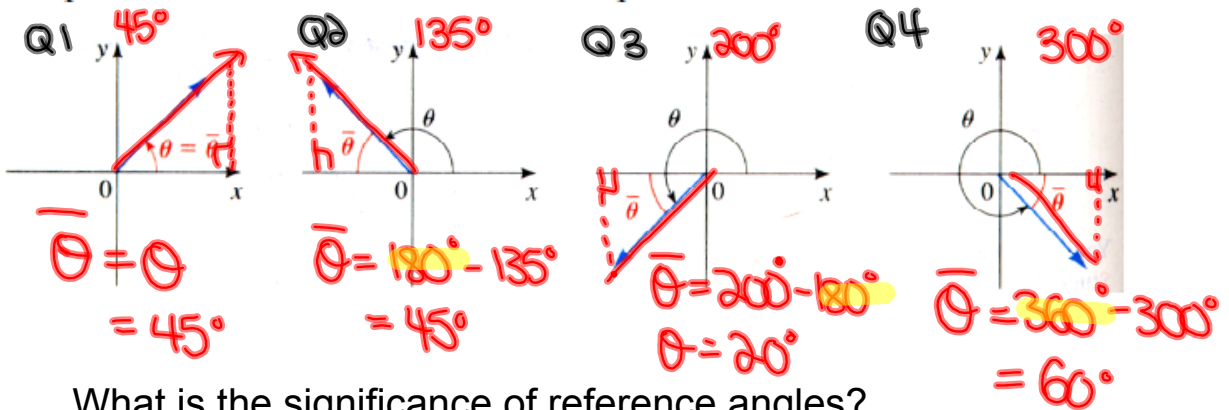


Reference Triangles:

Definition 17 The reference angle $\bar{\theta}$ of an angle θ in standard position is the acute angle (between 0 and 90°) the terminal side makes with the x-axis.

0 and 2π rads

The picture below illustrates this concept.

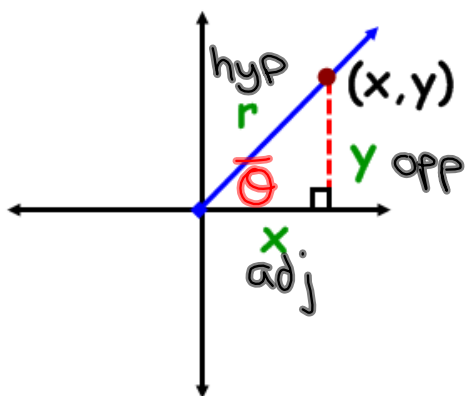


What is the significance of reference angles?

Angles on the Cartesian Plane



- **Reference Angle** - an acute angle formed between the terminal arm and the **x-axis**.
- **Reference Triangle** - a triangle formed by drawing a perpendicular line from a point on the terminal to the **x-axis**.



Notice what will happen if the rotation moves into other quadrants?

