$\theta = 3(2\pi) = 6\pi$ rads

Ex. A Ferris Wheel rotates 3 times each minute. The passengers sit in seats that are 5 m from the center of the wheel. What is the angular velocity of the wheel in radians per second? What distance do the passengers travel in 6.5 seconds?

radius

a) $V_a = \frac{\theta}{t} = \frac{6\pi \text{ rads}}{\text{min}} = \frac{6\pi \text{ rads}}{60 \text{ sec}} = \boxed{0.314 \text{ rads/sec}}$

b) (i) Find θ:

$\theta = 0.314 \frac{\text{rads}}{\cancel{\text{sec}}} \times 6.5 \cancel{\text{sec}}$

$\theta = 2.041$ rads

(ii) Find a:

$a = \theta r$

$a = (2.041)(5)$

$\boxed{a = 10.25\text{m}}$

Ex. A bicycle wheel has a radius of 36 cm and is turning at 4.8m/s. Determine the angu velocity of this wheel?

Given:

$r = 36cm = 0.36m$

arc length after 1sec:

$a = 4.8m$

(i) Find θ:

$\theta = \frac{a}{r} = \frac{4.8}{0.36} = \boxed{13.\overline{3} \text{ rads}}$

(ii) Find V_a:

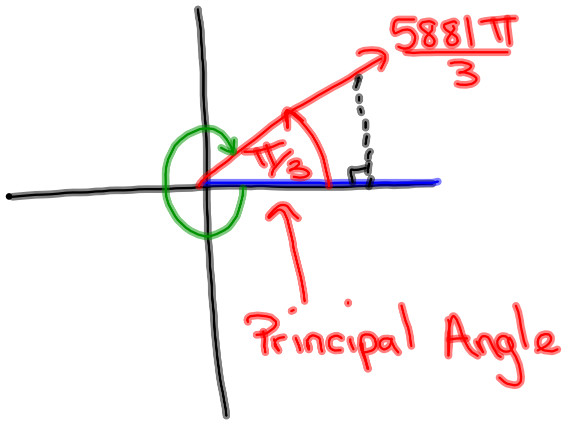
$V_a = \frac{\theta}{t} = \boxed{\frac{13.3 \text{rads}}{\text{sec}}}$

Sketch the following and determine a negative angle co-terminal with:

(i) $\frac{5881\pi}{3}$

$\frac{5880\pi}{3}$, $\frac{5881\pi}{3}$, $\frac{5882\pi}{3}$

1960π



Negative co-terminal angle:

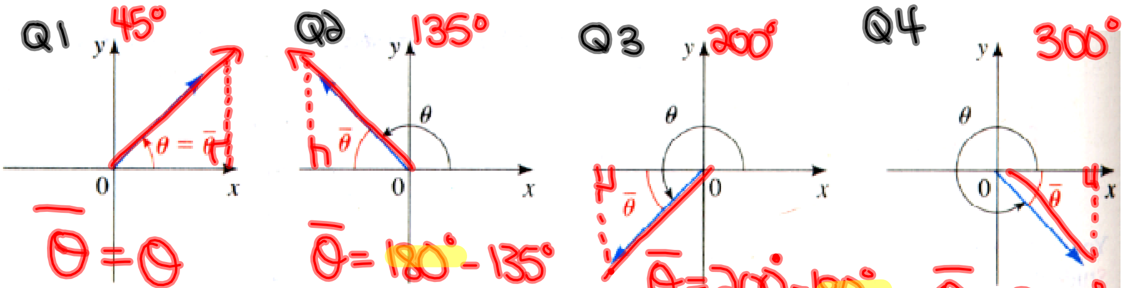
$$\frac{\pi}{3} - \frac{2\pi}{1} = \frac{\pi}{3} - \frac{6\pi}{3} = \boxed{-\frac{5\pi}{3}}$$

Reference Triangles:

Definition 17 *The reference angle $\bar{\theta}$ of an angle θ in standard position is the acute angle (between 0 and 90°) the terminal side makes with the x-axis.*

0 and $\pi/2$ rads

The picture below illustrates this concept.



