Chapter 7

Graph an Absolute Value Function of the Form $f(x) = |ax^2 + bx + c|$

Example 2

Consider the absolute value function $f(x) = |-x^2 + 2x + 8|$.

a) Determine the y-intercept and the x-intercepts.

b) Sketch the graph.

c) State the domain and range.

d) Express as a piecewise function.

Solution

a) Determine the y-intercept by evaluating the function at $x = 0$.

$f(x) = |-x^2 + 2x + 8|$
$f(0) = |-(0)^2 + 2(0) + 8|$
$f(0) = |8|$
$f(0) = 8$

The y-intercept occurs at (0, 8)

The x-intercepts are the real zeros of the function, since they correspond to the x-intercepts of the graph.

$f(x) = |-x^2 + 2x + 8|$
$0 = -x^2 + 2x + 8$
$0 = -(x^2 - 2x - 8)$
$0 = -(x + 2)(x - 4)$
$x + 2 = 0$ or $x - 4 = 0$
$x = -2$ $\quad$ $x = 4$

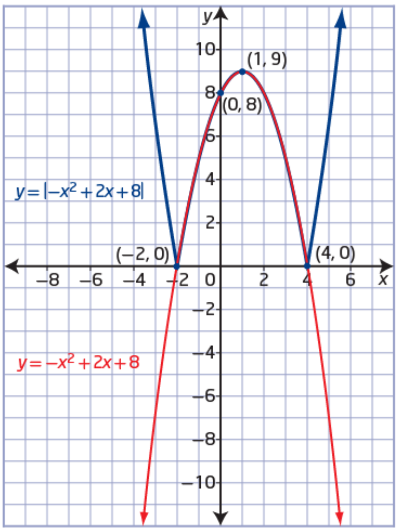
The x-intercepts occur at (−2, 0) and (4, 0).

b) Use the graph of $y = f(x)$ to graph $y = |f(x)|$.

Complete the square to convert the quadratic function $y = -x^2 + 2x + 8$ to vertex form, $y = a(x - p)^2 + q$.

$y = -x^2 + 2x + 8$
$y = -(x^2 - 2x) + 8$
$y = -(x^2 - 2x + 1 - 1) + 8$
$y = -[(x^2 - 2x + 1) - 1] + 8$
$y = -[(x - 1)^2 - 1] + 8$
$y = -(x - 1)^2 - 1(-1) + 8$
$y = -(x - 1)^2 + 9$

Since $p = 1$ and $q = 9$, the vertex is located at (1, 9). Since $a < 0$, the parabola opens downward. Sketch the graph.



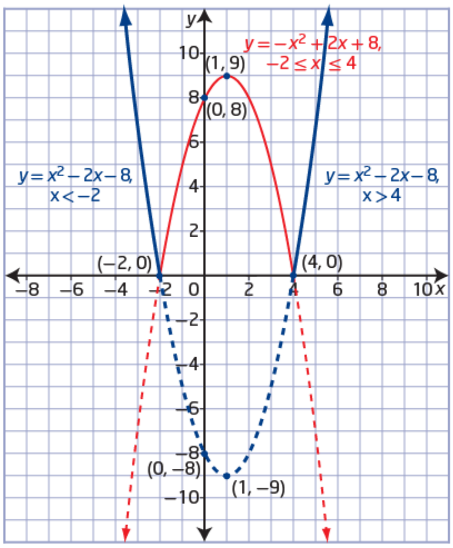
What other methods could you use to find the vertex of the quadratic function?

Reflect in the x-axis the part of the graph of $y = -x^2 + 2x + 8$ that lies below the x-axis.

c) The domain is all real numbers, or $\{x \mid x \in R\}$, and the range is all non-negative values of y, or $\{y \mid y \geq 0, y \in R\}$.

d) The graph of $y = |-x^2 + 2x + 8|$ consists of two separate quadratic functions. You can use the x-intercepts to identity each function's specific domain.