TRIG RATIOS on the CARTESIAN PLANE

$$\sin \theta = \frac{y}{r}$$

$$\csc \theta = \frac{r}{y}$$

$$\cos \theta = \frac{x}{r}$$

$$\sec \theta = \frac{r}{x}$$

$$\tan \theta = \frac{y}{x}$$

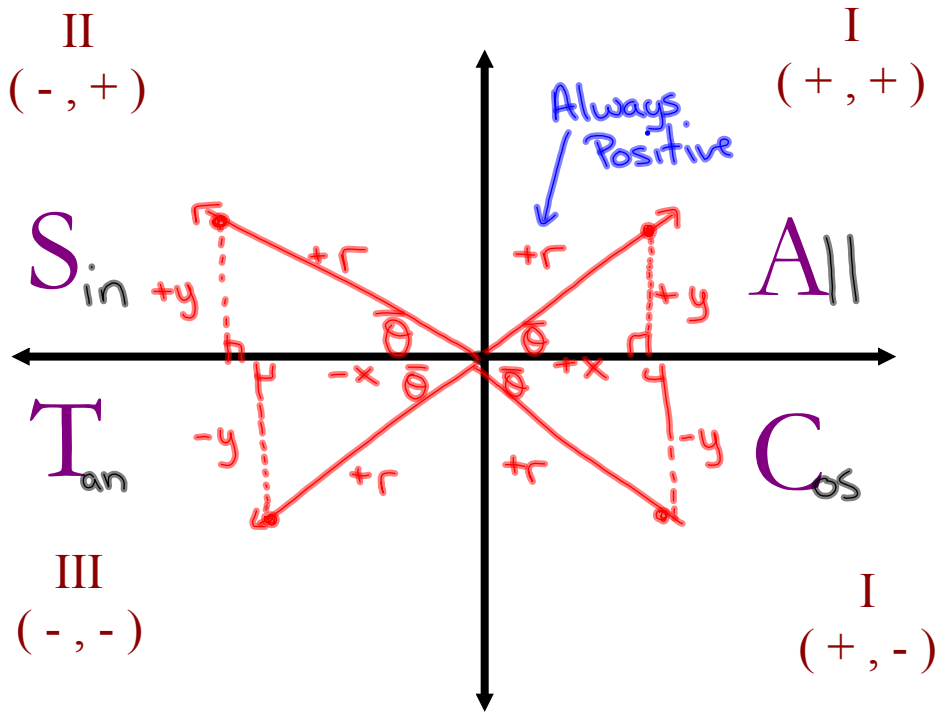
$$\cot \theta = \frac{x}{y}$$

"Primary"

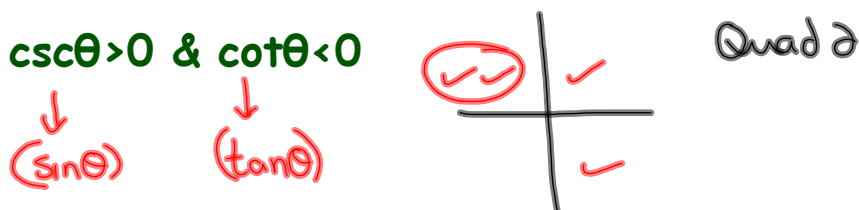
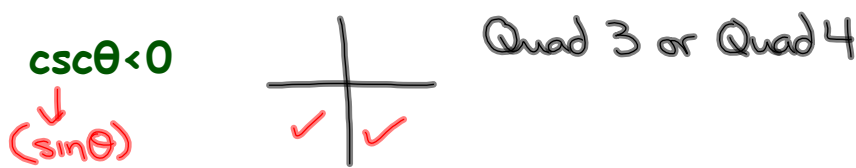
"Reciprocal"

TRIG RATIOS IN ALL 4 QUADRANTS

What primary trig ratios are **POSITIVE** in...



Where is θ if...

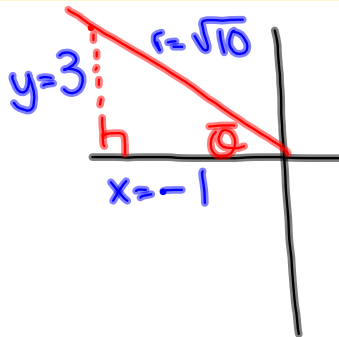


If $\sec \theta = -\sqrt{10}$ and $\sin \theta > 0$, determine the value of $\csc \theta$

$$\sec \theta = -\frac{\sqrt{10}}{1} = \frac{r}{x}$$

$$r = \sqrt{10}$$

$$x = -1$$



$$x^2 + y^2 = r^2$$

$$(-1)^2 + y^2 = (\sqrt{10})^2$$

$$1 + y^2 = 10$$

$$y^2 = 9$$

$$y = \pm 3$$

Choose $y = 3$

$$\csc \theta = \frac{\text{hyp}}{\text{opp}} = \frac{r}{y} = \frac{\sqrt{10}}{3}$$

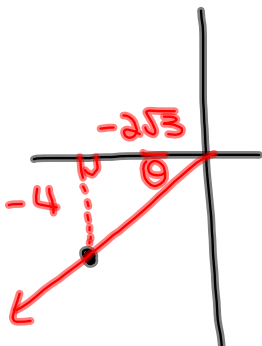
Example

Determine the measure (in radians) of an angle whose terminal arm passes through the ordered pair $(-2\sqrt{3}, -4)$

