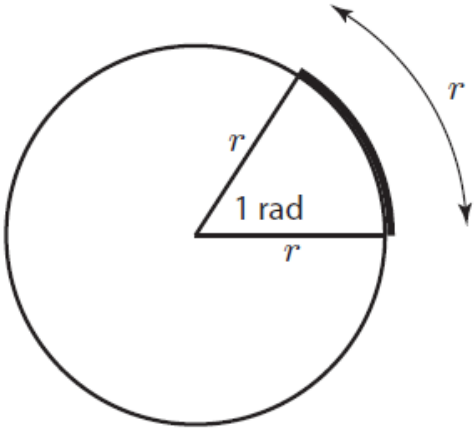
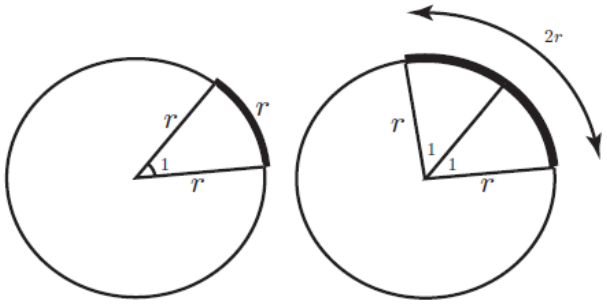
Radian Measure

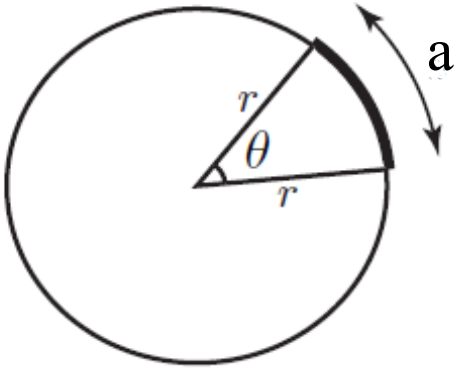
A radian is the angle subtended by an arc of length r (radius)





An angle of 2 rad subtends an arc of 2r...
7 rads subtends an arc of??? 7r

Use the above information to develop a formula to connect arc length, radius and the measure of an angle in radian measure...



has to be in radians

$$a = \theta r$$

Principal Angles

The smallest positive coterminal angle between 0 and 360° or 2π.

or 2π

1) **Divide By 360° (how many rotations?).**
2) **Get rid of # of full rotations.**
3) **Mulitply decimal by 360° to find principal angle.**

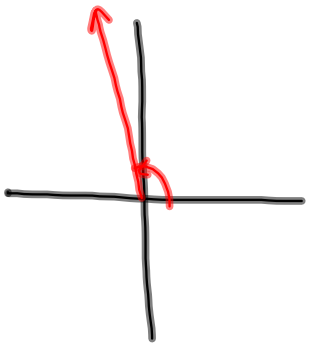
or 2π

Ex: 13784°

① $13784^\circ \div 360^\circ = 38.2\overline{8}$

② $38.2\overline{8} - 38 = 0.2\overline{8}$

③ $0.2\overline{8} \times 360^\circ = 104^\circ$



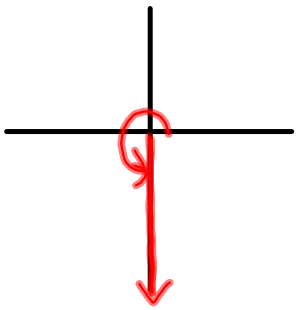
For each angle in standard position, determine one positive and one negative angle measure that is coterminal with it. (Make a sketch)

a) $270°$ **b)** $-\frac{5\pi}{4}$ **c)** $740°$

a) $270°$

① $270° + 360° = 630°$

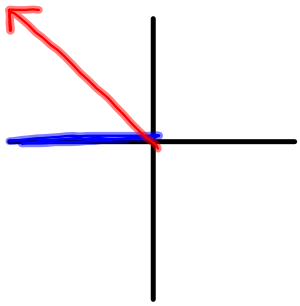
② $270° - 360° = -90°$



b) $-\frac{5\pi}{4}$

① $-\frac{5\pi}{4} + \frac{2\pi}{1} = -\frac{5\pi}{4} + \frac{8\pi}{4} = \frac{3\pi}{4}$

② $-\frac{5\pi}{4} - \frac{2\pi}{1} = -\frac{5\pi}{4} - \frac{8\pi}{4} = -\frac{13\pi}{4}$