- When $-2 \leq x \leq 4$, the graph of $y = |-x^2 + 2x + 8|$ is the graph of $y = -x^2 + 2x + 8$, which is a parabola opening downward with a vertex at (1, 9), a y-intercept of 8, and x-intercepts at −2 and 4.
- When $x < -2$ or $x > 4$, the graph of $y = |-x^2 + 2x + 8|$ is the graph of $y = -x^2 + 2x + 8$ reflected in the x-axis. The equation of the reflected graph is $y = -(-x^2 + 2x + 8)$ or $y = x^2 - 2x - 8$, which is a parabola opening upward with a vertex at (1, −9), a y-intercept of −8, and x-intercepts at −2 and 4.



Express the absolute value function $y = |-x^2 + 2x + 8|$ as the piecewise function

$$y = \begin{cases} -x^2 + 2x + 8, \text{ if } -2 \leq x \leq 4 \\ -(-x^2 + 2x + 8), \text{ if } x < -2 \text{ or } x > 4 \end{cases}$$

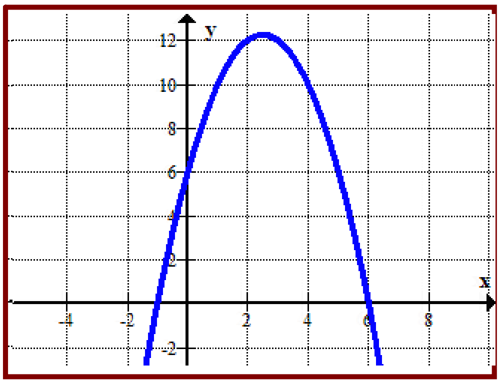
Chapter 7

Graphing an Absolute Value Function

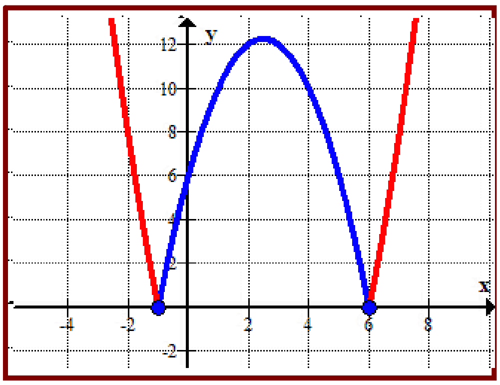
Sketch the graph of $y = |-x^2 + 5x + 6|$

The graph of $y = |-x^2 + 5x + 6|$ may be sketched using the graph of $y = -x^2 + 5x + 6$.

Use the pen tool to sketch the graph of the function $y = -x^2 + 5x + 6$ on the accompanying grid.



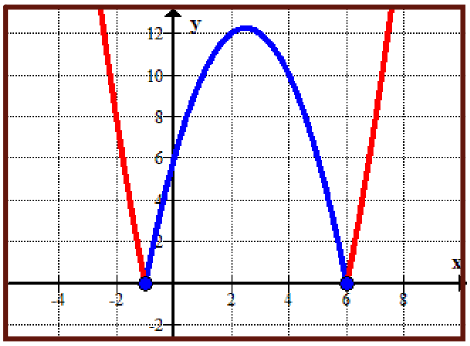
To graph the absolute value function, use the pen tool to reflect in the *x*-axis the sections of the graph that are below the *x*-axis.



Chapter 7

Graph an Absolute Value Function

Use the pen tool to express the absolute value function $y = |-x^2 + 5x + 6|$ as a piecewise function.



Click here for the solution.