Given:

$$x = -2\sqrt{3}$$

$$y = -4$$



x, y

$$\tan \bar{\theta} = \frac{y}{x}$$

$$\tan \bar{\theta} = \frac{-4}{-2\sqrt{3}}$$

$$\tan \bar{\theta} = \frac{2}{\sqrt{3}}$$

$$\bar{\theta} = \tan^{-1}\left(\frac{2}{\sqrt{3}}\right)$$

$$\bar{\theta} = 49.1$$

$$\theta = 49.1^\circ + 180^\circ$$

$$\theta = 229.1^\circ$$

```
tan-1(2/√(3))
49.10660535
```

To Convert to Radians

$$229.1 \left(\frac{\pi}{180}\right) = \boxed{3.99 \text{ rads}}$$

In Radians

```
tan-1(2/√(3))
.8570719479
```

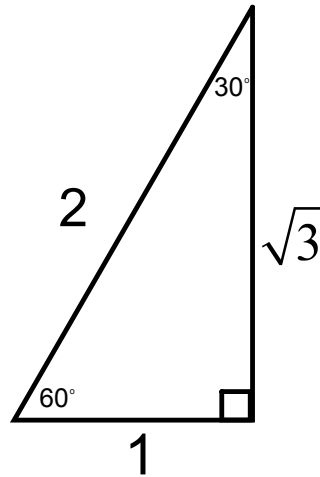
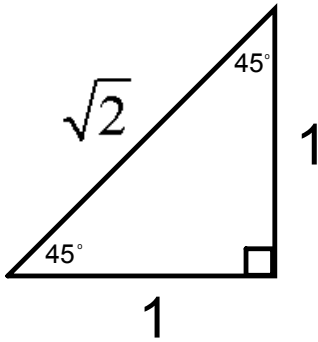
$$\theta = 0.857 + \pi$$

$$\theta = 0.857 + 3.14$$

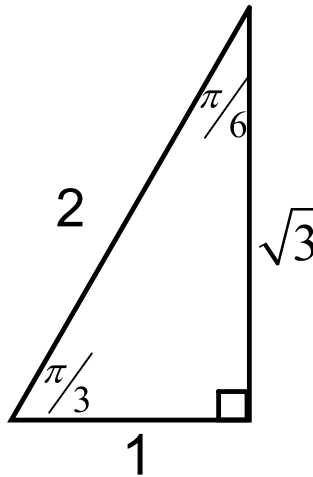
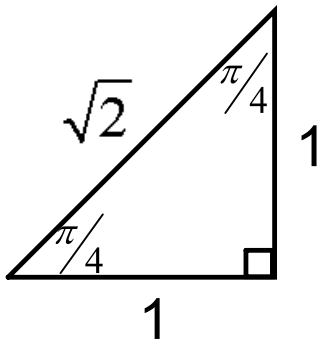
$$\theta = 3.99 \text{ rads}$$

Special Angles

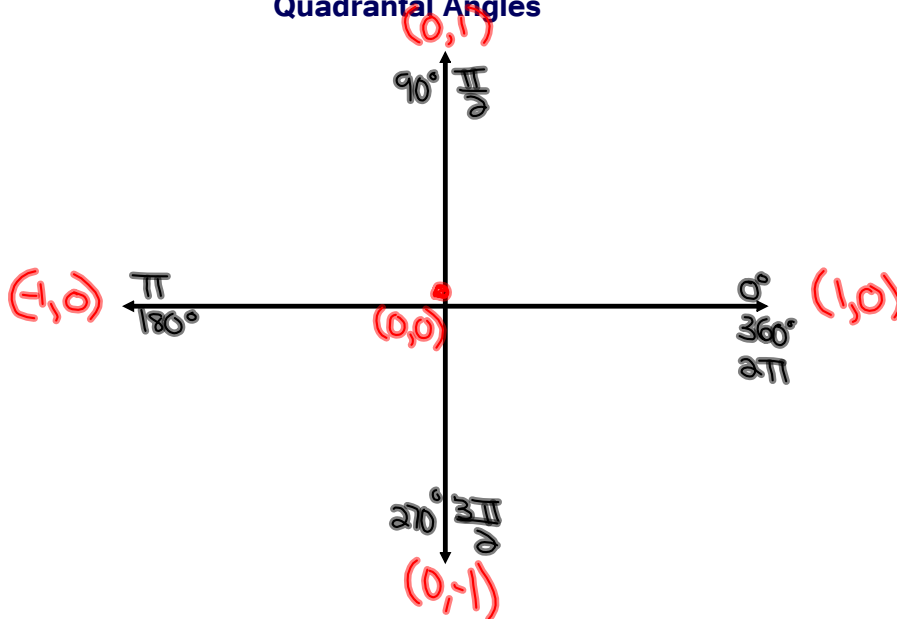
In Degrees:



In Radians:

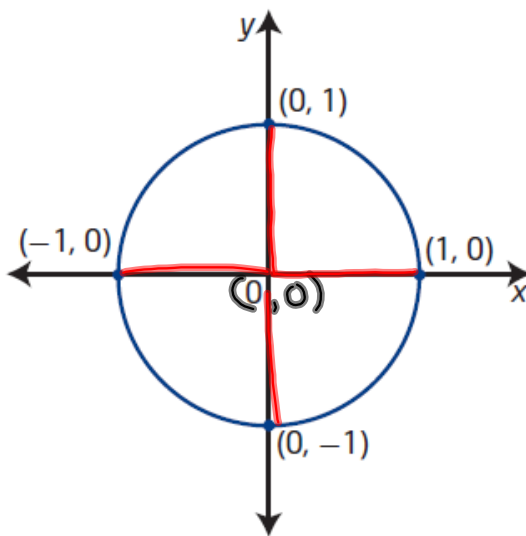


Quadrantal Angles



Unit Circle

(used for multiples of 90° or $\frac{\pi}{2}$ rads)



unit circle

- a circle with radius 1 unit
- a circle of radius 1 unit with centre at the origin on the Cartesian plane is known as *the* unit circle

$$\sin \theta = \frac{y}{r} = \frac{y}{1} = y$$

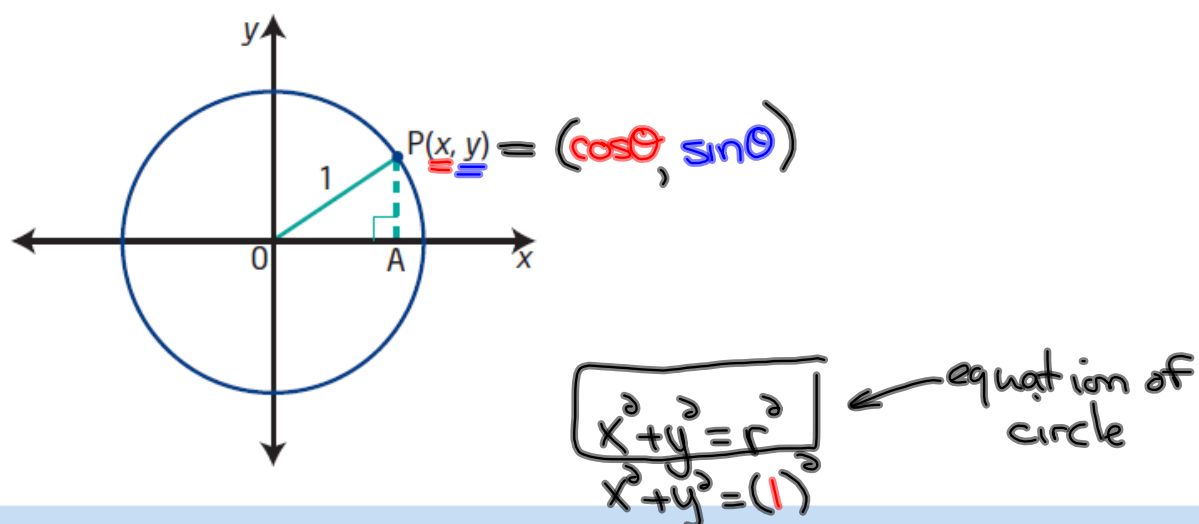
$$\csc \theta = \frac{1}{y}$$

$$\cos \theta = \frac{x}{r} = \frac{x}{1} = x$$

$$\sec \theta = \frac{1}{x}$$

$$\tan \theta = \frac{y}{x}$$

$$\cot \theta = \frac{x}{y}$$



The equation of the unit circle is $x^2 + y^2 = 1$.

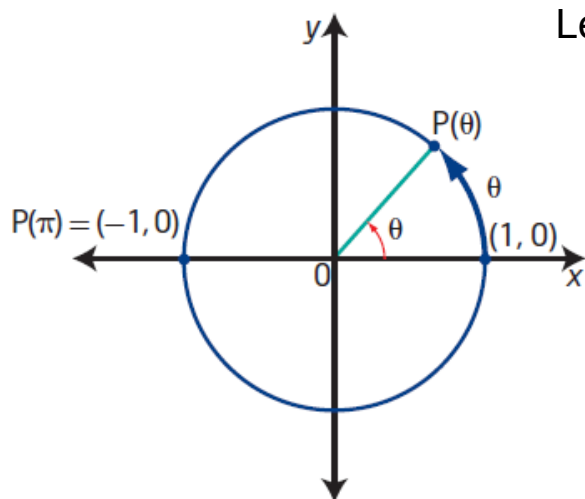
Determine the equation of a circle with centre at the origin and radius 6.