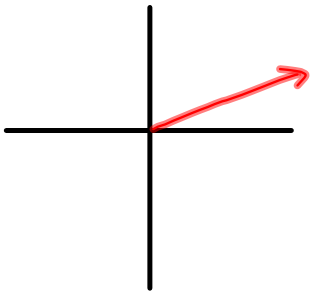


$\frac{2\pi}{4}, \frac{3\pi}{4}, \frac{4\pi}{4}$ ← 1π

c) $740°$

① $740° - 360° = 380°$

② $740° - 720° = 20°$



Extra Practice

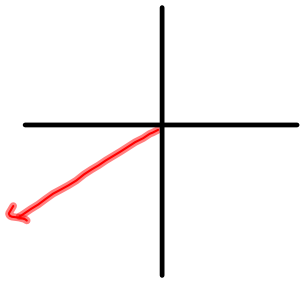
Sketch the following angles.

a) $\frac{7\pi}{6}$ b) $\frac{8\pi}{3}$ c) 5.7 d) $-\frac{11\pi}{4}$

Answers:

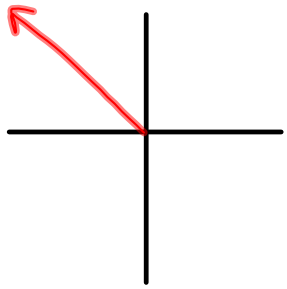
a) $\frac{6\pi}{6}$, $\frac{7\pi}{6}$, $\frac{8\pi}{6}$

1π

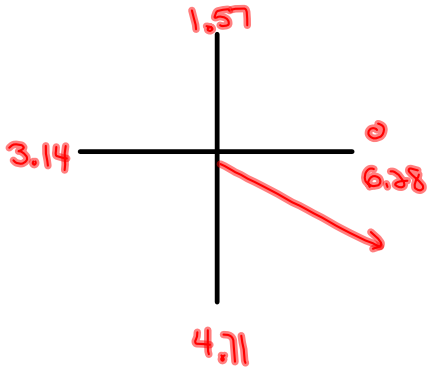


b) $\frac{7\pi}{3}$, $\frac{8\pi}{3}$, $\frac{9\pi}{3}$

3π

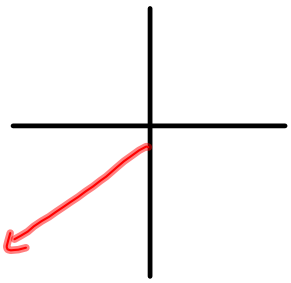


c) 5.7 rads



d) $-\frac{12\pi}{4}$, $-\frac{11\pi}{4}$, $-\frac{3\pi}{4}$

-3π

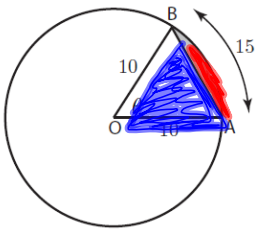


Questions from Homework

Example

Refer to Figure 8. Suppose we have a circle of radius 10cm and an arc of length 15cm. Suppose we want to find (a) the angle θ, (b) the area of the sector OAB, (c) the area of the minor segment (shaded).

Given:
$r = 10cm$
$a = 15cm$



a) Find θ

$a = \theta r$

$15 = \theta(10)$

$1.5 rads = \theta$

Figure 8. The shaded area is called the minor segment.

b) $\frac{\text{Area of Sector}}{\text{Area of Circle}} = \frac{\text{Central Angle}}{\text{Complete Rotation}}$

$$\frac{x}{\pi(10)^2} = \frac{1.5 \text{ rads}}{2\pi \text{ rads}}$$

$$x = \frac{150\pi}{2\pi}$$

$$x = 75\,cm^2$$

c) Area of Seg = Area of Sec − Area of Triangle

① Area of Triangle $= \frac{1}{2}r^2\sin\theta$

$A_\Delta = \frac{1}{2}(10)^2\sin(1.5)$ ← rads (Convert to rads on calculato

$A_\Delta = \frac{1}{2}(100)(0.997)$

$A_\Delta = 49.85\,cm^2$

② $A_{seg} = A_{sec} - A_\Delta$

$= 75cm^2 - 49.85cm^2$

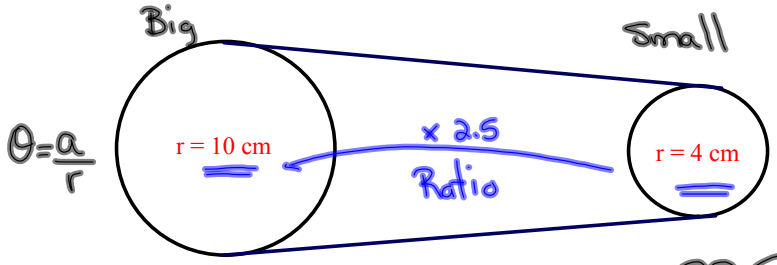
$= 25.15cm^2$

Applying our knowledge of rotations and radians...

