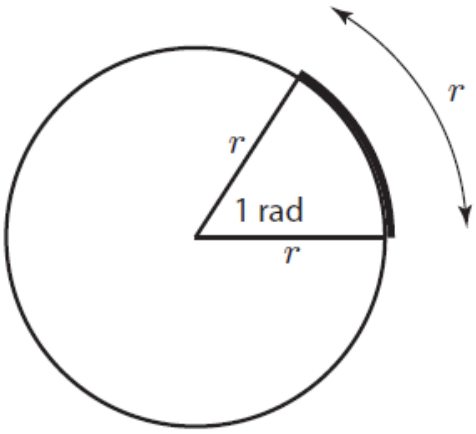
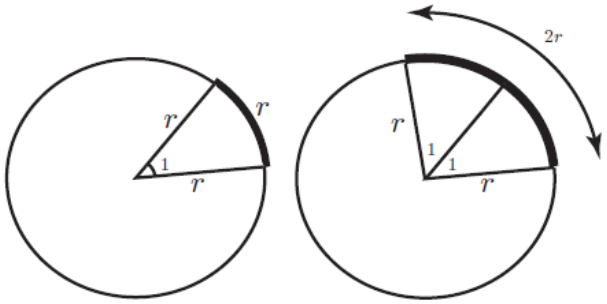
Radian Measure

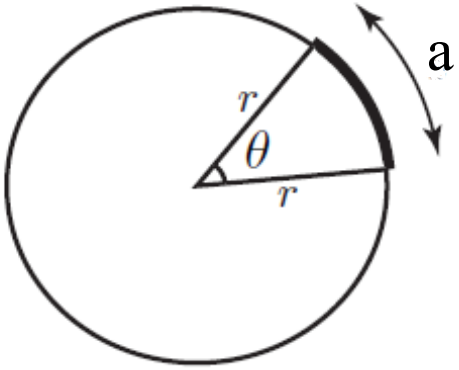
A radian is the angle subtended by an arc of length r (radius)





An angle of 2 rad subtends an arc of 2r...
7 rads subtends an arc of??? 7r

Use the above information to develop a formula to connect arc length, radius and the measure of an angle in radian measure...



has to be in radians

$$a = \theta r$$

Check-Up...

Arrange the following angles in descending order:

① $340°$ ③ $4.28\,rad$ ⑤ $\frac{9\pi}{5}$ ④ $(10\pi)°$

$31.4°$

$4.28\left(\frac{180}{\pi}\right)$ $\frac{9\pi}{5}\left(\frac{180}{\pi}\right)$

$245°$ $324°$

Find the angles co-terminal to θ on the given domain

$\theta = \frac{5\pi}{6}$, $-2\pi \le \theta \le 8\pi$

$-\frac{12\pi}{6} \le \theta \le \frac{48\pi}{6}$

$\theta = -\frac{7\pi}{6}, \frac{17\pi}{6}, \frac{29\pi}{6}, \frac{41\pi}{6}$

Homework

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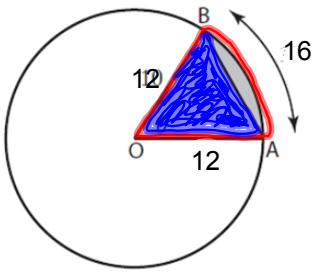
Find the area of the shaded region ← segment

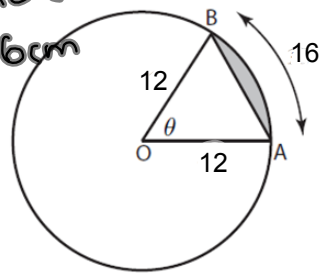


Area of Segment = Area of Sector − Area of Triangle

Questions from Homework

$r = 12\text{ cm}$

$a = 16\text{cm}$



① Find θ

$\theta = \frac{a}{r}$

$\theta = \frac{16}{12}$

$\theta = \frac{4}{3}$ rads

② $\frac{\text{Sector Area}}{\text{Area of Circle}} = \frac{\text{Central Angle}}{\text{Complete Rev}}$

$\frac{x}{\pi(12)^2} = \frac{4/3}{2\pi}$

$x = 96\text{cm}^2$

③ $A_\Delta = \frac{1}{2}r^2\sin\theta$

$A_\Delta = \frac{1}{2}(12)^2\sin\left(\frac{4}{3}\right)$

$A_\Delta = \frac{1}{2}(144)(0.972)$

$A_\Delta = 70\text{ cm}^2$

④ $A_{seg} = A_{sec} - A_\Delta$

$A_{seg} = 96\text{cm}^2 - 70\text{cm}^2$

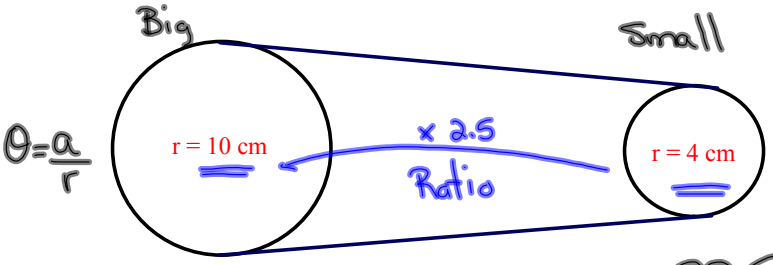
$A_{seg} = 26\text{cm}^2$

Applying our knowledge of rotations and radians...