$$x^2 + y^2 = r^2$$

$$x^2 + y^2 = (6)^2$$

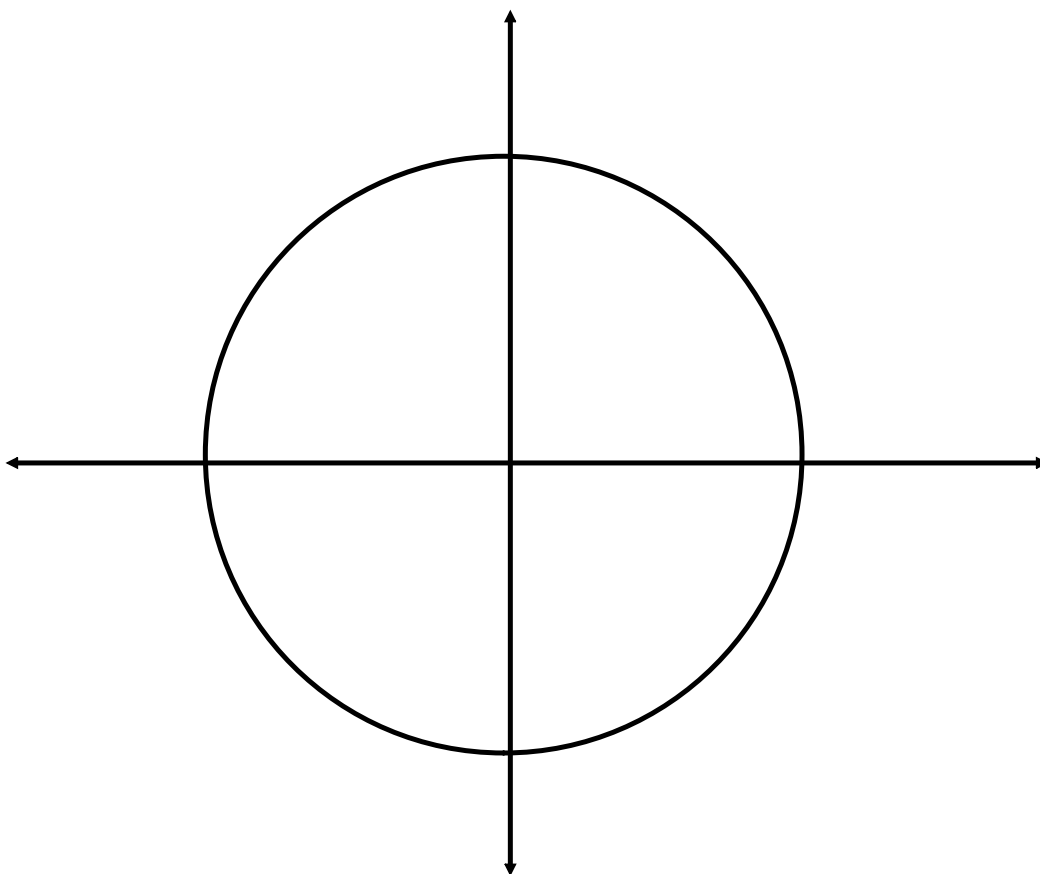
$$x^2 + y^2 = 36$$

Special Angles on the Unit Circle:

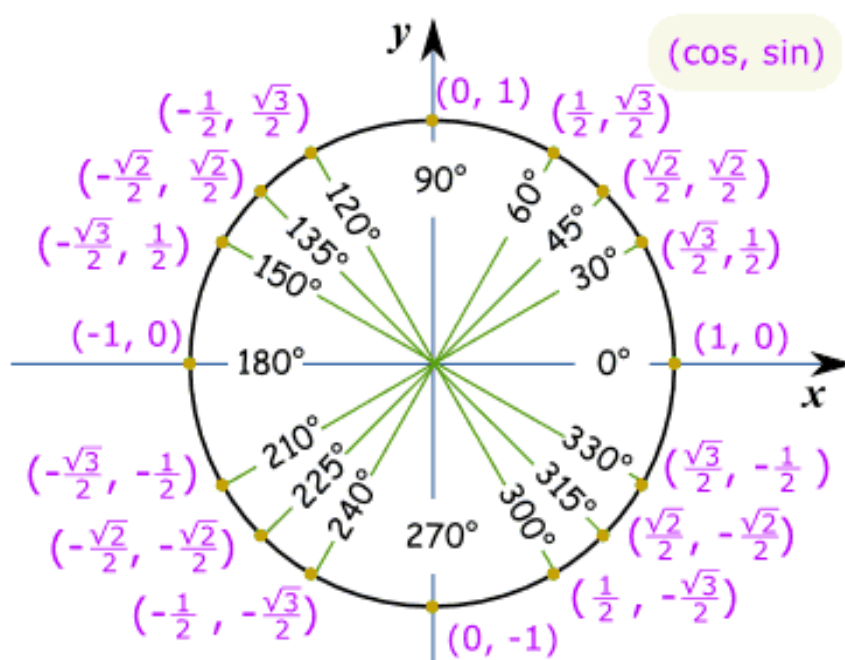


Let's use $\frac{\pi}{4}$ as our reference angle

Construct reference triangles
for all multiples of $\pi/4$
between 0 and 2π

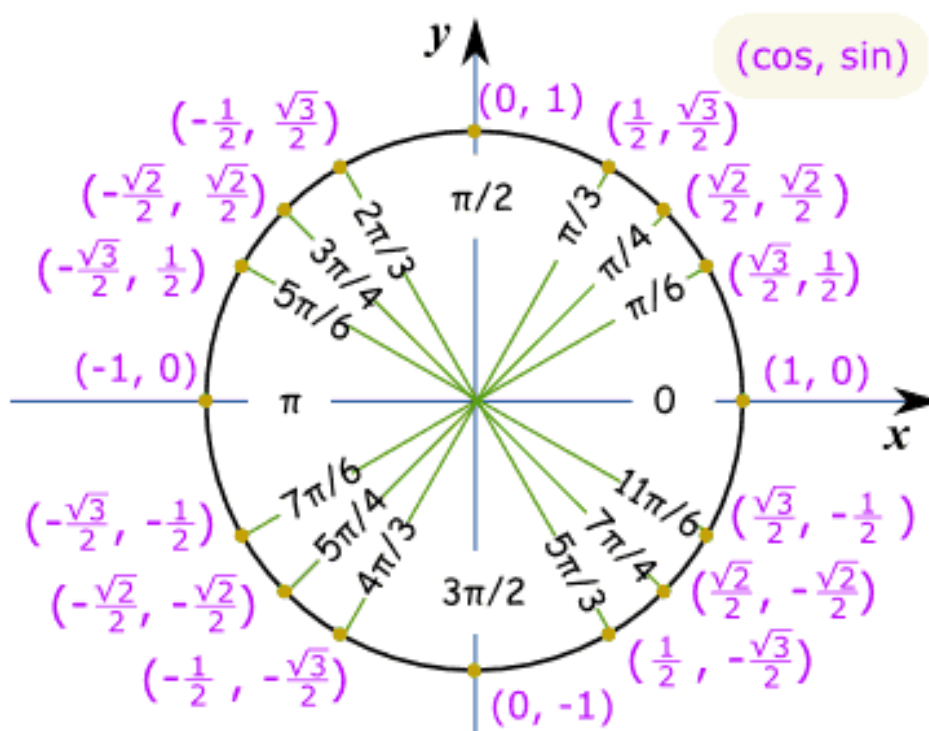


Unit Circle of Special Angles in Degrees



This is lovely...so what is it used for????

Unit Circle of Special Angles in Radians



Problems Involving the Unit Circle:

Determine Coordinates for Points of the Unit Circle

Determine the coordinates for all points on the unit circle that satisfy the conditions given. Draw a diagram in each case.

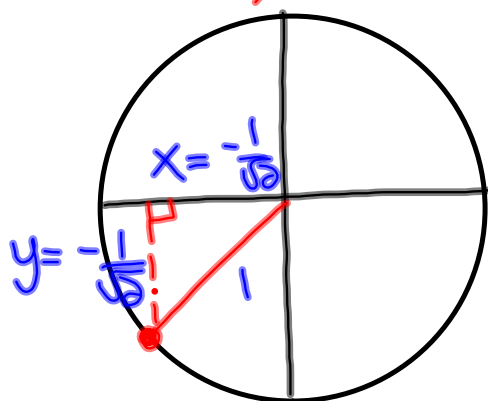
- the y-coordinate is $-\frac{1}{\sqrt{2}}$ and the point is in quadrant III