$\bar{\theta} = \theta = 45°$

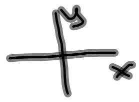
$\bar{\theta} = 180° - 135° = 45°$

$\bar{\theta} = 200° - 180°$
$\bar{\theta} = 20°$

$\bar{\theta} = 360° - 300° = 60°$

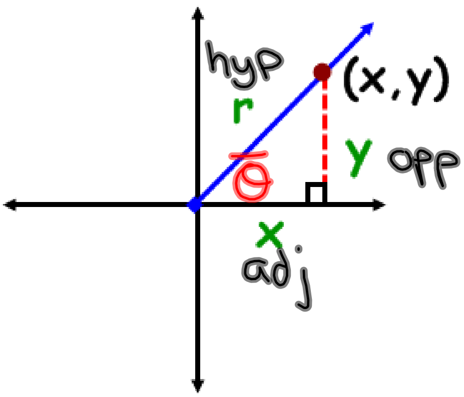
What is the significance of reference angles?

Angles on the Cartesian Plane



- **Reference Angle** - an acute angle formed between the terminal arm and the x-axis.

- **Reference Triangle** - a triangle formed by drawing a perpendicular line from a point on the terminal to the x-axis.



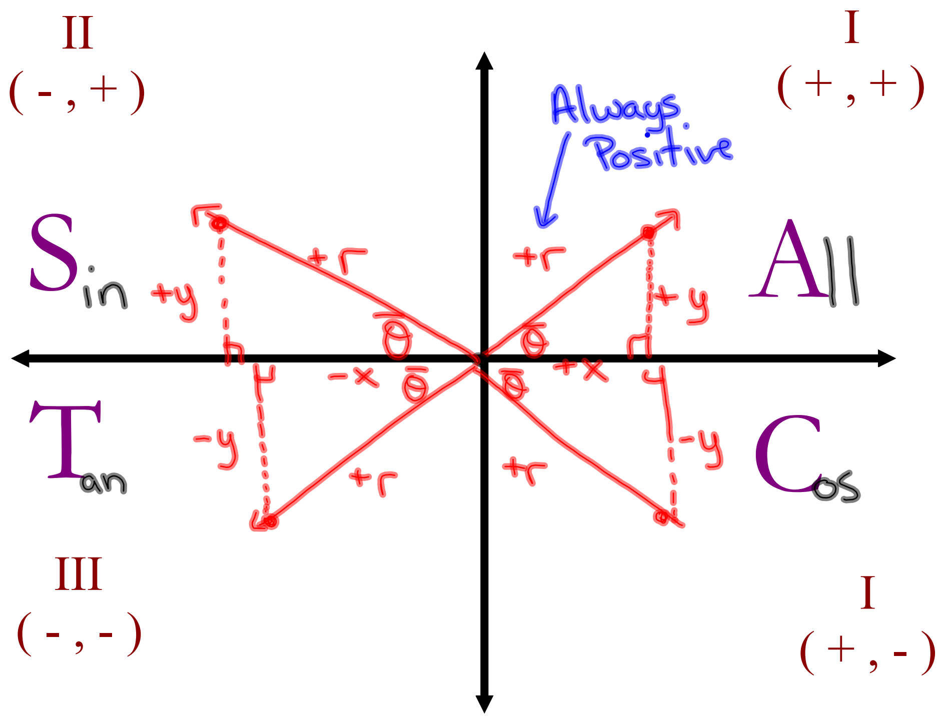
Notice what will happen if the rotation moves into other quadrants?

TRIG RATIOS on the CARTESIAN PLANE

$\sin\theta = \frac{y}{r}$	$\csc\theta = \frac{r}{y}$
$\cos\theta = \frac{x}{r}$	$\sec\theta = \frac{r}{x}$
$\tan\theta = \frac{y}{x}$	$\cot\theta = \frac{x}{y}$
"Primary"	"Reciprocal"

TRIG RATIOS IN ALL 4 QUADRANTS

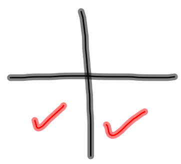
What primary trig ratios are POSITIVE in...



Where is θ if...

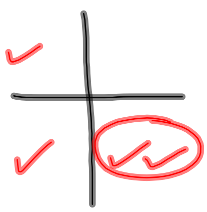
csc θ<0

(sin θ)



Quad 3 or Quad 4

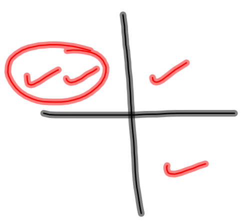
sin θ<0 & tan θ<0



Quad 4

csc θ>0 & cot θ<0

(sin θ) (tan θ)



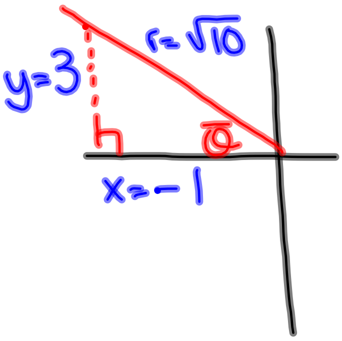
Quad 2

If $\sec\theta = -\sqrt{10}$ *and* $\sin\theta > 0$, determine the value of $\csc\theta$.