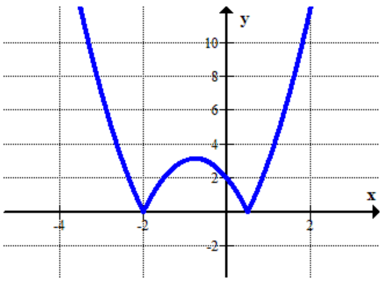
Chapter 7

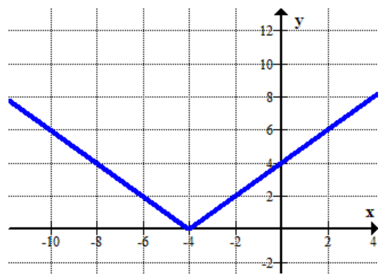
Graphs of Absolute Value Functions

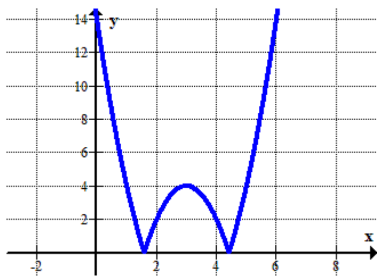
Drag each of the following absolute value functions to the matching graph. Pull the tabs to reveal the answers.

$y = |-2x^2 - 3x + 2|$ $y = |2(x-3)^2 - 4|$ $y = |-x - 2|$

$y = |x^2 - 9|$ $y = |2x - 5|$ $y = |-x - 4|$

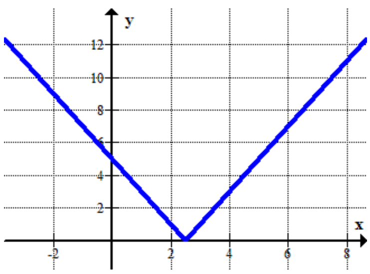


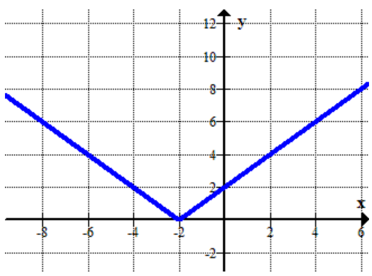


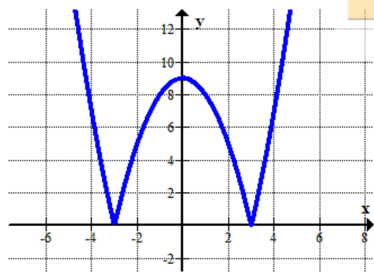


Pull

Pull to here







Pull

Pull to here

Chapter 7

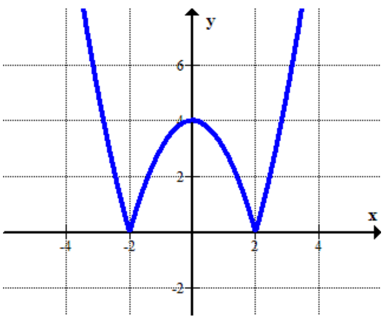
Absolute Value as a Piecewise Function

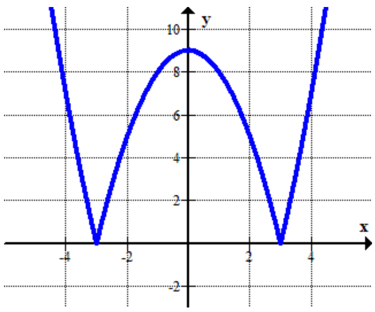
Match the piecewise definition with the graph of an absolute value function.

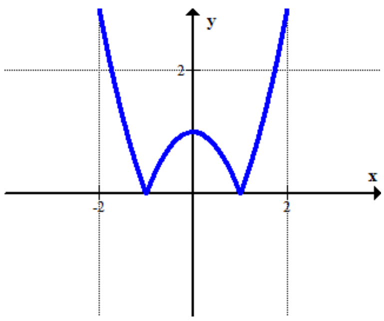
$$y = \begin{cases} x^2 - 1 & \text{if } x < -1 \text{ or } x > 1 \\ -(x^2 - 1) & \text{if } -1 \leq x \leq 1 \end{cases}$$

$$y = \begin{cases} x^2 - 4 & \text{if } x < -2 \text{ or } x > 2 \\ -(x^2 - 4) & \text{if } -2 \leq x \leq 2 \end{cases}$$

$$y = \begin{cases} x^2 - 9 & \text{if } x < -3 \text{ or } x > 3 \\ -(x^2 - 9) & \text{if } -3 \leq x \leq 3 \end{cases}$$