$$x^2 + y^2 = 1 \quad (\text{Equation of Unit Circle})$$

$$x^2 + \left(-\frac{1}{\sqrt{2}}\right)^2 = 1$$

$$x^2 + \frac{1}{2} = 1$$

$$x^2 = \frac{1}{2}$$



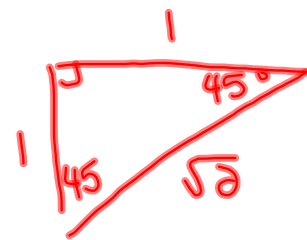
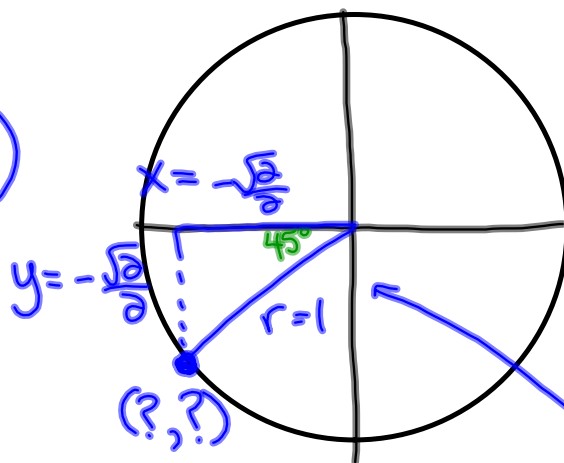
$$x = \pm\sqrt{\frac{1}{2}} = \pm\frac{\sqrt{1}}{\sqrt{2}} = \pm\frac{1}{\sqrt{2}} \quad \text{Choose } x = -\frac{1}{\sqrt{2}} \text{ (Q3)}$$

Problems Involving the Unit Circle:

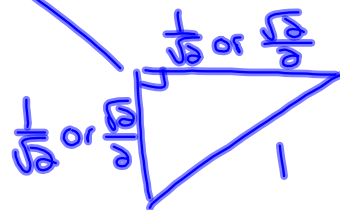
If $P(\theta)$ is the point at which the terminal arm of an angle θ in standard position intersects the unit circle, determine the exact coordinates of...

$P(225^\circ)$

$$\left(-\frac{\sqrt{2}}{2}, -\frac{\sqrt{2}}{2}\right)$$

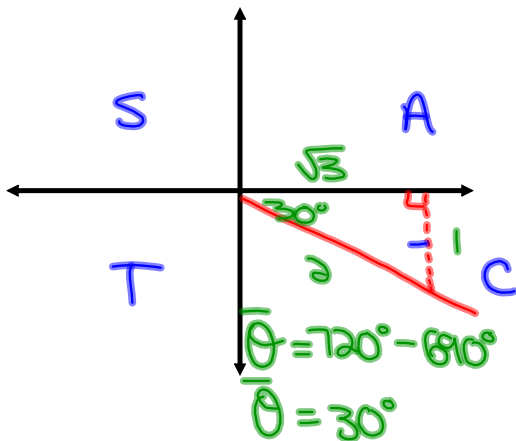


Scale above diagram to fit unit circle

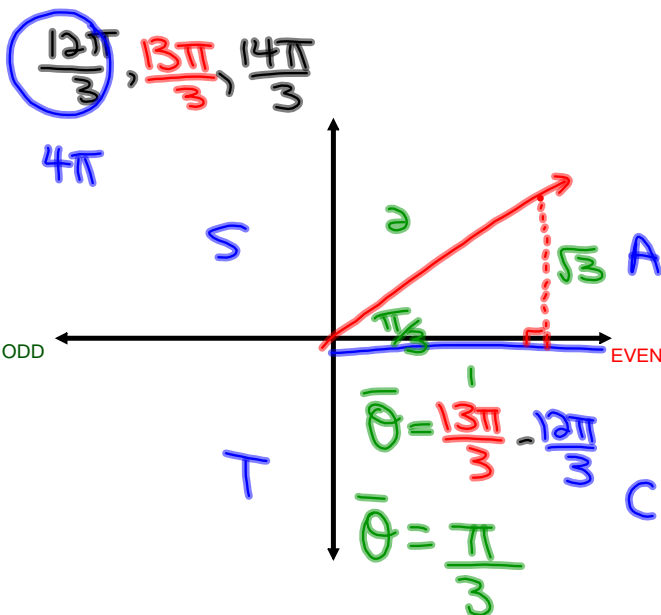


Solving Trig Expressions by Sketching Angles

Ex. Evaluate the $\sin 690^\circ = -\frac{1}{2}$



Ex. $\cos \frac{13\pi}{3} = +\frac{1}{2}$



Homework

Evaluate each Trig Expression (provide a sketch of each angle)