$\sec\theta = -\dfrac{\sqrt{10}}{1}$ $\quad \dfrac{r}{x}$

$r = \sqrt{10}$

$x = -1$



$x^2 + y^2 = r^2$

$(-1)^2 + y^2 = (\sqrt{10})^2$

$1 + y^2 = 10$

$y^2 = 9$

$y = \pm 3$

Choose $y = 3$

$\csc\theta = \dfrac{hyp}{opp} = \dfrac{r}{y} = \dfrac{\sqrt{10}}{3}$

Example

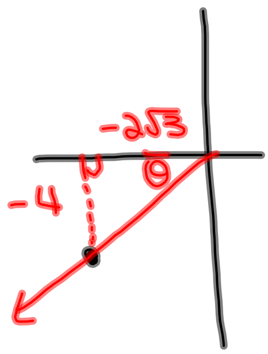
Determine the measure (in radians) of an angle whose terminal arm passes through the ordered pair $\left(-2\sqrt{3}, -4\right)$

x, y

Given:

$x = -2\sqrt{3}$

$y = -4$



$\tan\bar{\theta} = \frac{y}{x}$

$\tan\bar{\theta} = \frac{-4}{-2\sqrt{3}}$

$\tan\bar{\theta} = \frac{2}{\sqrt{3}}$

```
tan⁻¹(2/√(3))
     49.10660535
```

$\bar{\theta} = \tan^{-1}\left(\frac{2}{\sqrt{3}}\right)$

$\bar{\theta} = 49.1$

$\theta = 49.1^\circ + 180^\circ$

$\theta = 229.1^\circ$

To Convert to Radians

$229.1\left(\frac{\pi}{180}\right) = \boxed{3.99 \text{ rads}}$

In Radians

```
tan⁻¹(2/√(3))
     .8570719479
```

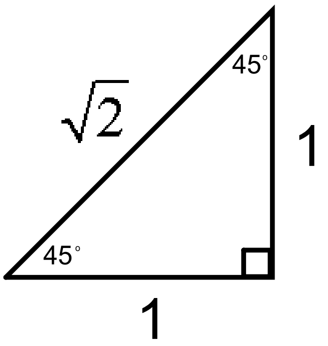
$\theta = 0.857 + \pi$

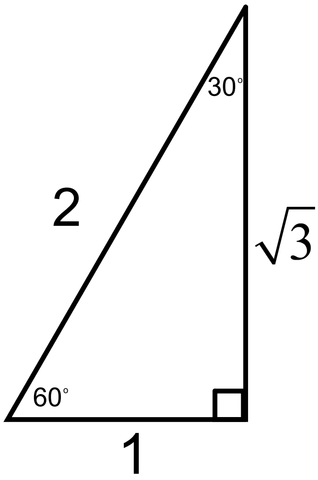
$\theta = 0.857 + 3.14$

$\theta = 3.99 \text{ rads}$

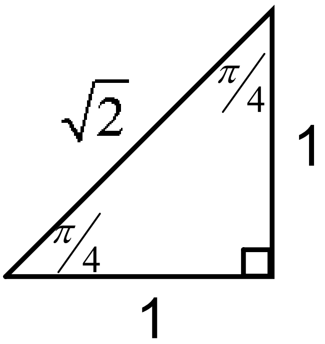
Special Angles

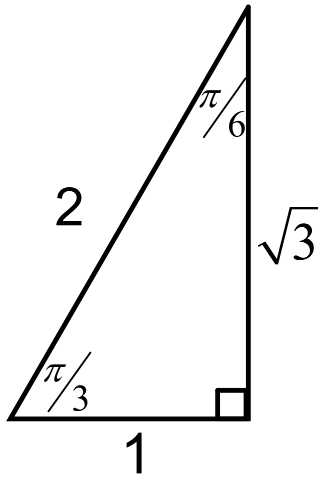
In Degrees:



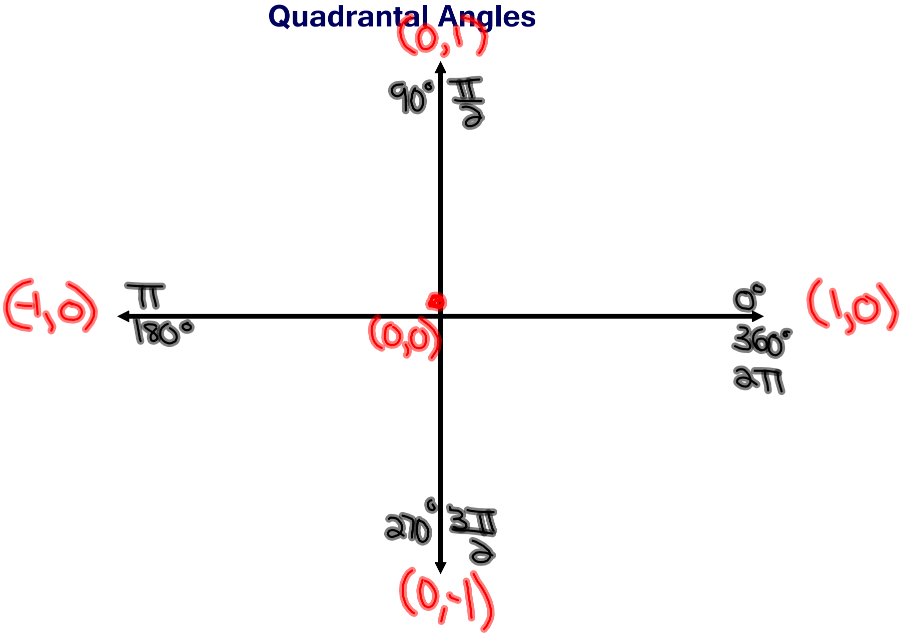


In Radians:



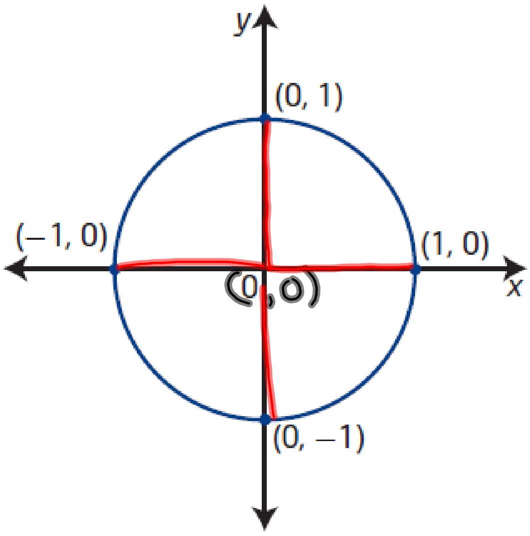


Quadrantal Angles



Unit Circle

(used for multiples of 90° or $\frac{\pi}{2}$ rads)



unit circle

- a circle with radius 1 unit
- a circle of radius 1 unit with centre at the origin on the Cartesian plane is known as *the* unit circle

$\sin\theta = \frac{y}{r} = \frac{y}{1} = y$

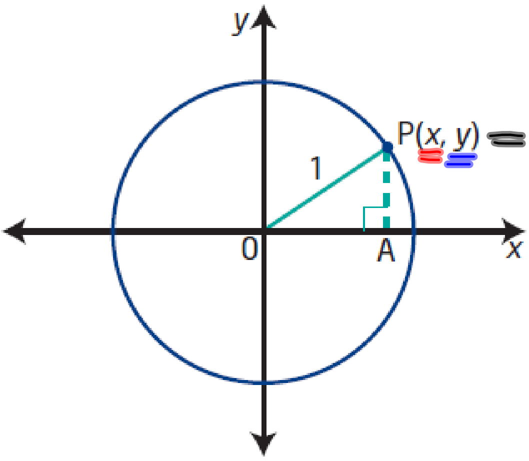
$\cos\theta = \frac{x}{r} = \frac{x}{1} = x$

$\tan\theta = \frac{y}{x}$

$\csc\theta = \frac{1}{y}$

$\sec\theta = \frac{1}{x}$

$\cot\theta = \frac{x}{y}$



$= (\cos\theta, \sin\theta)$

$\boxed{x^2+y^2=r^2}$ ← equation of circle

$x^2+y^2=(1)^2$

The equation of the unit circle is $x^2 + y^2 = 1$.

