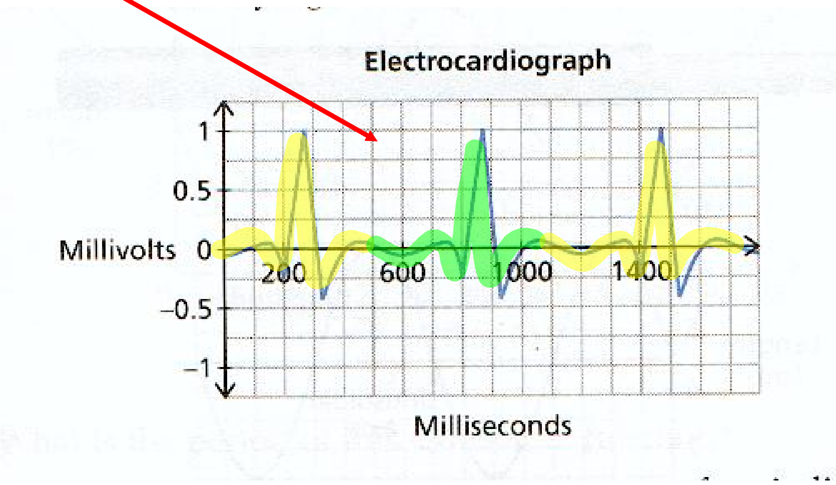
Sinusoidal Relations

Periodic Function: A function for which the dependent variable takes on the same set of values over and over again as the independent variable changes.

(A function that repeats)

Example of periodic behavior

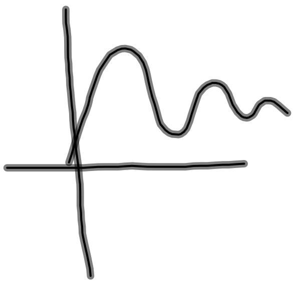


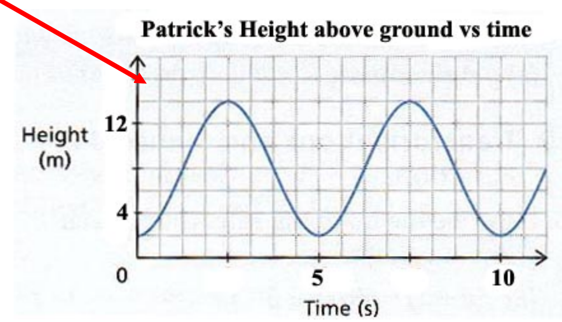
Sinusoidal Function: A periodic function that looks like waves, where any portion of the curve can be translated onto another portion of the curve.

(Repeats and looks like waves)

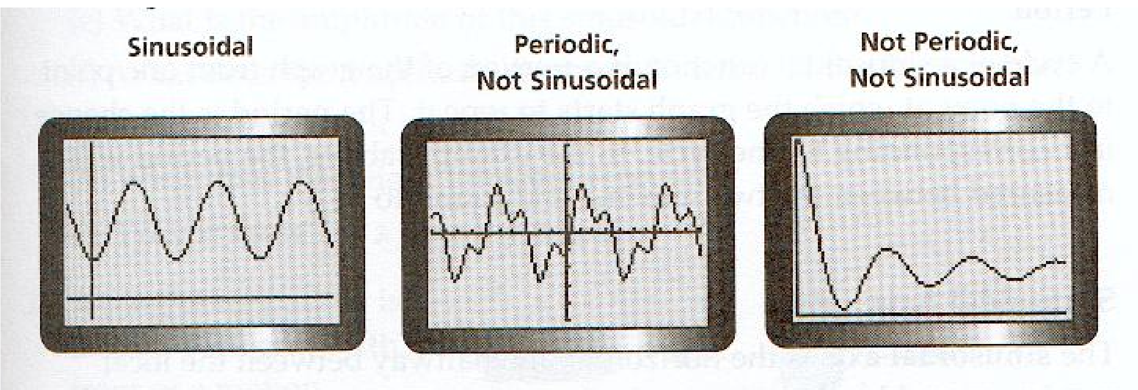
Example of sinusoidal behavior

Neither





These illustrations should summarize periodic and sinusoidal...

