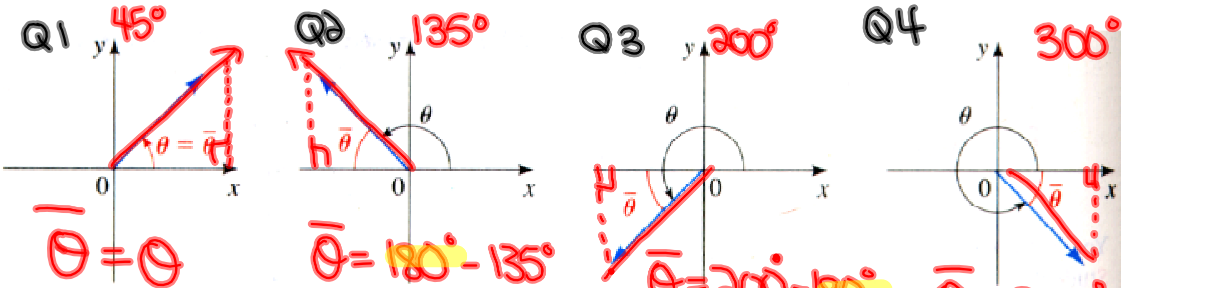
Reference Triangles:

Definition 17 *The reference angle $\bar{\theta}$ of an angle θ in standard position is the acute angle (between 0 and 90°) the terminal side makes with the x-axis.*

0 and $\frac{\pi}{2}$ rads

The picture below illustrates this concept.



$\bar{\theta} = \theta = 45°$

$\bar{\theta} = 180° - 135° = 45°$

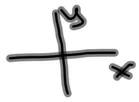
$\bar{\theta} = 200° - 180°$

$\bar{\theta} = 20°$

$\bar{\theta} = 360° - 300° = 60°$

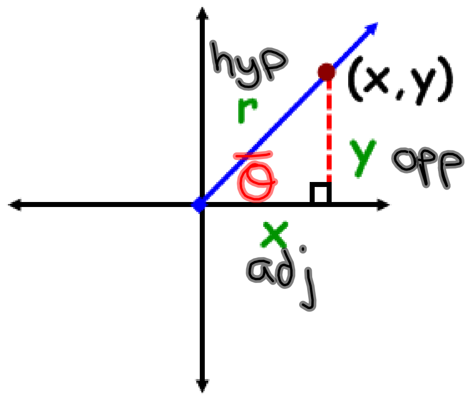
What is the significance of reference angles?

Angles on the Cartesian Plane



- **Reference Angle** - an acute angle formed between the terminal arm and the x-axis.

- **Reference Triangle** - a triangle formed by drawing a perpendicular line from a point on the terminal to the x-axis.



Notice what will happen if the rotation moves into other quadrants?

TRIG RATIOS on the CARTESIAN PLANE

$$\sin\theta = \frac{y}{r} \qquad \csc\theta = \frac{r}{y}$$

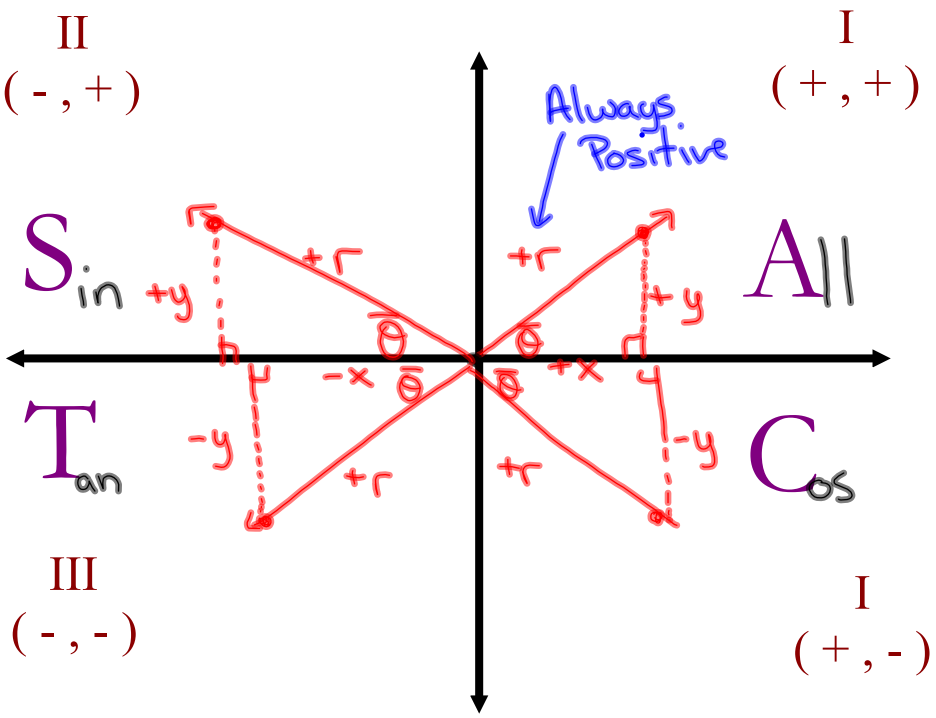
$$\cos\theta = \frac{x}{r} \qquad \sec\theta = \frac{r}{x}$$