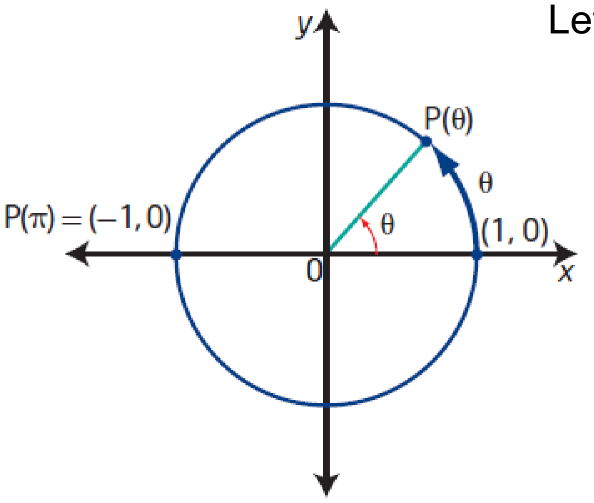
Determine the equation of a circle with centre at the origin and radius 6.

$x^2+y^2=r^2$

$x^2+y^2=(6)^2$

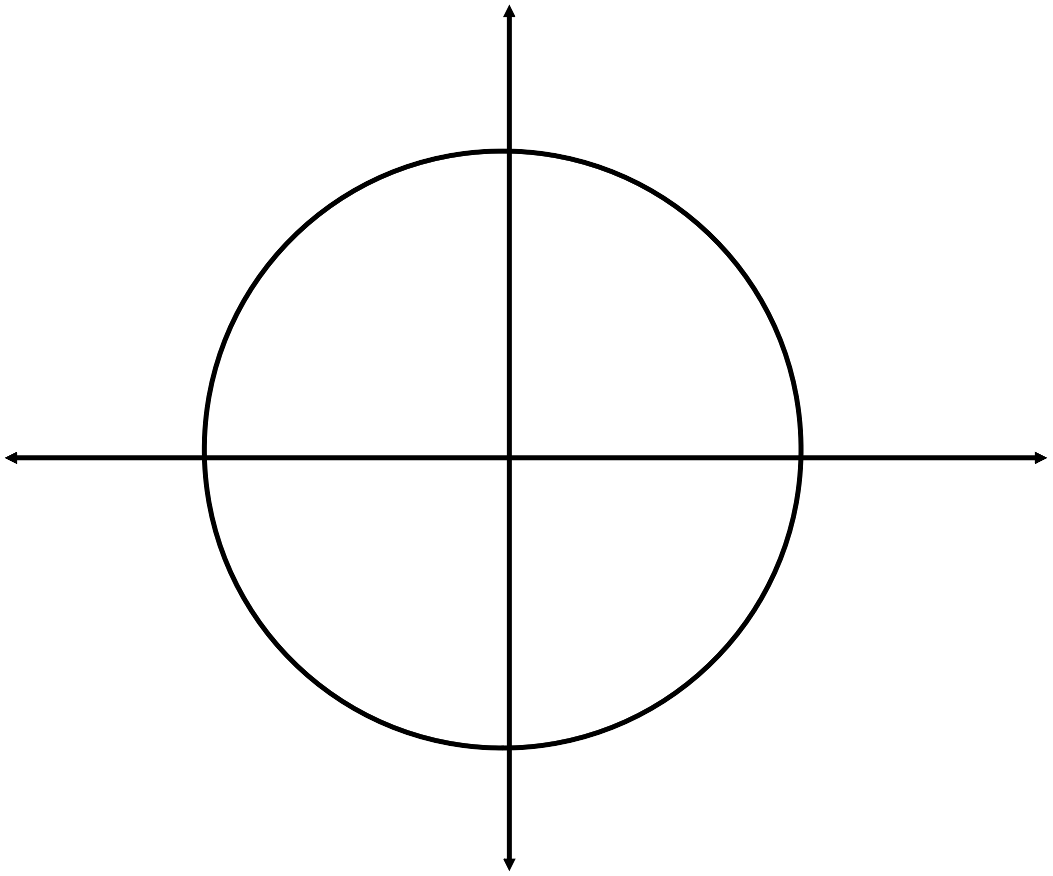
$x^2+y^2=36$

Special Angles on the Unit Circle:

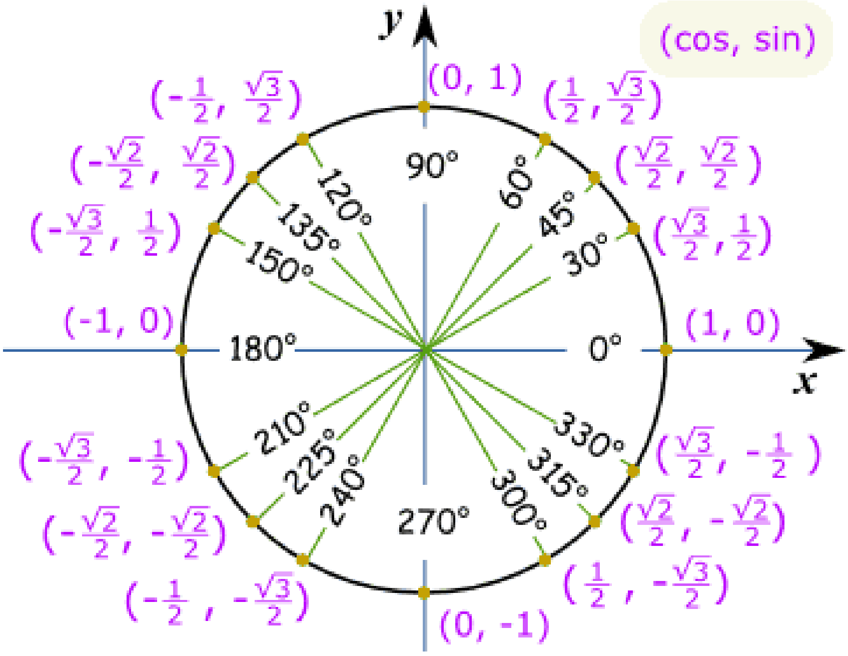


Let's use $\frac{\pi}{4}$ as our reference angle

Construct reference triangles for all multiples of $\pi/4$ between 0 and 2π

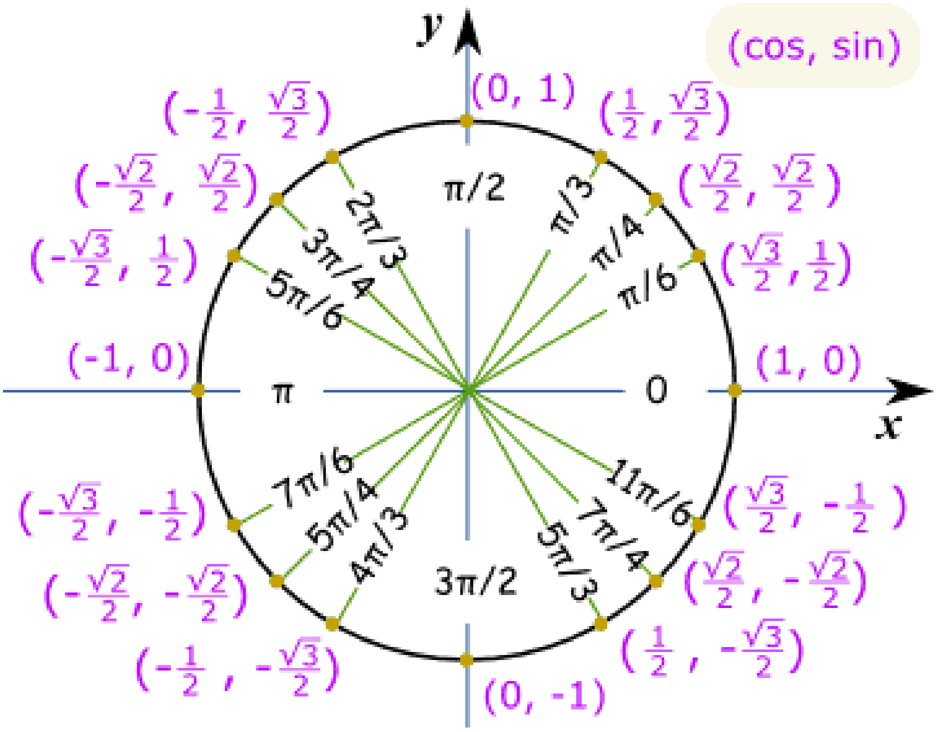


Unit Circle of Special Angles in Degrees



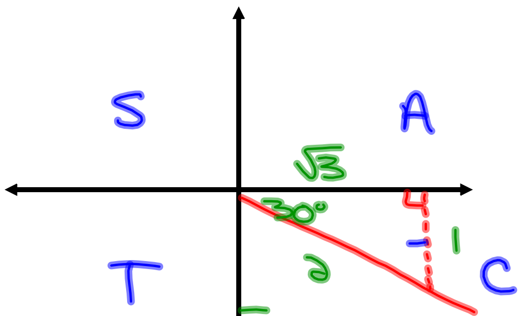
This is lovely...so what is it used for????

Unit Circle of Special Angles in Radians



Solving Trig Expressions by Sketching Angles

Ex. Evaluate the $\sin 690° = -\frac{1}{2}$



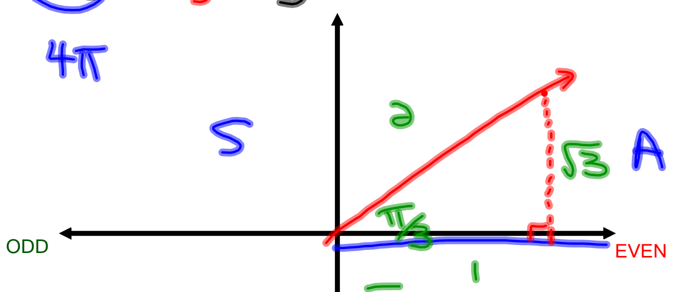
$\bar{\theta} = 720° - 690°$

$\bar{\theta} = 30°$

Ex. $\cos\frac{13\pi}{3} = +\frac{1}{2}$

$\frac{12\pi}{3}, \frac{13\pi}{3}, \frac{14\pi}{3}$

4π