1. $\tan \frac{17\pi}{6}$

$$-\frac{1}{\sqrt{3}} \text{ or } -\frac{\sqrt{3}}{3}$$

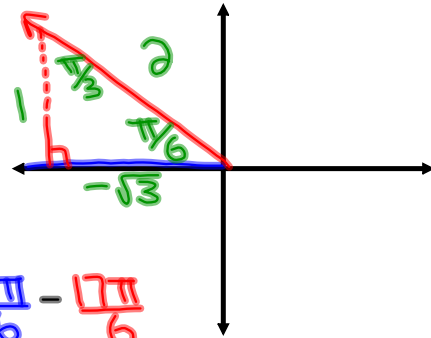
2. $\sin \frac{15\pi}{4}$

$$-\frac{1}{\sqrt{2}} \text{ or } -\frac{\sqrt{2}}{2}$$

3. $\cos \left(-\frac{21\pi}{4} \right)$

$$-\frac{1}{\sqrt{2}} \text{ or } -\frac{\sqrt{2}}{2}$$

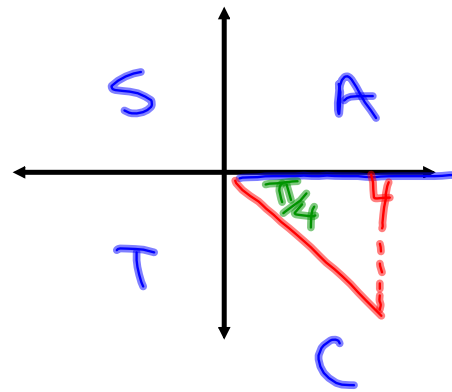
Ex. $\tan \frac{17\pi}{6} = -\frac{1\sqrt{3}}{\sqrt{3}\sqrt{3}} = \boxed{-\frac{\sqrt{3}}{3}}$



$\frac{16\pi}{6}, \frac{17\pi}{6}, \frac{18\pi}{6}$
 3π

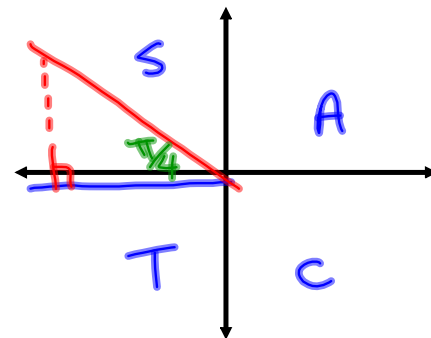
$\bar{\theta} = \frac{18\pi}{6} - \frac{17\pi}{6}$
 $= \frac{\pi}{6}$

Ex. $\sin \frac{15\pi}{4} = -\frac{1}{\sqrt{2}} = \boxed{-\frac{\sqrt{2}}{2}}$



$\frac{14\pi}{4}, \frac{15\pi}{4}, \frac{16\pi}{4}$
 4π

Ex. $\cos\left(-\frac{21\pi}{4}\right)$ *add 6π to unwind*



$\cos \frac{3\pi}{4} = -\frac{1}{\sqrt{2}} = \boxed{-\frac{\sqrt{2}}{2}}$

$\frac{2\pi}{4}, \frac{3\pi}{4}, \frac{4\pi}{4}$
 π

Evaluate without the use of a calculator:

$$\sin \frac{9\pi}{2} - \cos^2 \left(\frac{29\pi}{6} \right) \tan \left(\frac{15\pi}{4} \right)$$

Evaluate without the use of a calculator:

$$\cos\left(\frac{16\pi}{3}\right)\tan^2\left(\frac{23\pi}{6}\right) + \csc\left(\frac{11\pi}{2}\right) + \sin^2\left(\frac{27\pi}{4}\right)$$

Homework:

Worksheet - Sketching Angles in Radians.doc

Solutions...

1. $-\frac{5}{3}$

5. $\frac{4+3\sqrt{3}}{6}$

2. $\frac{-\sqrt{6}}{3}$

6. $\frac{-10}{3}$

3. $-2-\sqrt{3}$

7. 0

4. $\frac{-5}{3}$

8. $\frac{3+3\sqrt{3}}{-2}$

Attachments

Worksheet - Sketching Angles in Radians.doc