Questions from Quiz

① a) $\lim_{x\to 3} \frac{(\sqrt{x^2+7}-\sqrt{x+13})(\sqrt{x^2+7}+\sqrt{x+13})}{(x-3)(\sqrt{x^2+7}+\sqrt{x+13})}$

$\lim_{x\to 3} \frac{x^2+7-x-13}{(x-3)(\sqrt{x^2+7}+\sqrt{x+13})}$

$\lim_{x\to 3} \frac{\cancel{(x-3)}(x+2)}{\cancel{(x-3)}(\sqrt{x^2+7}+\sqrt{x+13})} = \boxed{\frac{5}{8}}$

g) $\lim_{x\to 0} \frac{\frac{1}{x+5}-\frac{1}{5}}{x^2+5x}$

L'Hopital's Rule

$\lim_{x\to 0} \frac{\frac{-1}{(x+5)^2}}{2x+5}$ $\quad \lim_{x\to 0} \frac{\frac{-1}{25}}{5} = \boxed{\frac{-1}{125}}$

g) $\lim_{x\to 0} \frac{5\cancel{(x+5)}\frac{1}{\cancel{x+5}} - \frac{1}{\cancel{5}}5(x+5)}{x^2+5x\,(5)(x+5)}$

$\lim_{x\to 0} \frac{5-x-5}{5x(x+5)(x+5)}$

$\lim_{x\to 0} \frac{-\cancel{x}}{5\cancel{x}(x+5)^2} = \boxed{-\frac{1}{125}}$

① h) $\lim_{x\to b} \frac{x^2-b^2}{x^8-b^8}$

$\lim_{x\to b} \frac{(x^2-b^2)}{(x^4-b^4)(x^4+b^4)}$

$\lim_{x\to b} \frac{\cancel{(x^2-b^2)}}{\cancel{(x^2-b^2)}(x^2+b^2)(x^4+b^4)} = \frac{1}{(2b^2)(2b^4)} = \boxed{\frac{1}{4b^6}}$

① f) $\lim_{x\to 0} \frac{\sin 7x}{2x}$

$\lim_{x\to 0} \left(\frac{\sin 7x}{7x}\right)\left(\frac{7}{2}\right) = \boxed{\frac{7}{2}}$

④ $\lim_{x\to 1} \frac{\sqrt[6]{x}-1}{\sqrt[4]{x}-1}$ L'Hopital's Rule

$\lim_{x\to 1} \frac{\frac{1}{6}x^{-5/6}}{\frac{1}{4}x^{-3/4}}$ $\quad \lim_{x\to 1} \frac{\frac{1}{6}}{\frac{1}{4}} \cdot \frac{4}{6} = \boxed{\frac{2}{3}}$

Bonus:

$y = ax^2 + bx + c$ $\quad$ $4 = 2a(1) + b$ $\quad$ $8 = 2a(-1) + b$

$y' = 2ax + b$ $\quad$ $2a + b = 4$ $\quad$ $-2a + b = 8$

$$\begin{aligned} 2a + b &= 4 \\ (+)\ -2a + b &= 8 \\ \hline 2b &= 12 \\ \boxed{b = 6} \end{aligned}$$

$$\begin{aligned} 2a + 6 &= 4 \\ 2a &= -2 \\ \boxed{a = -1} \end{aligned}$$

$y = -x^2 + 6x + c$ passes through $(2, 15)$

$15 = -4 + 12 + c$

$\boxed{7 = c}$

$\therefore\ y = -x^2 + 6x + 7$ is the equation

d) $\lim\limits_{x \to -1^-} \dfrac{|x+1|}{x^2 - 1}$

$\lim\limits_{x \to -1^-} \dfrac{|x+1|}{(x+1)(x-1)}$

$\lim\limits_{x \to -1^-} \dfrac{\cancel{|-1.0001+1|}^{\,-1}}{\cancel{(-1.0001+1)}(-1.0001-1)} = \dfrac{-1}{-2} = \boxed{\dfrac{1}{2}}$