$$\tan\theta = \frac{y}{x} \qquad \cot\theta = \frac{x}{y}$$

"Primary" **"Reciprocal"**

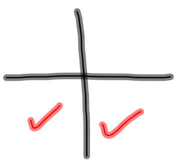
TRIG RATIOS IN ALL 4 QUADRANTS

What primary trig ratios are POSITIVE in...



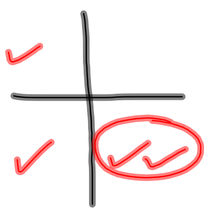
Where is θ if...

csc θ<0
(sin θ)



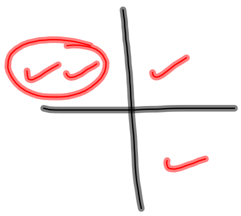
Quad 3 or Quad 4

sin θ<0 & tan θ<0



Quad 4

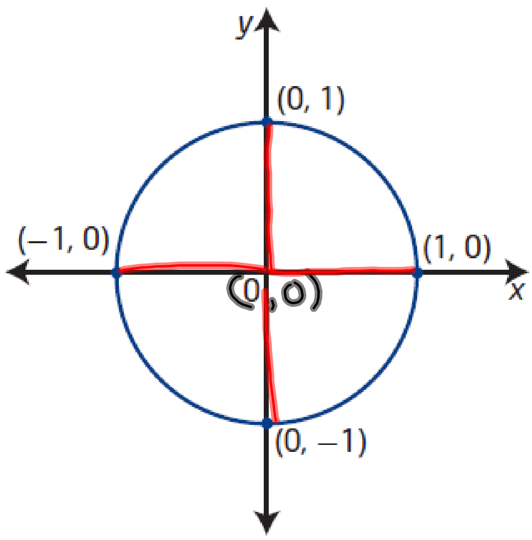
csc θ>0 & cot θ<0
(sin θ) (tan θ)



Quad 2

Unit Circle

(used for multiples of 90° or $\frac{\pi}{2}$ rads)



unit circle

- a circle with radius 1 unit
- a circle of radius 1 unit with centre at the origin on the Cartesian plane is known as *the* unit circle

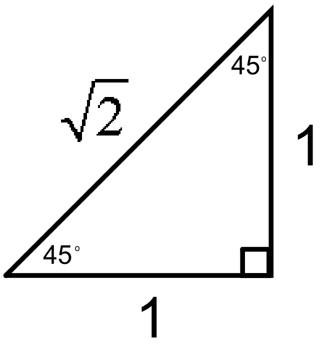
$$\sin\theta = \frac{y}{r} = \frac{y}{1} = y \qquad \csc\theta = \frac{1}{y}$$