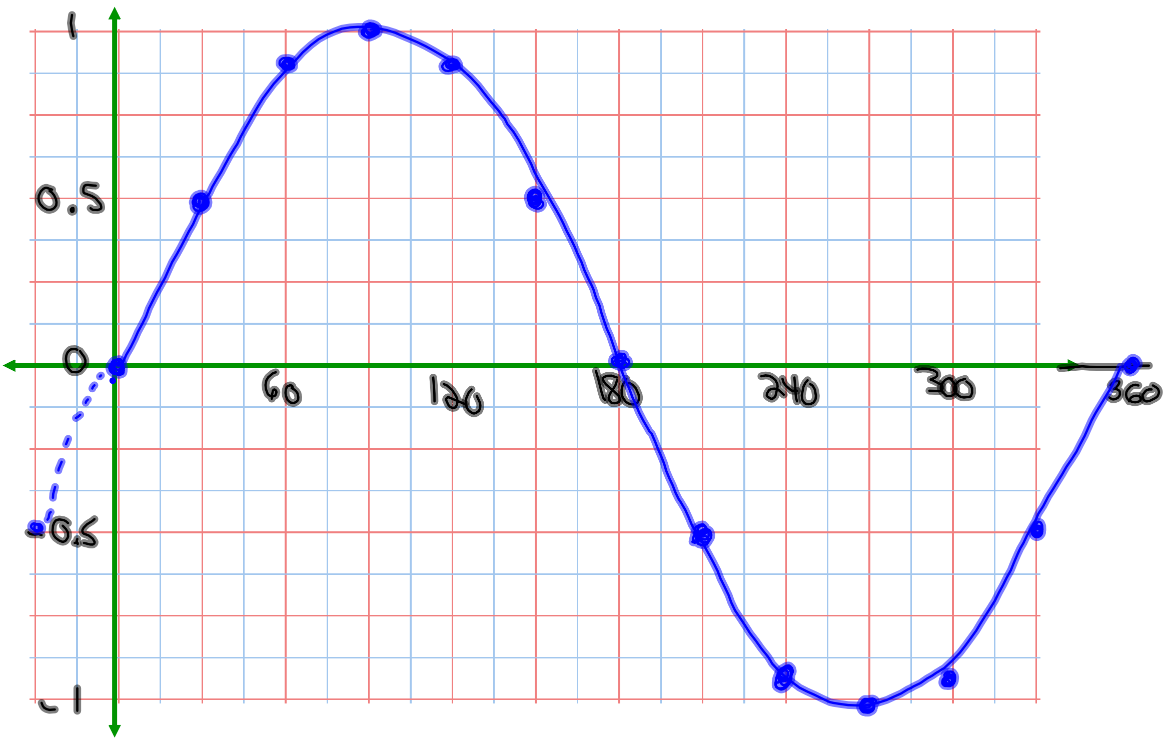


Let's examine the graph of $y = \sin\theta$

$y = \sin x$

θ	$0°$	$30°$	$60°$	$90°$	$120°$	$150°$	$180°$	$210°$	$240°$	$270°$	$300°$	$330°$	$360°$
y	0	0.5	0.9	1	0.9	0.5	0	-0.5	-0.9	-1	-0.9	-0.5	0

Now plot the above points...

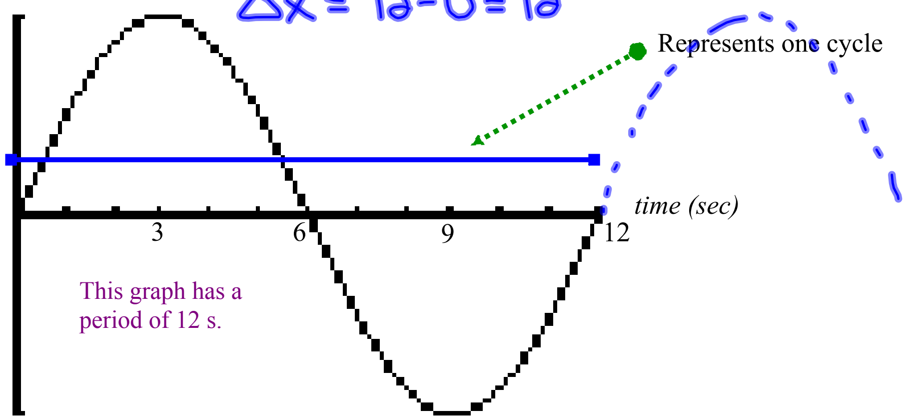


Vocabulary of Sinusoidal Functions

I. Period:

The change in x corresponding to one cycle. (max → max) (min → min)