Ex. (a) If the large wheel rotates $2\pi/3$ radians, how many radians does the smaller wheel rotate? Find θ

(b) If the large wheel completes three revolutions, how much does the small wheel rotate in radians?

(c) If the small wheel rotates $-15\pi/4$ radians, how many radians does the larger wheel rotate?



a) Big $\theta = \frac{a}{r}$

$(10)\ \frac{2\pi}{3} = \frac{a}{10}\ (10)$

$\frac{20\pi}{3}\text{ cm} = a$

Small $\theta = \frac{a}{r}$

$\theta = \frac{\frac{20\pi}{3}\text{ cm}}{4\text{ cm}}$

$\theta = \frac{20\pi}{3} \times \frac{1}{4}$

$\theta = \frac{20\pi}{12} = \frac{5\pi}{3}\text{ rads}$

Using Ratio: $\frac{2\pi}{3} \cdot 2.5 = \frac{5\pi}{3}$

b) 3 revs = 6π rads

Big $\theta = \frac{a}{r}$

$(10)\ 6\pi = \frac{a}{10}\ (10)$

$60\pi\text{ cm} = a$

Small $\theta = \frac{a}{r}$

$\theta = \frac{60\pi\text{ cm}}{4\text{ cm}}$

$\theta = 15\pi$ rads

Using Ratio: $6\pi \cdot 2.5 = 15\pi$

c) Small $\theta = \frac{a}{r}$

$\frac{-15\pi}{4} = \frac{a}{4}$

$-15\pi\text{ cm} = a$

Big $\theta = \frac{a}{r}$

$\theta = \frac{-15\pi\text{ cm}}{10\text{ cm}}$

$\theta = -\frac{3\pi}{2}$ or -1.5π rads

Using Ratio $-\frac{15\pi}{4} \div 2.5 = -1.5\pi$

Angular Velocity

Angular velocity - amount of rotation around a central point per unit of time

$$v_a = \frac{\theta}{t} \qquad \theta = \frac{a}{r}$$

θ = angle (radians)

v_a = angular velocity

t = time

a = arc length

r = radius

Ex. The roller on a computer printer makes 2200 rpm (revolution per minute). Find the roller's angular velocity.

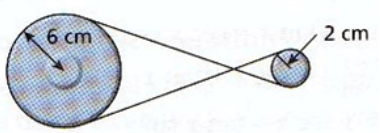
$2200 \frac{\text{revs}}{\text{min}} \times 2\pi \frac{\text{rads}}{\text{rev}} = 4400\pi \frac{\text{rads}}{\text{min}}$ ← angular velocity (V_a)

$= \frac{4400\pi \text{ rads}}{\text{min}}$

Convert to rads/sec → $\frac{4400\pi \text{ rads}}{60\text{s}} = 73.\overline{3} \frac{\text{rads}}{\text{sec}}$

Two flywheels are connected by a belt, as shown in the diagram below. The larger one has a radius of 6 cm and the smaller one has a radius of 2 cm.

Ratio 3:1



(a) If the small wheel rotates –300°, then through how many radians does the large wheel rotate?

(b) If the large wheel rotates $\frac{7\pi}{6}$ radians, what distance would a point on the circumference of the small wheel rotate?

a) $-300°\left(\frac{\pi}{180}\right) = \boxed{-\frac{5\pi}{3} \text{ rads}}$ ← θ

When small rotates clockwise, large wheel rotates ccw.

Using Ratio (3:1)

$-\frac{5\pi}{3} \div 3 = -\frac{5\pi}{3} \times \frac{1}{3} = +\frac{5\pi}{9}$ rads

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a) Small

$\theta = \frac{a}{r}$

$-\frac{5\pi}{3} = \frac{a}{2\text{cm}}$ (at)

$\boxed{-\frac{10\pi}{3}\text{ cm} = a}$

Large

$\theta = \frac{a}{r}$

$\theta = \frac{-\frac{10\pi}{3}\text{ cm}}{6\text{ cm}}$

$\theta = -\frac{10\pi}{3} \cdot \frac{1}{6} = -\frac{10\pi}{18} = \boxed{+\frac{5\pi}{9}}$

↑ Same on both wheels

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b) Large

$\theta = \frac{a}{r}$

$\frac{7\pi}{6} = \frac{a}{6}$

$\boxed{a = 7\pi \text{ cm}}$

← arcs are equal for both wheels

Ex. A small electrical motor turns at 2200 rpm. $2200 \times 2\pi = 4400\pi$

(a) Express the angular velocity in rad/s.
(b) Find the distance a point 0.8cm from the center of rotation travels in 0.008 s.

r

a) $V_a = \frac{\theta}{t} = 4400\pi \frac{rads}{min} \cdot \frac{min}{60s} = 73.3\pi \text{ rads/sec}$

b) The point would rotate $73.3\pi \frac{rads}{sec} \times 0.008 sec = 1.843 rad$ (θ)

$a = \theta r$

$a = (1.843)(0.8)$

$a = 1.47 \text{ cm}$

Homework

Ex. A Ferris Wheel rotates 3 times each minute. The passengers sit in seats that are 5 m from the center of the wheel. What is the angular velocity of the wheel in radians per second? What distance do the passengers travel in 6.5 seconds?

Answer: a) $V_a = 0.314$ rads/sec

b) $a = 10.2$ m

Ex. A bicycle wheel has a radius of 36 cm and is turning at 4.8m/s. Determine the angu velocity of this wheel?

Answer: $V_a = 13.\overline{3}$ rads/sec