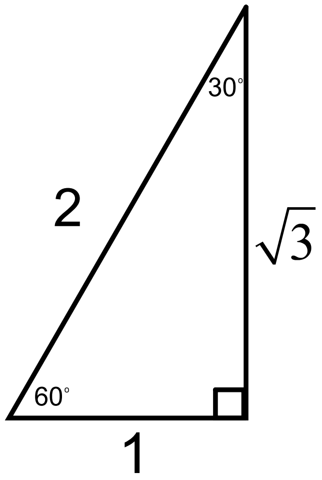
$$\cos\theta = \frac{x}{r} = \frac{x}{1} = x \qquad \sec\theta = \frac{1}{x}$$

$$\tan\theta = \frac{y}{x} \qquad \cot\theta = \frac{x}{y}$$

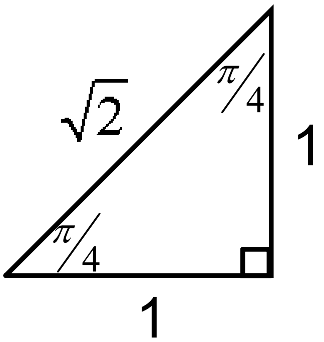
Special Angles

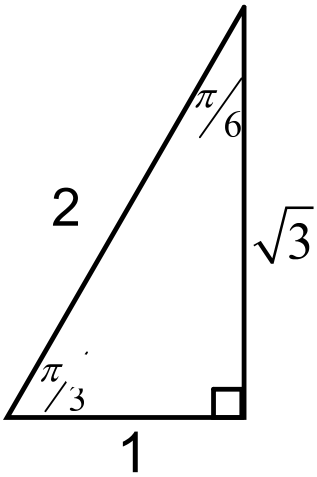
In Degrees:



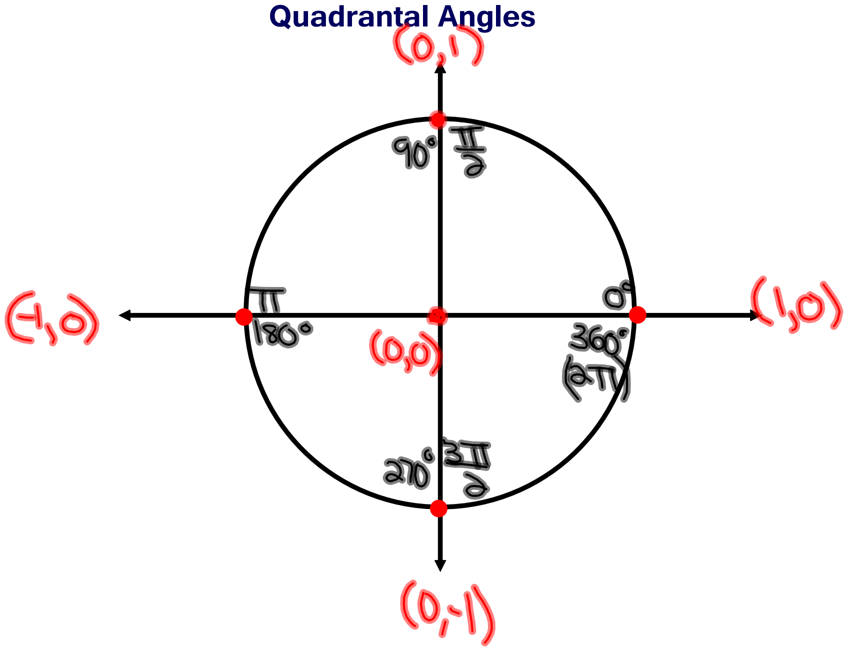


In Radians:

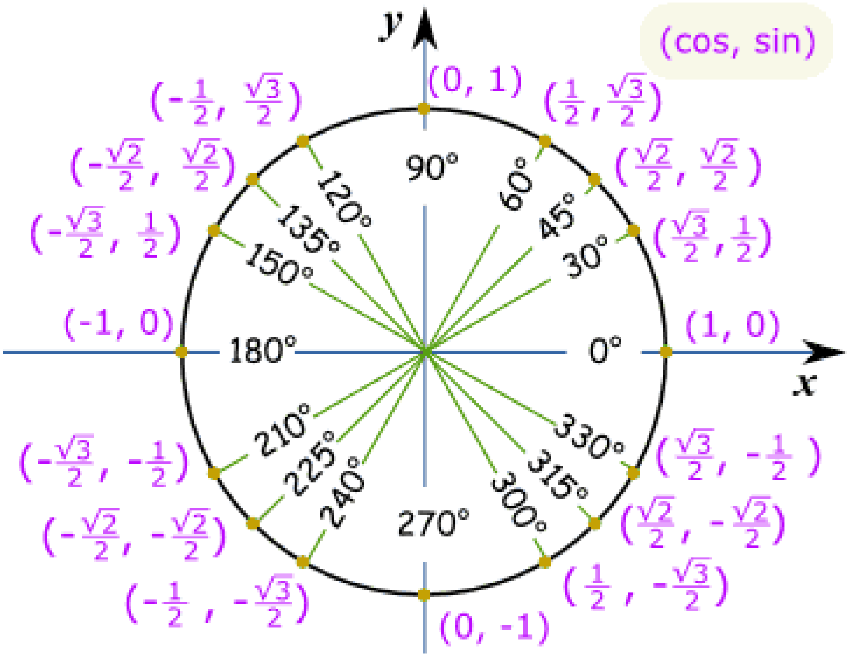




Quadrantal Angles

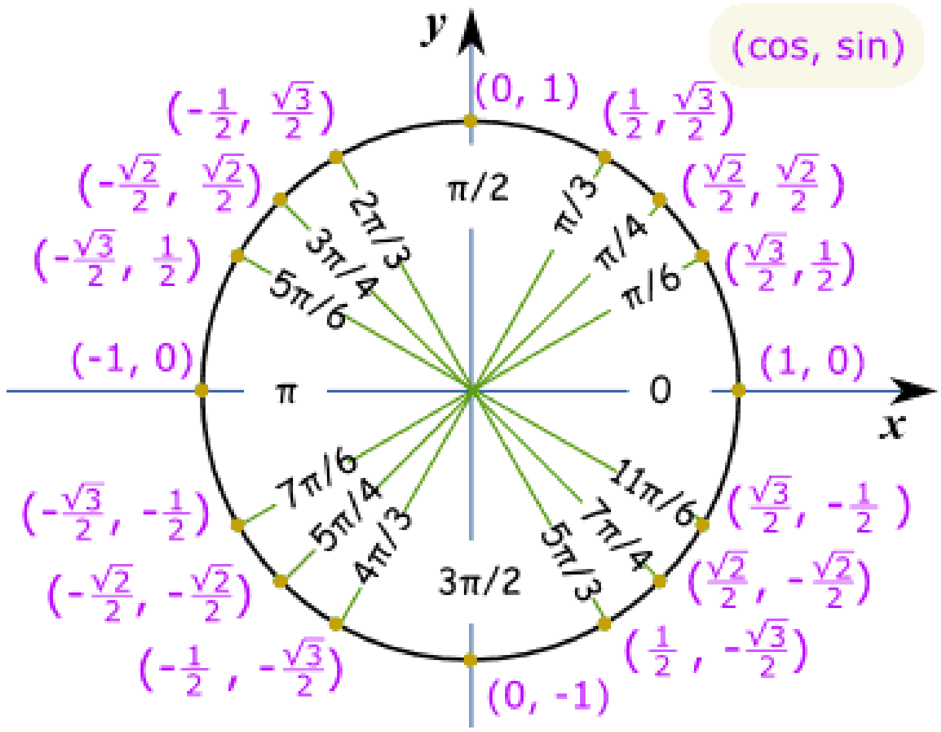


Unit Circle of Special Angles in Degrees



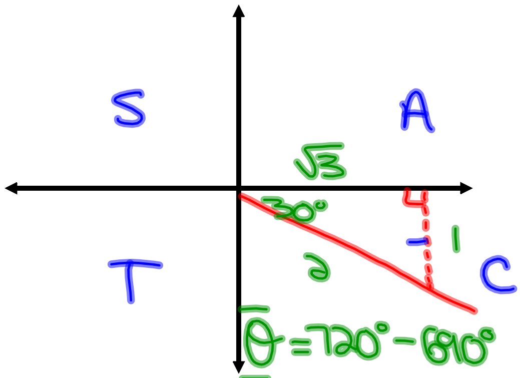
This is lovely...so what is it used for????