$\Delta x = 12 - 0 = 12$



II. Sinusoidal Axis:

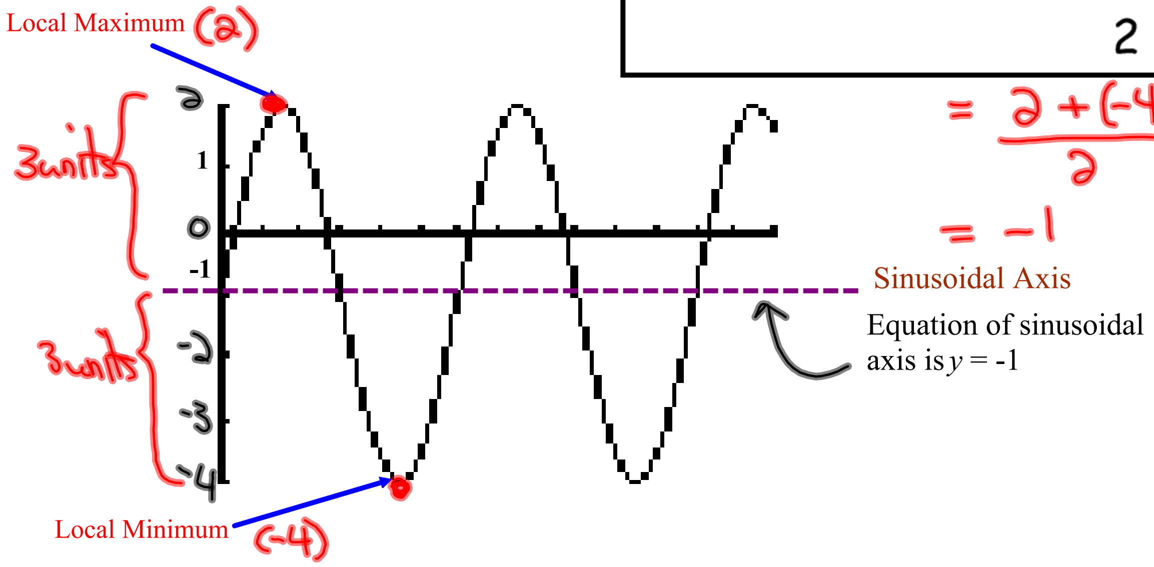
The y = horizontal line halfway between the local maximum and local minimum.

(highest point) (lowest)

$$\boxed{\text{Sinusoidal Axis} = \frac{\text{Max} + \text{Min}}{2}}$$

$= \frac{2 + (-4)}{2}$

$= -1$

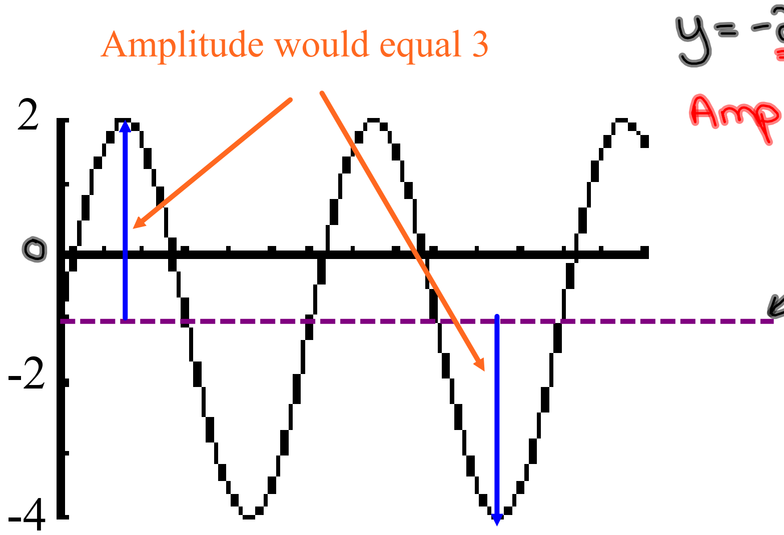


III. Amplitude:

The vertical distance from the sinusoidal axis to a local maximum or local minimum. (Amplitude is always (+))