Questions from Homework

⑥ a) $f(x) = \sqrt{x} - \sqrt{1-x} = x^{1/2} - (1-x)^{1/2}$

$$F(x) = \frac{2}{3}x^{3/2} - \frac{2}{3}(1-x)^{3/2}(-1) + C$$

$$F(x) = \frac{2}{3}x^{3/2} + \frac{2}{3}(1-x)^{3/2} + C$$

⑥ b) $f(x) = \frac{1}{x} - \frac{1}{1-x}$

$$F(x) = \ln|x| - \ln|1-x|(-1) + c$$

$$F(x) = \ln(x) + \ln(1-x) + c$$

$$F(x) = \ln(x - x^2) + C$$

Warm Up

Determine the general antiderivative of the following:

$$f(x)=2x^2-x+7$$

$$F(x)=\frac{2x^3}{3}-\frac{x^2}{2}+7x+C$$

$$f(x)=\cos x-\sin x$$

$$F(x)=\sin x+\cos x+C$$

$$f(x)=-3e^{-x}+6e^{2x}$$

$$F(x)=\frac{-3e^{-x}}{-1}+\frac{6e^{2x}}{2}+C$$

$$F(x)=3e^{-x}+3e^{2x}+C$$

$$f(x)=\frac{2}{x^2}-\frac{5}{x}+x \quad = 2x^{-2}-\frac{5}{x}+x$$

$$F(x)=\frac{2x^{-1}}{-1}-5\ln|x|+\frac{x^2}{2}+C$$

$$F(x)=-\frac{2}{x}-5\ln(x)+\frac{1}{2}x^2+C$$

$f(x)=\cos 5x - x^2\csc^2 x^3+ 5x\sin 2x^2$

$$F(x)=\frac{1}{5}\sin 5x+\frac{1}{3}\cot x^3-\frac{5}{4}\cos 2x^2+C$$

$$-\csc^2 x^3(3x^2) \qquad -\sin 2x^2(4x)$$

$$\frac{-3x^2\csc^2 x^3}{3} \qquad -4x\sin 2x^2$$

Differential Equations

An equation that involves the derivative of a function is called a differential equation:

As discussed previously, in applications of calculus it is very common to have a situation where it is required to find a function, given knowledge about its derivatives.

Find all functions g such that:

$$g'(x) = 4\sin x - 3x^5 + 6\sqrt[4]{x^3}$$

$$f(x) = -4\cos x - \frac{x^6}{2} + \frac{24}{7}x^{7/4} + C$$

$$g'(x) = 4\sin x - 3x^5 + 6x^{3/4}$$

$$g(x) = -4\cos x - \frac{3x^6}{6} + \frac{6x^{7/4}}{7/4} + C$$

$$g(x) = -4\cos x - \frac{1}{2}x^6 + \frac{24}{7}x^{7/4} + C$$

Identifying a unique solution for an antiderivative

Examples:

Determine the function with the given derivative whose graph satisfies the initial condition provided.

Find f if given f '(x): and f (0) = - 2 $\therefore$ when $x=0$ $y=-2$

$$f'(x) = e^x + \frac{20}{1+x^2} \longrightarrow f(x) = e^x + 20\tan^{-1}x - 3$$

$$f(x) = e^x + 20\tan^{-1}x + C$$

$$-2 = e^0 + 20\tan^{-1}(0) + C$$

$$-2 = 1 + 20(0) + C$$

$$-3 = C$$

$$\boxed{f(x) = e^x + 20\tan^{-1}x - 3}$$

$x=0, y=4$ $x=1, y=1$

Find f if given f ''(x): and f (0) = 4, and f (1) = 1

$$f''(x) = 12x^2 + 6x - 4 \longrightarrow f(x) = x^4 + x^3 - 2x^2 - 3x + 4$$

$$f'(x) = 4x^3 + 3x^2 - 4x + C$$

$$f(x) = x^4 + x^3 - 2x^2 + Cx + D$$

$$4 = (0)^4 + (0)^3 - 2(0)^2 + C(0) + D$$

$$4 = D$$

$$f(x) = x^4 + x^3 - 2x^2 + Cx + 4$$

$$1 = (1)^4 + (1)^3 - 2(1)^2 + C(1) + 4$$

$$1 = 1 + 1 - 2 + C + 4$$

$$-3 = C$$

$$\boxed{f(x) = x^4 + x^3 - 2x^2 - 3x + 4}$$

Practice Problems...

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Antiderivatives involving chain rule...

Remember how the Chain Rule works...

$$f(x) = [g(x)]^n$$

$$f'(x) = n[g(x)]^{n-1} g'(x)$$

Let's look at the following:

$$f'(x) = (x^2 - 3)^5 (2x)$$

$$f'(x) = x^2\sqrt{x^3 - 1}$$

$$f'(x) = \frac{3x}{\sqrt{1 - 5x^2}}$$

$$f'(x) = \frac{\cos 8x}{(1 + \sin 8x)^4}$$