{-3} + \frac{5}{12}x^{12/5} + 12x^{-1/2} + xe^2 + C$

$$F(x) = 2\sqrt{x^3} + \frac{2}{15x^3} + \frac{5\sqrt[5]{x^{12}}}{12} + \frac{12}{\sqrt{x}} + xe^2 + C$$

- ✓ constants
- ✓ power rules
- logarithmic functions
- trigonometric functions
- exponential functions
- inverse trigonometric functions
- chain rules

# Table of some of the Most General Antiderivatives

*where a is a constant!*

| Function, f(x) | Most General Antiderivative, F(x) |
|---|---|
| $a$ | $ax + C$ |
| $ax^n \ (n \neq -1)$ | $\frac{a}{n+1}x^{n+1} + C$ |
| $\frac{a}{x} \ (x \neq 0)$ | $a \ln\lvert x\rvert + C$ |
| $ae^{kx}$ | $\frac{a}{k}e^{kx} + C$ |
| $a^{kx}$ | $\frac{a^x}{k \ln a} + c$ |
| $a \cos kx$ | $\frac{a}{k}\sin kx + C$ |
| $a \sin kx$ | $-\frac{a}{k}\cos kx + C$ |
| $a \sec^2 kx$ | $\frac{a}{k}\tan kx + C$ |
| $a \sec kx \tan kx$ | $\frac{a}{k}\sec kx + C$ |
| $a \csc kx \cot kx$ | $-\frac{a}{k}\csc kx + C$ |
| $a \csc^2 kx$ | $-\frac{a}{k}\cot kx + C$ |
| $\frac{a}{\sqrt{1-(kx)^2}}$ | $\frac{a}{k}\sin^{-1} kx + C$ |
| $\frac{a}{1+(kx)^2}$ | $\frac{a}{k}\tan^{-1} kx + C$ |

Practice:

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