$y = -2\sin[3(x+30°)]-1$

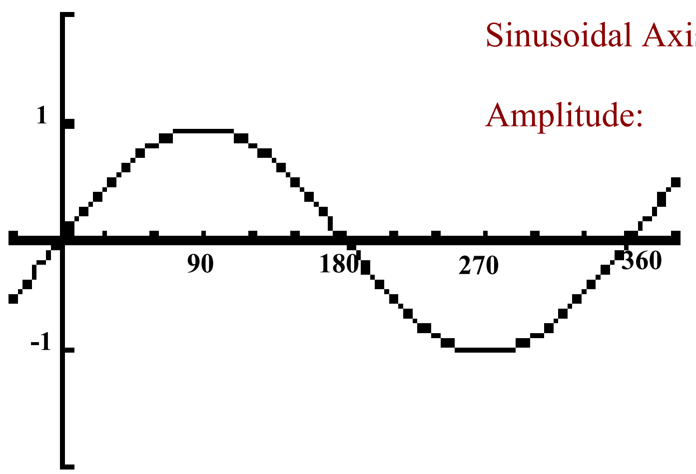
Amp = 2



sinusoidal axis: $\boxed{y=-1}$

Summarize...

Here is the graph of $y = \sin\theta$



Period :

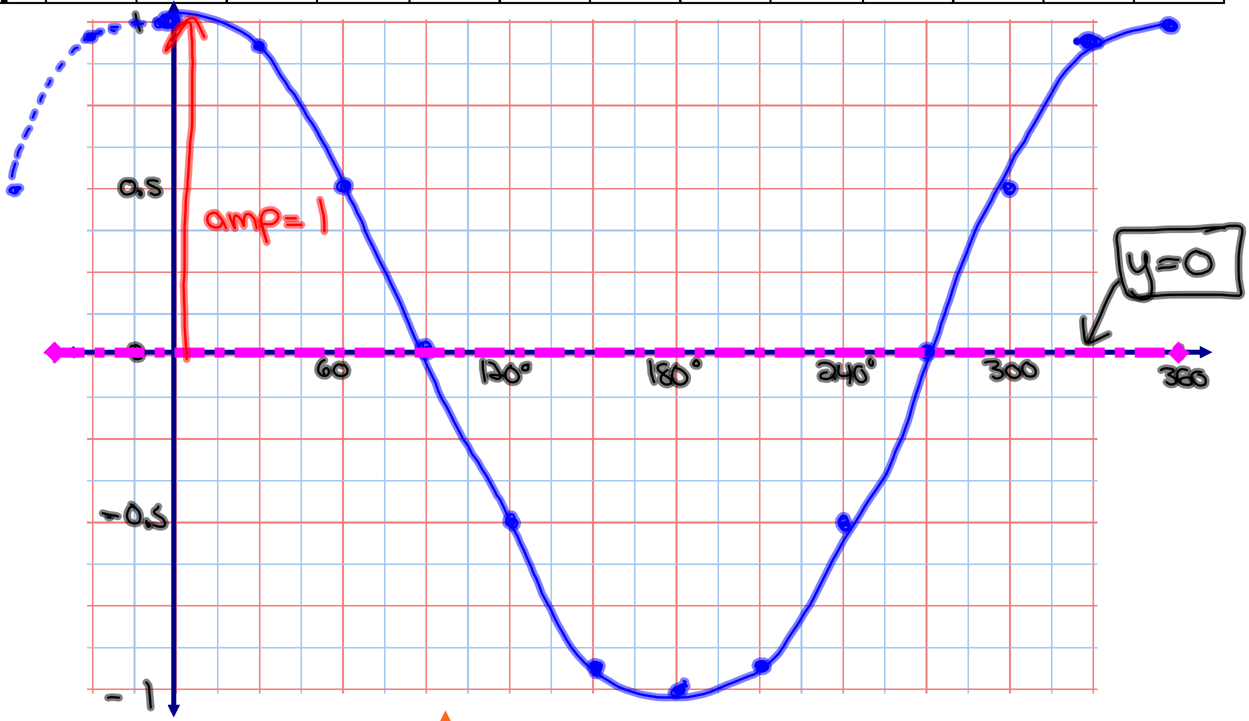
Sinusoidal Axis:

Amplitude:

What about $y = \cos\theta$?

Complete the table of values and sketch below

θ	$0°$	$30°$	$60°$	$90°$	$120°$	$150°$	$180°$	$210°$	$240°$	$270°$	$300°$	$330°$	$360°$
y	1	0.9	0.5	0	-0.5	-0.9	-1	-0.9	-0.5	0	0.5	0.9	1



Is this a sinusoidal function? Yes

What about the period, sinusoidal axis, and amplitude?

