

$$\textcircled{1} \text{ i) } y = x^3 + 8$$

x-intercept (y=0)

$$0 = x^3 + 8 \quad (\text{Sum of Cubes})$$

$$0 = (x+2)(x^2-2x+4)$$

$$x+2=0$$

$$\boxed{x=-2}$$

$(-2, 0)$

$$x^2 - 2x + 4 = 0 \quad a=1 \quad b=-2 \quad c=4$$

$$x = \frac{-(-2) \pm \sqrt{(-2)^2 - 4(1)(4)}}{2(1)}$$

$$x = \frac{2 \pm \sqrt{4-16}}{2}$$

$$x = \frac{2 \pm \sqrt{-12}}{2}$$

No Solution (2 imaginary roots)

y-intercept (x=0)

$$y = (0)^3 + 8 = 8 \quad (0, 8)$$

Curve Sketching

In this chapter we look at further aspects of curves such as **vertical and horizontal asymptotes**, concavity, and inflections points. Then we use them, together with intervals of increase and decrease and maximum and minimum values, to develop a procedure for curve sketching.

Asymptotes only exist with rational functions: $\left(y = \frac{x+2}{x^2+7x+12}\right)$

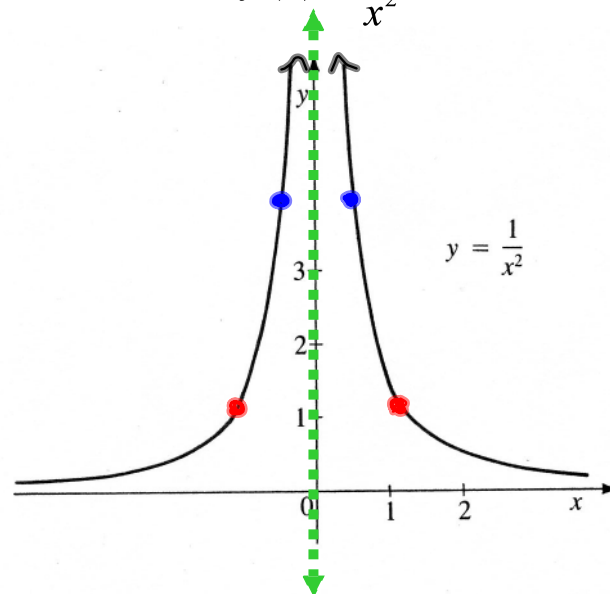
Types of Asymptotes:

- Vertical $\left(\updownarrow\right) \rightarrow$ (zeros of the denominator)
- Horizontal $\left(\leftarrow\text{---}\rightarrow\right)$
- Oblique / Slant $\left(\swarrow\text{---}\searrow\right)$

Vertical Asymptotes

Let us examine the behaviour of the function $f(x) = \frac{1}{x^2}$ for x close to 0.

x	$f(x) = \frac{1}{x^2}$
± 1	1
± 0.5	4
± 0.2	25
± 0.1	100
± 0.05	400
± 0.01	10000
± 0.001	1000000



The values in the table and the graph show that the closer we take x to 0, the larger $\frac{1}{x^2}$ becomes. In fact, it appears that by taking x close

enough to 0, we can make $f(x)$ as large as we like. We indicate this type of behaviour by writing