Ex. (a) If the large wheel rotates $2\pi/3$ radians, how many radians does the smaller wheel rotate? Find θ

(b) If the large wheel completes three revolutions, how much does the small wheel rotate in radians?

(c) If the small wheel rotates $-15\pi/4$ radians, how many radians does the larger whell rotate?



a) Big $\theta = \frac{a}{r}$

$(10)\ \frac{2\pi}{3} = \frac{a}{10}\ (10)$

$\frac{20\pi}{3}\text{ cm} = a$

Small $\theta = \frac{a}{r}$

$\theta = \frac{\frac{20\pi}{3}\text{ cm}}{4\text{ cm}}$

$\theta = \frac{20\pi}{3} \times \frac{1}{4}$

$\theta = \frac{20\pi}{12} = \frac{5\pi}{3}$ rads

Using Ratio: $\frac{2\pi}{3} \cdot 2.5 = \frac{5\pi}{3}$

b) 3 revs = 6π rads

Big $\theta = \frac{a}{r}$

$(10)\ 6\pi = \frac{a}{10}\ (10)$

$60\pi\text{ cm} = a$

Small $\theta = \frac{a}{r}$

$\theta = \frac{60\pi\text{ cm}}{4\text{ cm}}$

$\theta = 15\pi$ rads

Using Ratio: $6\pi \cdot 2.5 = 15\pi$

c) Small $\theta = \frac{a}{r}$

$\frac{-15\pi}{4} = \frac{a}{4}$

$-15\pi\text{ cm} = a$

Big $\theta = \frac{a}{r}$

$\theta = \frac{-15\pi\text{ cm}}{10\text{ cm}}$

$\theta = -\frac{3\pi}{2}$ or -1.5π rads

Using Ratio $-\frac{15\pi}{4} \div 2.5 = -1.5\pi$

Angular Velocity

Angular velocity - amount of rotation around a central point per unit of time

$$v_a = \frac{\theta}{t} \qquad \theta = \frac{a}{r}$$

θ = angle (radians)

v_a = angular velocity

t = time

a = arc length

r = radius

Ex. The roller on a computer printer makes 2200 rpm (revolution per minute). Find the roller's angular velocity.

$2200 \frac{\text{revs}}{\text{min}} \times 2\pi \frac{\text{rads}}{\text{rev}} = 4400\pi \frac{\text{rads}}{\text{min}}$ ← angular velocity (V_a)

$= \frac{4400\pi \text{ rads}}{\text{min}}$

Convert to rads/sec → $\frac{4400\pi \text{ rads}}{60\text{s}} = 73.\overline{3} \frac{\text{rads}}{\text{sec}}$

Homework

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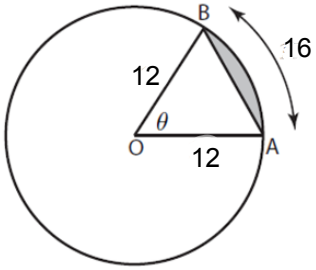
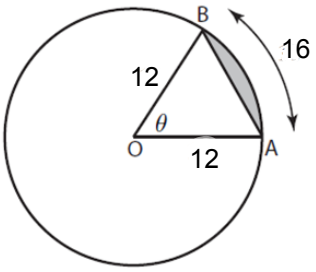
Find the area of the shaded region





① Find θ

$\theta = \frac{a}{r}$

$\theta = \frac{16}{12}$

$\theta = \frac{4}{3}$ rads

② $\frac{\text{Sector Area}}{\text{Area of Circle}} = \frac{\text{Central Angle}}{\text{Complete Rev}}$

$\frac{x}{\pi(12)^2} = \frac{4/3}{2\pi}$

$x = 96\text{cm}^2$

③ $A_\Delta = \frac{1}{2}r^2 \sin\theta$

$A_\Delta = \frac{1}{2}(12)^2 \sin\left(\frac{4}{3}\right)$

$A_\Delta = \frac{1}{2}(144)(0.972)$

$A_\Delta = 70\text{ cm}^2$

④ $A_{seg} = A_{sec} - A_\Delta$

$A_{seg} = 96\text{cm}^2 - 70\text{cm}^2$

$A_{seg} = 26\text{cm}^2$