$P = 360°$

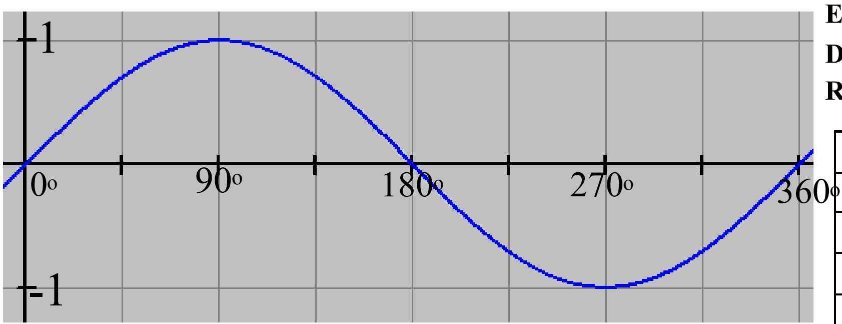
$$\text{sin axis} = \frac{1+(-1)}{2} = \frac{0}{2} = 0$$

$\boxed{y=0}$

Amp = 1

Basic Trig Graphs

y = sin θ

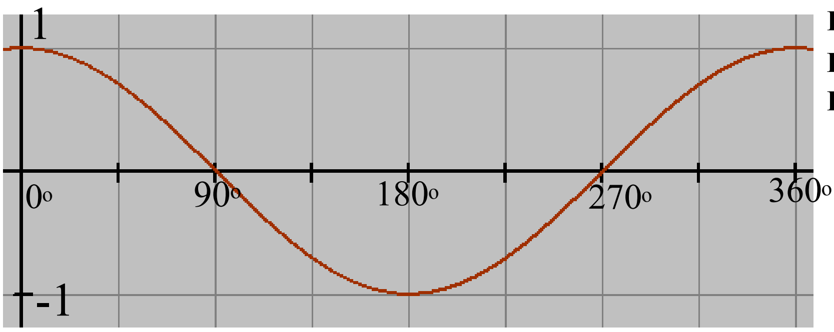


Period = 360°
Amplitude = 1
Eq'n of Sinusoidal Axis: y = 0
Domain: {θ ∈ R }
Range: { -1 ≤ y ≤ 1 }

θ	y
0°	0
90°	1
180°	0
270°	−1
360°	0

---

y = cos θ



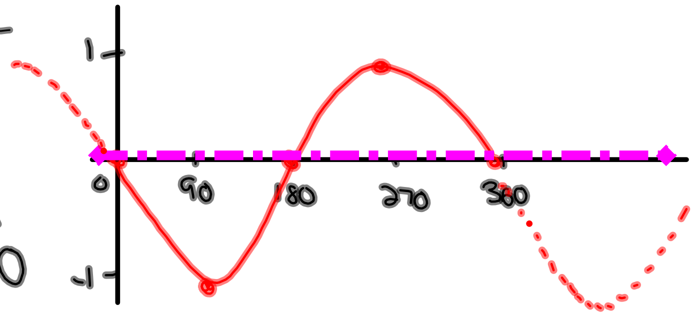
Period = 360°
Amplitude = 1
Eq'n of Sinusoidal Axis: y = 0
Domain: {θ ∈ R }
Range: { -1 ≤ y ≤ 1 }

θ	y
0°	1
90°	0
180°	−1
270°	0
360°	1

Homework

$y = -\sin\theta$ (reflected in x-axis)

θ	y
0°	0
90°	-1
180°	0
270°	1
360°	0



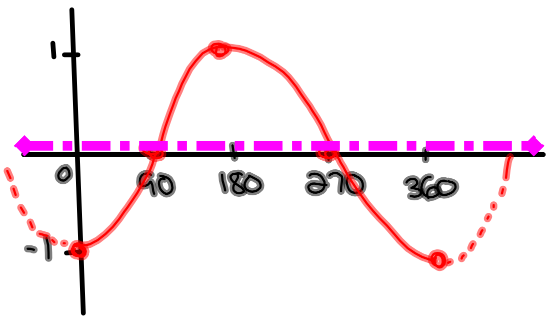
'Period': 360°
Amp: 1
equation of sin axis: $y=0$