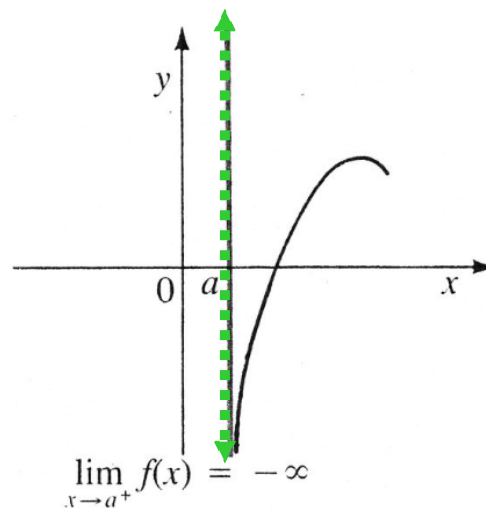
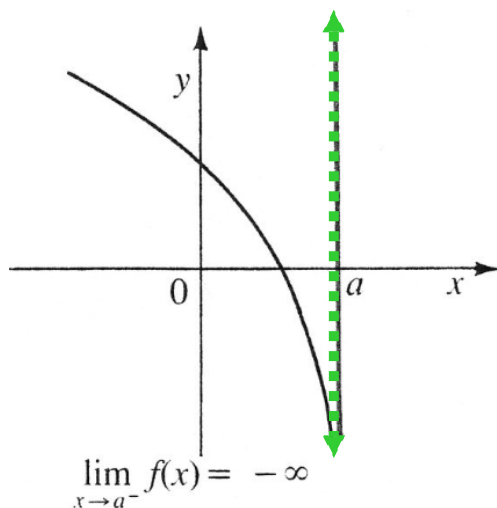
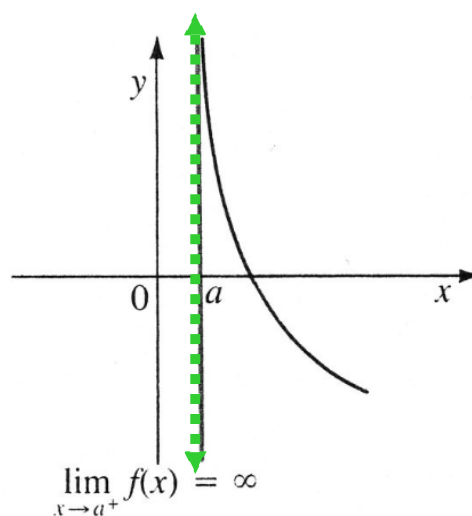
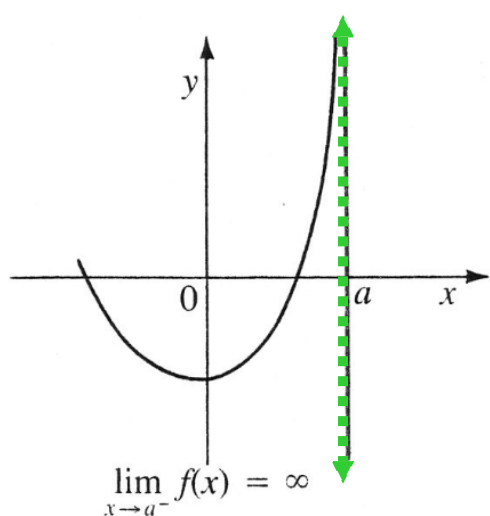
$$\lim_{x \rightarrow 0} \frac{1}{x^2} = \infty$$

and we say that the line $x = 0$ is a **vertical asymptote** of $y = \frac{1}{x^2}$

Vertical Asymptote

The line $x = a$ is a vertical asymptote of the graph of a function $y = f(x)$ if either

$$\lim_{x \rightarrow a^-} f(x) = \pm\infty \quad \text{or} \quad \lim_{x \rightarrow a^+} f(x) = \pm\infty$$



To find the vertical asymptotes of any rational function, we find the values of x where the denominator is zero and compute the limits of the function from the right and left.

Example

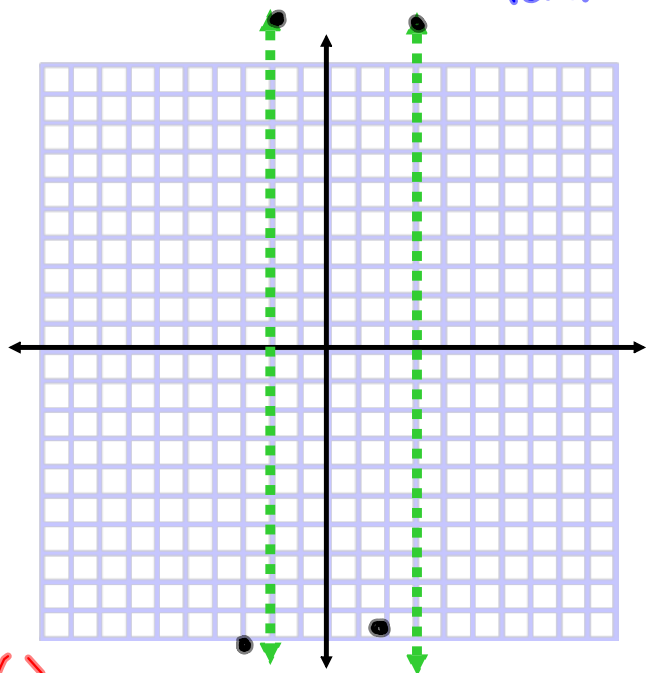
(Rational Function)