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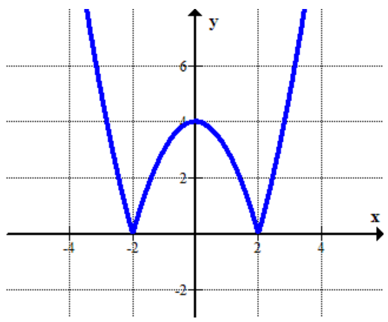
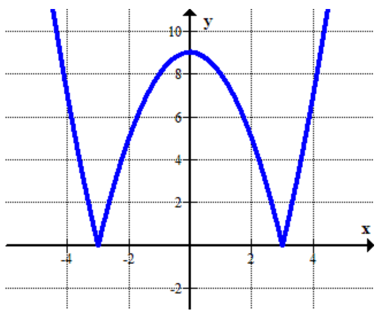
Click here for the solution.

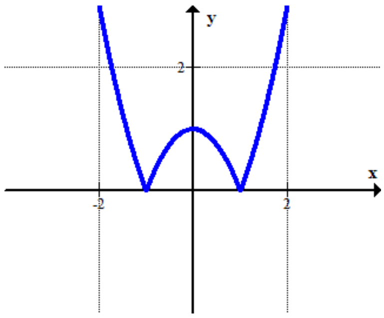
Solution



$$y=\begin{cases}x^2-4 & \text{if } x<-2 \text{ or } x>2\\ -(x^2-4) & \text{if } -2\le x\le 2\end{cases}$$



$$y=\begin{cases}x^2-9 & \text{if } x<-3 \text{ or } x>3\\ -(x^2-9) & \text{if } -3\le x\le 3\end{cases}$$



$$y=\begin{cases}x^2-1 & \text{if } x<-1 \text{ or } x>1\\ -(x^2-1) & \text{if } -1\le x\le 1\end{cases}$$

Go back to the question.

Chapter 7

Key Ideas

- You can analyse absolute value functions in several ways:
  - graphically, by sketching and identifying the characteristics of the graph, including the x-intercepts and the y-intercept, the minimum values, the domain, and the range
  - algebraically, by rewriting the function as a piecewise function
  - In general, you can express the absolute value function $y = |f(x)|$ as the piecewise function

$$y = \begin{cases} f(x), \text{ if } f(x) \geq 0 \\ -f(x), \text{ if } f(x) < 0 \end{cases}$$

- The domain of an absolute value function $y = |f(x)|$ is the same as the domain of the function $y = f(x)$.
- The range of an absolute value function $y = |f(x)|$ depends on the range of the function $y = f(x)$. For the absolute value of a linear or quadratic function, the range will generally, but not always, be $\{y \mid y \geq 0, y \in \mathrm{R}\}$.

Assignment

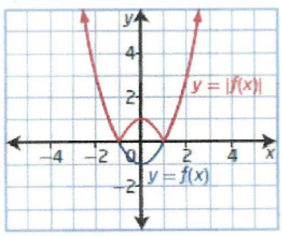
Complete pgs. 376 - 377

Questions 7, 8(abe), 9, 10

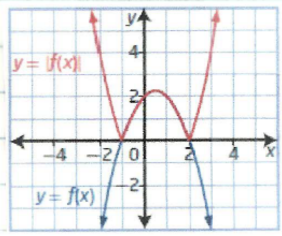
Solutions

7. Copy the graph of $y=f(x)$. On the same set of axes, sketch the graph of $y=|f(x)|$.

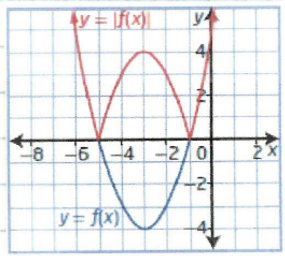
a)



b)



c)



Solutions

b) $y = |x^2 + 5x + 6|$

① Determine the y-int:

$y = |(0)^2 + 5(0) + 6|$
$y = |0 + 0 + 6|$
$y = |6|$
$y = 6$

② Determine the x-ints:

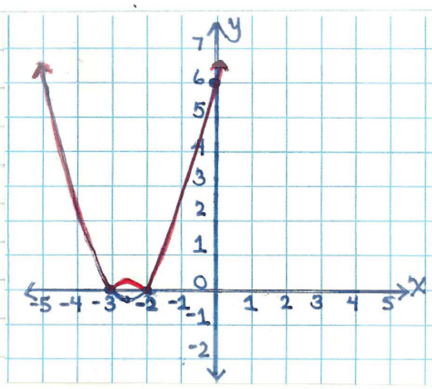
$0 = |x^2 + 5x + 6|$
$\Rightarrow 0 = x^2 + 5x + 6$
$0 = (x+2)(x+3)$
$x + 2 = 0$ or $x + 3 = 0$
$x = -2 \quad x = -3$

③ Complete the square to determine the vertex:

$y = |x^2 + 5x + 6|$
$\Rightarrow y = x^2 + 5x + 6$
$y = (x^2 + 5x) + 6$
$y = \left(x^2 + 5x + \frac{25}{4} - \frac{25}{4}\right) + 6$
$y = \left(x^2 + 5x + \frac{25}{4}\right) - \frac{25}{4} + \frac{6}{1}$
$y = \left(x + \frac{5}{2}\right)^2 - \frac{25}{4} + \frac{24}{4}$
$y = \left(x + \frac{5}{2}\right)^2 - \frac{1}{4}$

Vertex: $\left(-\frac{5}{2}, -\frac{1}{4}\right)$

④ Graph:



⑤ Domain:
$\{x \mid x \in R\}$

Range:
$\{y \mid y \geq 0, y \in R\}$

Solutions

8. Sketch the graph of each function. State the intercepts and the domain and range.

a) $y = |x^2 - 4|$

① Determine the y-int:

$y = |(0)^2 - 4|$

$y = |0 - 4|$

$y = |-4|$

$y = 4$

② Determine the x-ints:

$0 = |x^2 - 4|$

$\Rightarrow 0 = x^2 - 4$

$0 = (x-2)(x+2)$

$x - 2 = 0$ or $x + 2 = 0$

$x = 2$ $\quad$ $x = -2$

③ Complete the square to determine the vertex:

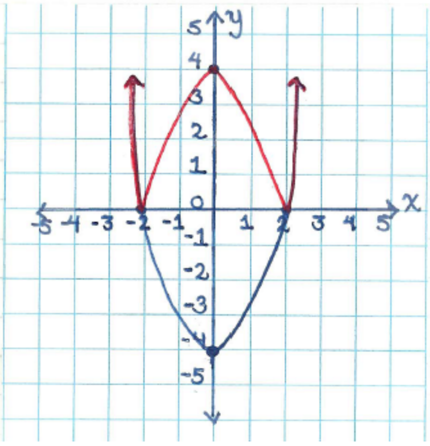
$y = |x^2 - 4|$

$\Rightarrow y = x^2 - 4$

$y = (x-0)^2 - 4$ (We just need to rewrite in this case)

Vertex: $(0, -4)$

④ Graph:



⑤ Domain:

$\{x \mid x \in R\}$

Range:

$\{y \mid y \geq 0, y \in R\}$

Solutions

e) $g(x) = |(x-3)^2 + 1|$

① Determine the y-int:

$y = |(0-3)^2 + 1|$

$y = |(-3)^2 + 1|$

$y = |9 + 1|$

$y = |10|$

$y = 10$

② Determine the x-ints:

$0 = |(x-3)^2 + 1|$

$\Rightarrow 0 = (x-3)^2 + 1$

$0 = (x-3)(x-3) + 1$

$0 = x^2 - 3x - 3x + 9 + 1$

$0 = x^2 - 6x + 10$

* Cannot be factored

↳ No x-ints.