D: $\{\theta \mid \theta \in R\}$

R: $\{y \mid -1 \le y \le 1, y \in R\}$

$y = -\cos\theta$

θ	y
0°	-1
90°	0
180°	1
270°	0
360°	-1



'Period': 360°
Amp: 1
equation of sin axis: $y=0$

D: $\{\theta \mid \theta \in R\}$

R: $\{y \mid -1 \le y \le 1, y \in R\}$

Transformations of the Sinusoidal Function

Recall... $y=-2(x-3)^2+4$ (Quadratic)

reflection in the x-axis (vertical)

vertical stretch by a factor of 2

$a=-2$

horizontal translation

$h=3$ (right 3)

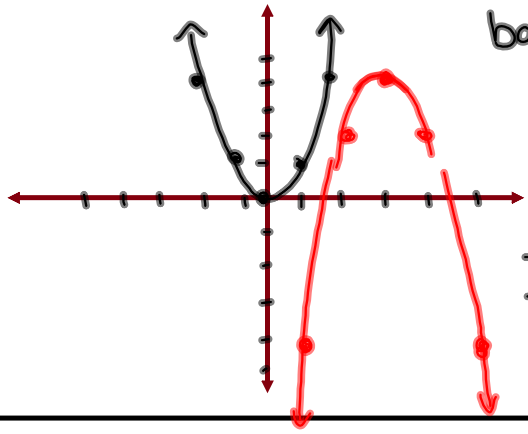
vertical translation

$K=4$ (up 4)

(h, K)

Vertex $\Rightarrow$ $(3, 4)$

Sketch $\Rightarrow$



base: $y=x^2$

x	y
-2	4
-1	1
0	0
1	1
2	4

$y=-2(x-3)^2+4$

x	y
1	-4
2	2
3	4
4	2
5	-4

Now, let's look at a sinusoidal function...

$$y=-2\sin[3(\theta-60^\circ)]-1$$

Reflection

Amplitude (v. stretch)

$a=2$

Horizontal Stretch

$b=3$

Phase Shift (h. translation)

$h=60^\circ$

Vertical Translation

$K=-1$

Equations in Standard Form

$$y = a\sin\left[b(x-c)\right] + d \quad \text{or} \quad y = a\cos\left[b(x-h)\right] + k$$

a = ***Amplitude*** → influences how tall the sine curve is. (always positive)

$b = \dfrac{360°}{P}$ → influences how often the pattern repeats. $\left(P = \dfrac{360°}{b}\right)$

Period

c = ***Horizontal Translation*** → Influences how far to the left or the right that the graph will shift.

- If C is positive → Shift Left
- If C is negative → Shift Right

} Inside Brackets

d = ***Vertical Translation*** → influences how far up and down the graph will shift.

- If d is positive → Shift Up
- If d is negative → Shift Down
- equal to the sinusoidal axis:
  ↳ equation of sinusoidal axis: $y=d$

Example:

$$2y+5 = -6\sin\left(\tfrac{1}{3}x-30°\right)-3$$

$$\frac{2y}{2} = \frac{-6}{2}\sin\left(\tfrac{1}{3}x-30°\right)-\frac{8}{2} \quad \text{(Subtract 5 from both sides)}$$

$$y = -3\sin\left(\tfrac{1}{3}x-30°\right)-4 \quad \text{(Divide by 2)}$$

$$y = -3\sin\left[\tfrac{1}{3}(x-90°)\right]-4 \quad \text{(Factor out a } \tfrac{1}{3}\text{)}$$

$a = 3$ $\quad b = \frac{1}{3}$ $\quad c = 90°$ $\quad d = -4$

$$P = \frac{360°}{b} = \frac{360°}{\frac{1}{3}} = 1080°$$

equation of sinusoidal axis: $y=-4$

Homework

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g) $y+5 = -2\sin\left(4x+\frac{\pi}{3}\right)$

$y = -2\sin\left(4x+\frac{\pi}{3}\right)-5$

$y = -2\sin\left[4\left(x+\frac{\pi}{12}\right)\right]-5$

$a = 2$ $\quad c = -\frac{\pi}{12}$ equation of sin axis: $y = -5$

$b = 4$ $\quad d = -5$ $\quad P = \frac{360}{4} = 90^\circ$

Attachments

worksheet-sketching in radian measure.doc

Worksheet - Finding the Equation.doc

Worksheet - Sketching Trigonometric Functions.doc

Worksheet Solns - Sketching Sinusoidal Relations.doc

Worksheet - Sketching Sinusoidal relations (sept06).pdf

Bonus Soln - Fox Population.doc

Worksheet Solns - Applications of Sinusoidal Relations.doc

Review - Practice Test for Sinusoidal Functions.doc

Review - Trigonometric Functions(3)(4).doc