Unit Circle of Special Angles in Radians



Solving Trig Expressions by Sketching Angles

Ex. Evaluate the $\sin 690° = -\frac{1}{2}$



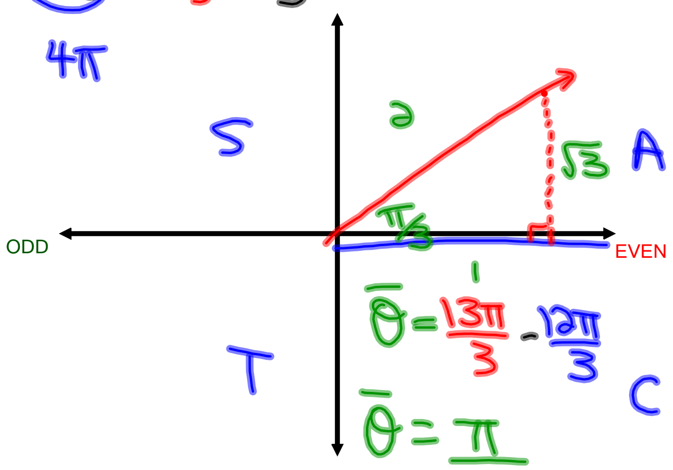
$\bar{\theta} = 720° - 690°$

$\bar{\theta} = 30°$

Ex. $\cos\frac{13\pi}{3} = +\frac{1}{2}$

$\frac{12\pi}{3}, \frac{13\pi}{3}, \frac{14\pi}{3}$

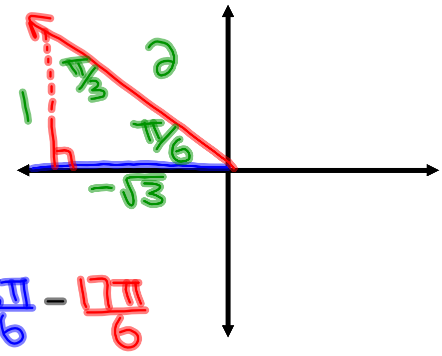
4π



$\bar{\theta} = \frac{13\pi}{3} - \frac{12\pi}{3}$

$\bar{\theta} = \frac{\pi}{3}$

Ex. $\tan\frac{17\pi}{6} = -\frac{1}{\sqrt{3}}\cdot\frac{\sqrt{3}}{\sqrt{3}} = \boxed{-\frac{\sqrt{3}}{3}}$



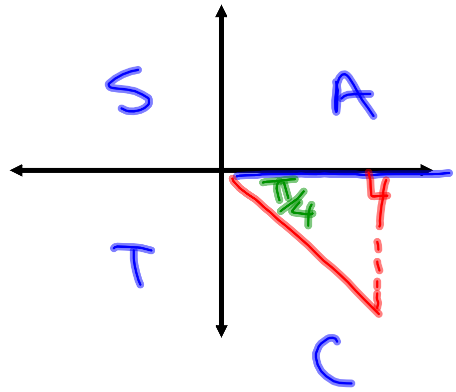
$\frac{16\pi}{6}, \frac{17\pi}{6}, \frac{18\pi}{6}$