a) Find the vertical asymptotes of $y = \frac{x}{x^2 - x - 6} = \frac{x}{(x+2)(x-3)}$

↑
factored form

b) Sketch the graph near the asymptotes

$$\begin{aligned} \text{a) } x^2 - x - 6 &= 0 \\ (x+2)(x-3) &= 0 \\ x(x+2) - 3(x+2) &= 0 \\ (x+2)(x-3) &= 0 \\ x+2=0 \quad | \quad x-3=0 \\ x=-2 \quad | \quad x=3 \end{aligned}$$



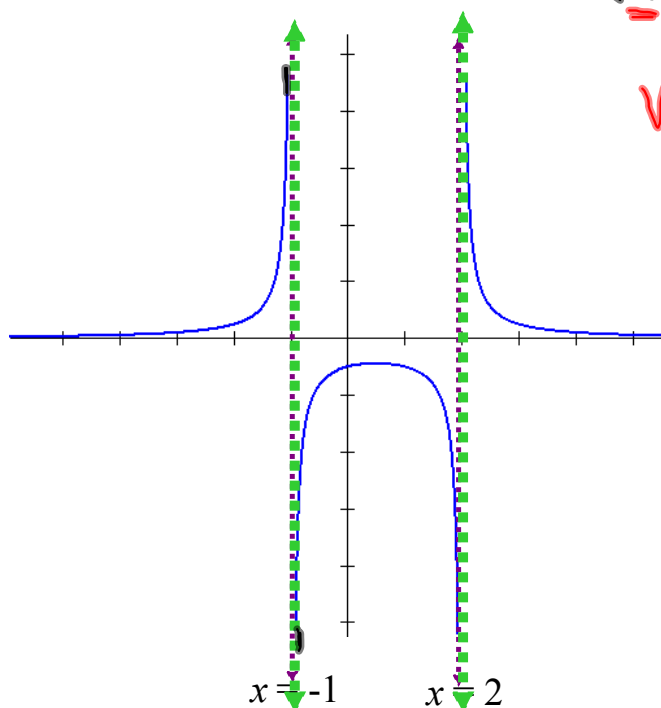
$$\begin{aligned} \lim_{x \rightarrow -2^-} \frac{(-)}{(-)(-)} &= -\infty & \lim_{x \rightarrow -2^+} \frac{(-)}{(+)(-)} &= +\infty \\ (-2.01) & & (-1.99) & \end{aligned}$$

$$\begin{aligned} \lim_{x \rightarrow 3^-} \frac{(+)}{(+)(-)} &= -\infty & \lim_{x \rightarrow 3^+} \frac{(+)}{(+)(+)} &= +\infty \\ (2.99) & & (3.01) & \end{aligned}$$

Example:

$$f(x) = \frac{1}{x^2 - x - 2} = \frac{1}{\underline{(x-2)}\underline{(x+1)}}$$

$$\text{VA: } x = -1, 2$$



$$\lim_{x \rightarrow -1^-} f(x) = +\infty$$

$$\lim_{x \rightarrow -1^+} f(x) = -\infty$$

$$\lim_{x \rightarrow 2^-} f(x) = -\infty$$

$$\lim_{x \rightarrow 2^+} f(x) = +\infty$$

Use limits to examine the behaviour of the function near the asymptotes

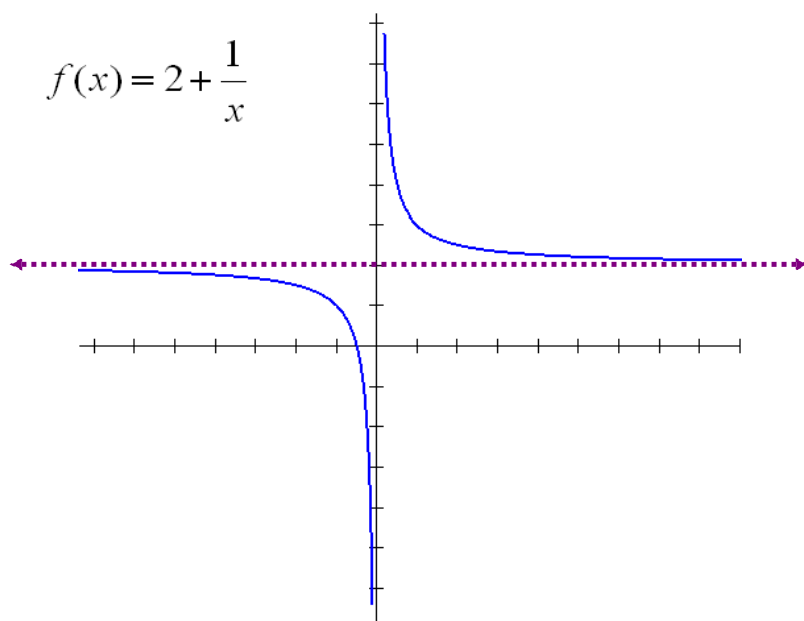
Homework

Asymptotes

Horizontal Asymptote

The line $y = b$ is a horizontal asymptote of the graph of a function $y = f(x)$ if either

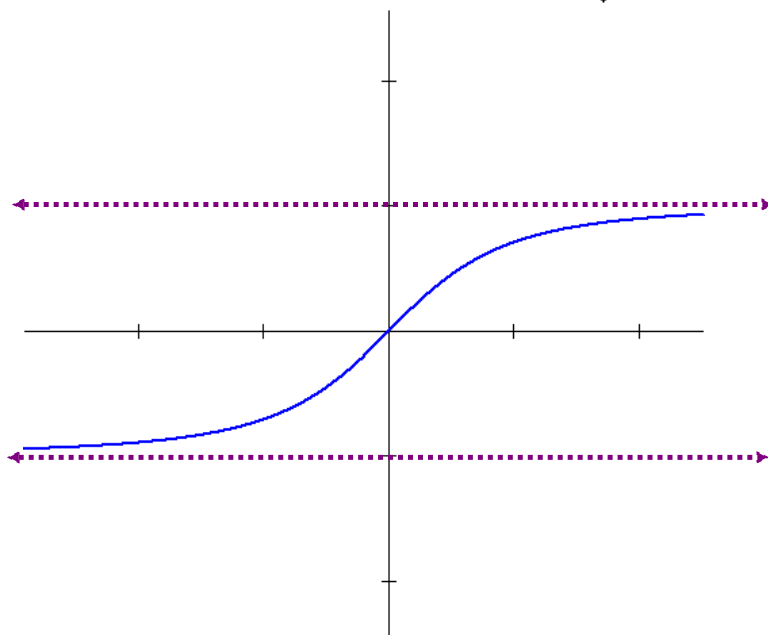
$$\lim_{x \rightarrow \infty} f(x) = b \quad \text{or} \quad \lim_{x \rightarrow -\infty} f(x) = b$$



Examine the limits of $f(x)$ as x approaches $\pm \infty$

There can be more than one horizontal asymptote.

Examine the function $f(x) = \frac{x}{\sqrt{x^2 + 1}}$



Examine the limits of $f(x)$ as x approaches $\pm \infty$

Sketch the following function:

$$f(x) = \frac{8(x-2)}{x^2} \quad f'(x) = \frac{-8(x-4)}{x^3} \quad f''(x) = \frac{16(x-6)}{x^4}$$

Be sure to examine...

- Intercepts
- Asymptotes (*vertical and horizontal*)
- Regions of increase/decrease
- Local extrema
- Regions where concave up/down
- Inflection points