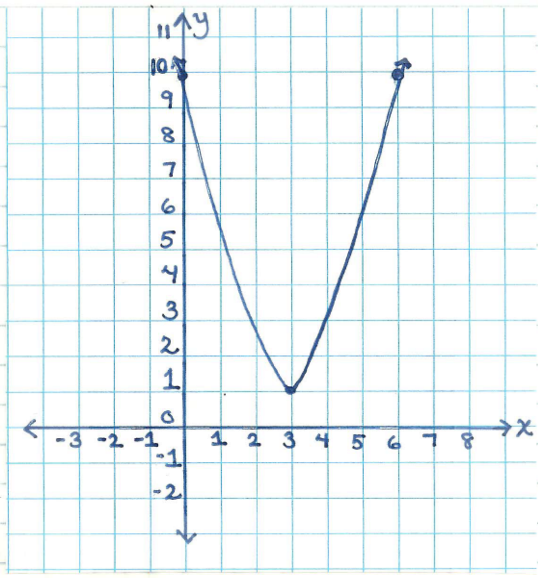
③ Complete the square to determine the vertex:

$y = |(x-3)^2 + 1|$

$\Rightarrow y = (x-3)^2 + 1$ (Already in vertex form ☺)

Vertex: (3, 1)

④ Graph:



⑤ Domain:

$\{x \mid x \in R\}$

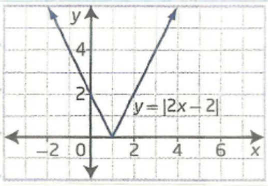
Range:

$\{y \mid y \geq 1, y \in R\}$

Solutions

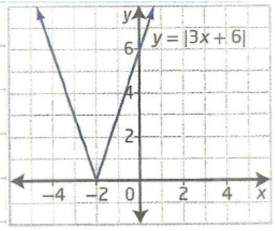
9. Write the piecewise function that represents each graph.

a)



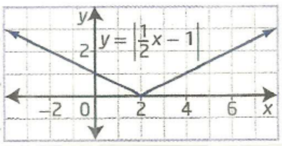
$$y = \begin{cases} 2x-2, & \text{if } x \ge 1 \\ -(2x-2), & \text{if } x < 1 \end{cases}$$

b)



$$y = \begin{cases} 3x+6, & \text{if } x \ge -2 \\ -(3x+6), & \text{if } x < -2 \end{cases}$$

c)

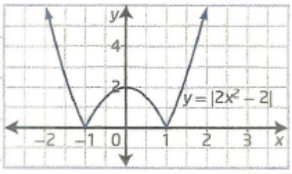


$$y = \begin{cases} \frac{1}{2}x-1, & \text{if } x \ge 2 \\ -(\frac{1}{2}x-1), & \text{if } x < 2 \end{cases}$$

Solutions

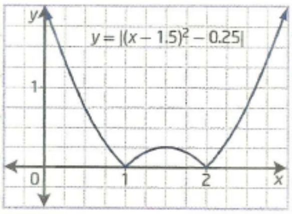
10. What piecewise function could you use to represent each graph of an absolute value function?

a)



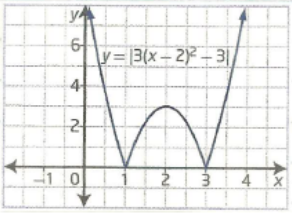
$$y=\begin{cases}2x^2-2, & \text{if } x\le -1 \text{ or } x\ge 1\\ -(2x^2-2), & \text{if } -1<x<1\end{cases}$$

b)



$$y=\begin{cases}(x-1.5)^2-0.25, & \text{if } x\le 1 \text{ or } x\ge 2\\ -[(x-1.5)^2-0.25], & \text{if } 1<x<2\end{cases}$$

c)



$$y=\begin{cases}3(x-2)^2-3, & \text{if } x\le 1 \text{ or } x\ge 3\\ -[3(x-2)^2-3], & \text{if } 1<x<3\end{cases}$$