

Questions from Homework

$$\rightarrow y - y_1 = m(x - x_1)$$

③ Find the equation of the tangent line to the curve at the given point:

b) $y = 7\sqrt{x} - 3x$, $(1, 4)$
 x_1, y_1

① Differentiate

$$y = 7x^{1/2} - 3x$$

$$y' = \frac{7}{2}x^{-1/2} - 3$$

$$y' = \frac{7}{2\sqrt{x}} - 3$$

② Sub $x=1$

$$y'(1) = \frac{7}{2\sqrt{1}} - 3$$

$$y'(1) = \frac{7}{2} - \frac{6}{2} = \left(\frac{1}{2}\right)$$

↑
m

③ Find Equation:

$$y - y_1 = m(x - x_1)$$

$$y - 4 = \frac{1}{2}(x - 1)$$

$$2 \cdot y - 4 = 2 \cdot \frac{1}{2}x - \frac{1}{2} \cdot 2$$

$$2y - 8 = x - 1$$

$0 = x - 2y + 7$

(x, y)

⑧ Find the point on the curve $y = x\sqrt{x}$ where the tangent line is parallel to $6x - y = 4$
 (same slope)

① Find slope of $6x - y = 4$

$$-y = -6x + 4$$

$$y = 6x - 4$$

↑

(Slope) $m = 6$

② Differentiate

$$y = x\sqrt{x} = x(x^{1/2}) = x^{3/2}$$

$$y' = \frac{3}{2}x^{1/2} = \frac{3\sqrt{x}}{2}$$

③ Solve for x

$$\frac{3\sqrt{x}}{2} = 6$$

$$3\sqrt{x} = 12$$

$$\sqrt{x} = 4$$

$x = 16$

④ Solve for y

$$y = x\sqrt{x}$$

$$y = (16)\sqrt{16} = 64$$

∴ The point is $(16, 64)$

$$\textcircled{6} \text{ b) } y = 2x^2 - 6\sqrt{x} \text{ at } (4, 20)$$

$\uparrow \quad \uparrow$
 $x_1 \quad y_1$

① Differentiate:

$$y = 2x^2 - 6x^{1/2}$$

$$y' = 4x - 3x^{-1/2}$$

$$y' = 4x - \frac{3}{x^{1/2}}$$

$$y' = 4x - \frac{3}{\sqrt{x}}$$

② Sub in x-value

$$y' = 4(4) - \frac{3}{\sqrt{4}}$$

$$y' = 16 - \frac{3}{2}$$

$$y' = \frac{32}{2} - \frac{3}{2} = \frac{29}{2}$$

Slope of
tangent
"m"

③ Find equation:

$$y - y_1 = m(x - x_1)$$

$$y - 20 = \frac{29}{2}(x - 4)$$

$$y - 20 = \frac{29x}{2} - \frac{116}{2}$$

$$\text{a. } y - 20 = \frac{29x}{2} - 58$$

$$2y - 40 = 29x - 116$$

$$\boxed{0 = 29x - 2y - 76}$$

$$\textcircled{7} \text{ c) } g(x) = 4x^3 - \frac{6}{x^3} + 14x$$

$$g(x) = 4x^3 - 6x^{-3} + 14x$$

$$g'(x) = 12x^2 + 12x^{-3} + 14x^0$$

$$g'(x) = 12x^2 + \frac{12}{x^3} + 14$$

Warm Up

Differentiate the following:

$$f(x) = -4x^2 - 5x(x^3 + 7)^2 + 2\sqrt[5]{x^9} - \frac{5}{x^{10}} + \frac{7x^2}{\sqrt{x}}$$

$$f(x) = -4x^2 - 5x(x^6 + 14x^3 + 49) + 2x^{9/5} - 5x^{-10} + 7x^2(x^{-1/2})$$

$$f(x) = -4x^2 - 5x^7 - 70x^4 - 245x + 2x^{9/5} - 5x^{-10} + 7x^{3/2}$$

$$f'(x) = -8x - 35x^6 - 280x^3 - 245 + \frac{18}{5}x^{4/5} - 50x^{-11} + \frac{21}{2}x^{1/2}$$

Differentiation Rules

Product Rule:

The Product Rule If f and g are both differentiable, then

$$\frac{d}{dx} [f(x)g(x)] = f(x) \frac{d}{dx} [g(x)] + g(x) \frac{d}{dx} [f(x)]$$

Express the product rule verbally if you are considering a function of the form...

$$f(x) = (\text{First}) \times (\text{Second})$$

In words, *the Product Rule* says that the *derivative of a product of two functions is: the first function times the derivative of the second function, plus the derivative of the first function times the second function*

$$(fg)' = fg' + f'g$$

Get in the habit of verbalizing the rule as you differentiate...it will help when the functions get more complicated.

Examples: $y' = f'g + fg'$

$$y = (2x^3 + 5)(3x^2 - x)$$

$$f(x) = 2x^3 + 5$$

$$f'(x) = 6x^2$$

$$g(x) = 3x^2 - x$$

$$g'(x) = 6x - 1$$

$$y' = 6x^2(3x^2 - x) + (2x^3 + 5)(6x - 1)$$

$$y' = 18x^4 - 6x^3 + 12x^4 - 2x^3 + 30x - 5$$

$$y' = 30x^4 - 8x^3 + 30x - 5$$

$$f(x) = \sqrt{x}(2 - 3x)$$

$$f(x) = \sqrt{x} = x^{1/2}$$

$$f'(x) = \frac{1}{2}x^{-1/2}$$

$$g(x) = 2 - 3x$$

$$g'(x) = -3$$

$$y' = \frac{1}{2}x^{-1/2}(2 - 3x) + (x^{1/2})(-3)$$

$$y' = \frac{1}{2}x^{-1/2} - \frac{3}{2}x^{1/2} - 3x^{1/2}$$

$$2y' = 2x^{-1/2} - 3x^{1/2} - 6x^{1/2}$$

$$2y' = 2x^{-1/2} - 9x^{1/2}$$

$$2y' = \frac{2}{x^{1/2}} - \frac{9x^{1/2}}{1}$$

$$2y' = \frac{2}{x^{1/2}} - \frac{9x}{x^{1/2}}$$

$$\left(\frac{1}{2}\right) 2y' = \frac{2 - 9x}{x^{1/2}} \left(\frac{1}{2}\right)$$

$$y' = \frac{2 - 9x}{2\sqrt{x}}$$

Examples:

$$f(x) = (7x^3 - x^2 + 5)(x^9 + 3x - 5)$$

$$f'(x) = (21x^2 - 2x)(x^9 + 3x - 5) + (7x^3 - x^2 + 5)(9x^8 + 3)$$

$$h(t) = (t^3 - 5t)(6\sqrt{t} - t^{-5})$$

Homework