$\bar{\theta} = \frac{13\pi}{3} - \frac{12\pi}{3}$

$\bar{\theta} = \frac{\pi}{3}$

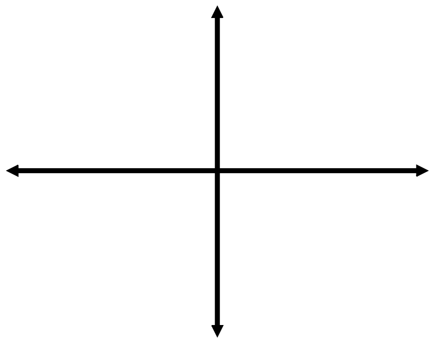
Homework

Evaluate each Trig Expression (provide a sketch of each angle)

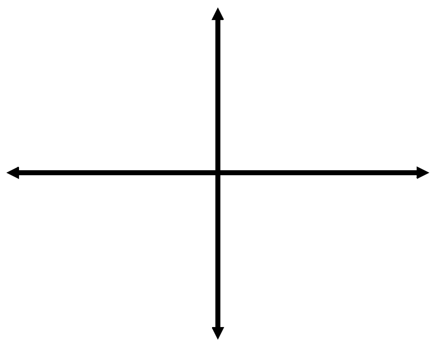
1. $\tan\frac{17\pi}{6}$ 2. $\sin\frac{15\pi}{4}$ 3. $\cos\left(-\frac{21\pi}{4}\right)$

$-\frac{1}{\sqrt{3}}$ or $-\frac{\sqrt{3}}{3}$ $-\frac{1}{\sqrt{2}}$ or $-\frac{\sqrt{2}}{2}$ $-\frac{1}{\sqrt{2}}$ or $-\frac{\sqrt{2}}{2}$

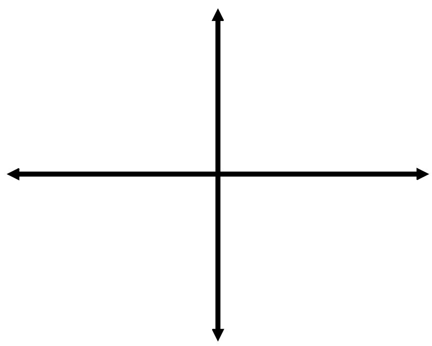
Ex. $\tan\dfrac{17\pi}{6}$



Ex. $\sin\dfrac{15\pi}{4}$



Ex. $\cos\left(-\dfrac{21\pi}{4}\right)$



Attachments

Worksheet - Sketching Angles in Radians.doc