3π

$\bar{\theta} = \frac{18\pi}{6} - \frac{17\pi}{6}$

$= \frac{\pi}{6}$

Ex. $\sin\frac{15\pi}{4} = -\frac{1}{\sqrt{2}} = \boxed{-\frac{\sqrt{2}}{2}}$

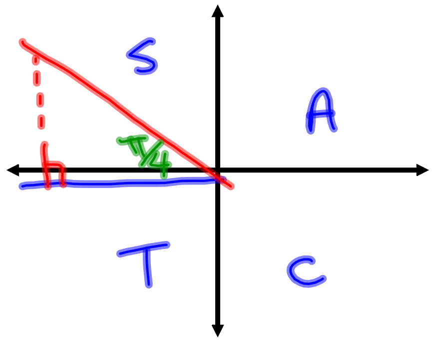


$\frac{14\pi}{4}, \frac{15\pi}{4}, \frac{16\pi}{4}$

4π

add 6π to unwind

Ex. $\cos\left(-\frac{21\pi}{4}\right)$

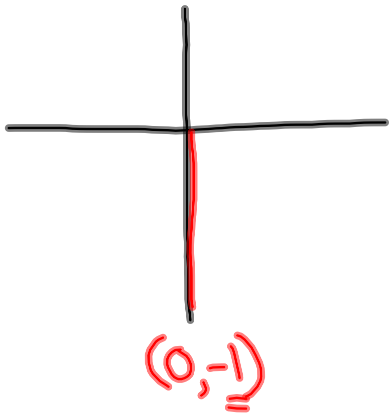


$\cos\frac{3\pi}{4} = -\frac{1}{\sqrt{2}} = \boxed{-\frac{\sqrt{2}}{2}}$

$\frac{2\pi}{4}, \frac{3\pi}{4}, \frac{4\pi}{4}$

π

① $\sin\frac{3\pi}{2} = -1$



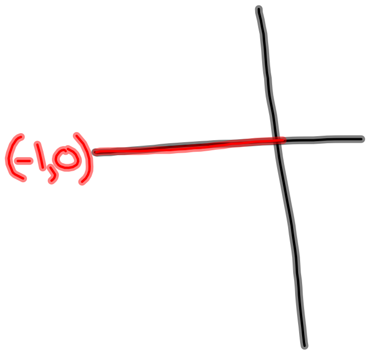
$\frac{2\pi}{2}, \frac{3\pi}{2}, \frac{4\pi}{2}$

π 2π

② $\tan\frac{3\pi}{2} = \frac{-1}{0} =$ undefined

③ $\csc\frac{3\pi}{2} = \frac{1}{-1} = -1$

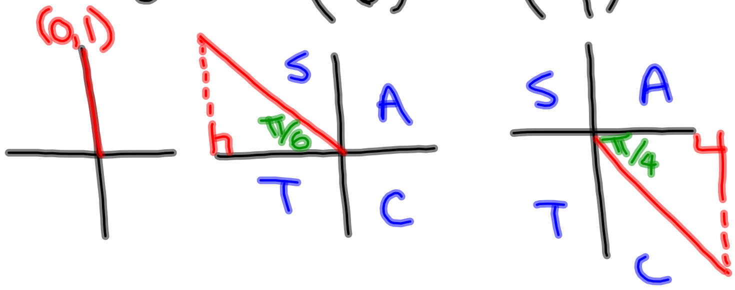
④ $\cos 5\pi = -1$



Evaluate without the use of a calculator:

$$\sin\frac{9\pi \overset{-4\pi}{}}{2} - \cos^2\left(\frac{29\pi \overset{-4\pi}{}}{6}\right)\tan\left(\frac{15\pi \overset{-2\pi}{}}{4}\right)$$

$$\sin\frac{\pi}{2} - \cos^2\left(\frac{5\pi}{6}\right)\tan\left(\frac{7\pi}{4}\right)$$



$$(1) - \left(-\frac{\sqrt{3}}{2}\right)^2\left(-\frac{1}{1}\right)$$

$$1 - \left(\frac{3}{4}\right)(-1)$$

$$1 + \frac{3}{4}$$

$$\frac{4}{4} + \frac{3}{4}$$

$$\boxed{\frac{7}{4}}$$

Evaluate without the use of a calculator:

$$\cos\left(\frac{16\pi}{3}\right)\tan^2\left(\frac{23\pi}{6}\right)+\csc\left(\frac{11\pi}{2}\right)+\sin^2\left(\frac{27\pi}{4}\right)$$

Homework: Worksheet - Sketching Angles in Radians.doc

Solutions...

1. $-\frac{5}{3}$

2. $\frac{-\sqrt{6}}{3}$

3. $-2-\sqrt{3}$

4. $\frac{-5}{3}$

5. $\frac{4+3\sqrt{3}}{6}$

6. $\frac{-10}{3}$

7. 0

8. $\frac{3+3\sqrt{3}}{-2}$

Attachments

Worksheet - Sketching Angles in Radians.doc