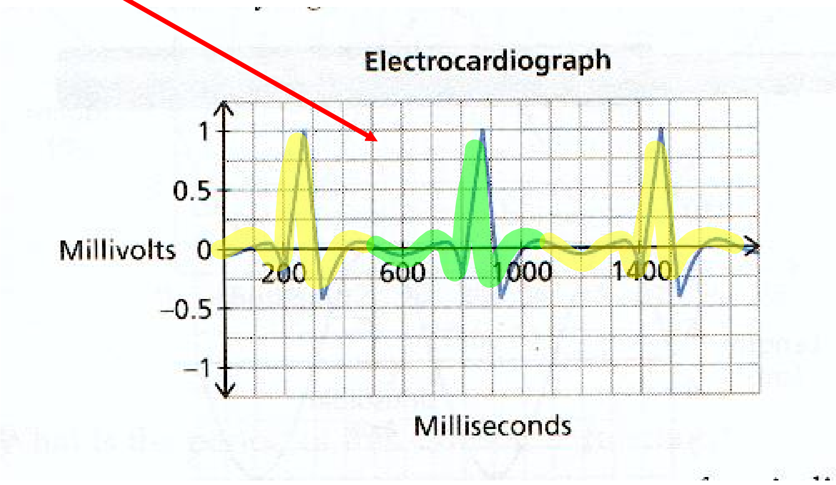
Sinusoidal Relations

Periodic Function: A function for which the dependent variable takes on the same set of values over and over again as the independent variable changes.

(A function that repeats)

Example of periodic behavior

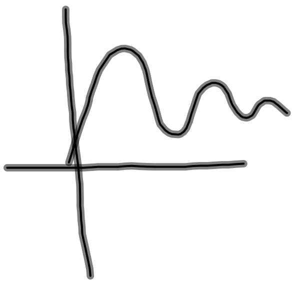


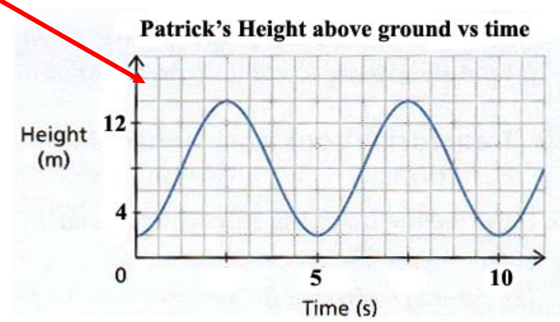
Sinusoidal Function: A periodic function that looks like waves, where any portion of the curve can be translated onto another portion of the curve.

(Repeats and looks like waves)

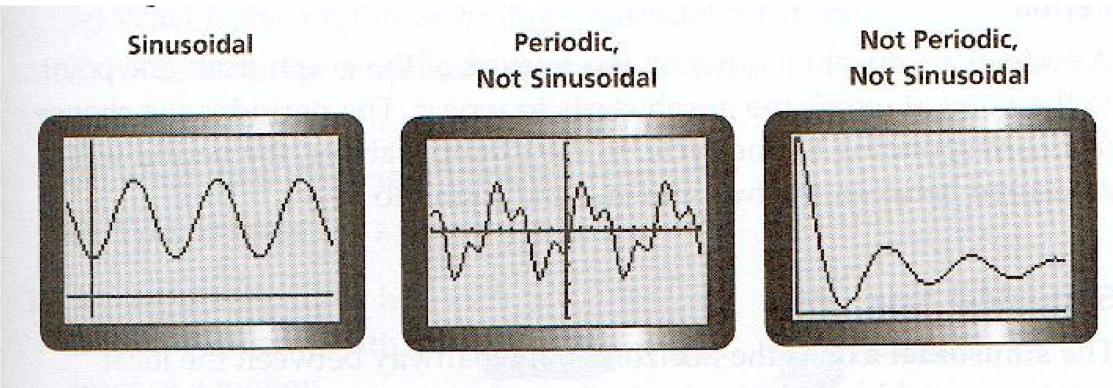
Example of sinusoidal behavior

Neither





These illustrations should summarize periodic and sinusoidal...

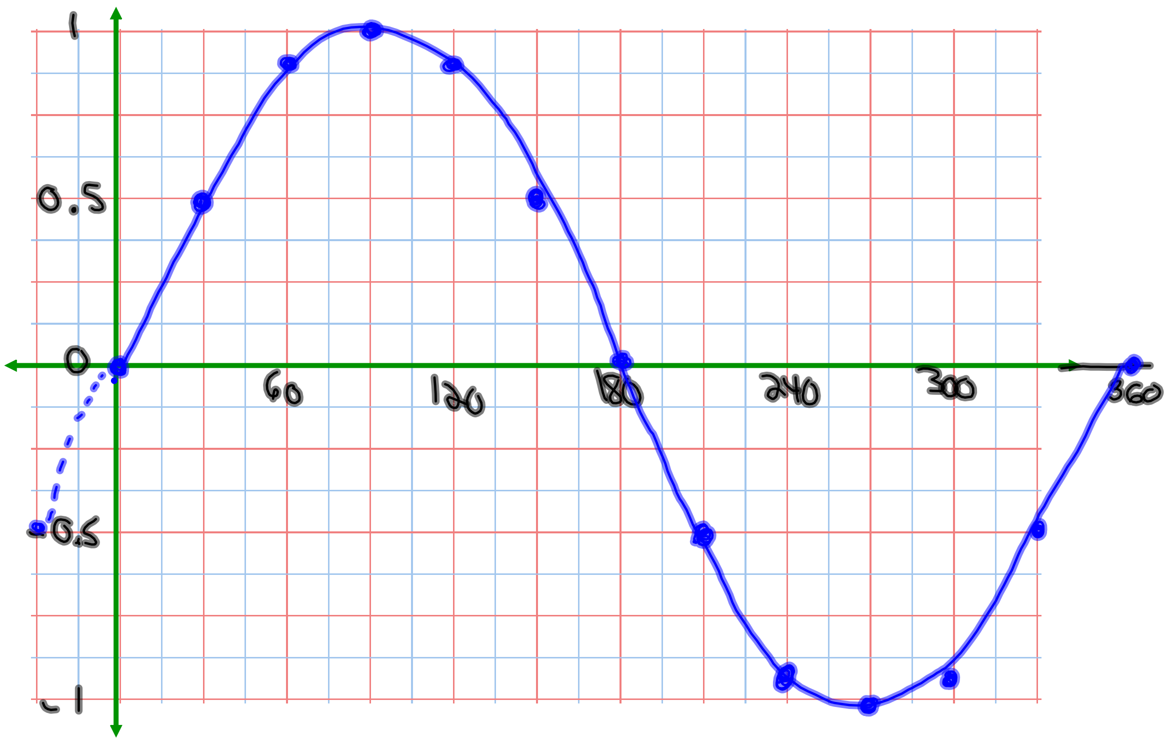


Let's examine the graph of $y = \sin\theta$

$y = \sin x$

θ	$0°$	$30°$	$60°$	$90°$	$120°$	$150°$	$180°$	$210°$	$240°$	$270°$	$300°$	$330°$	$360°$
y	0	0.5	0.9	1	0.9	0.5	0	-0.5	-0.9	-1	-0.9	-0.5	0

Now plot the above points...

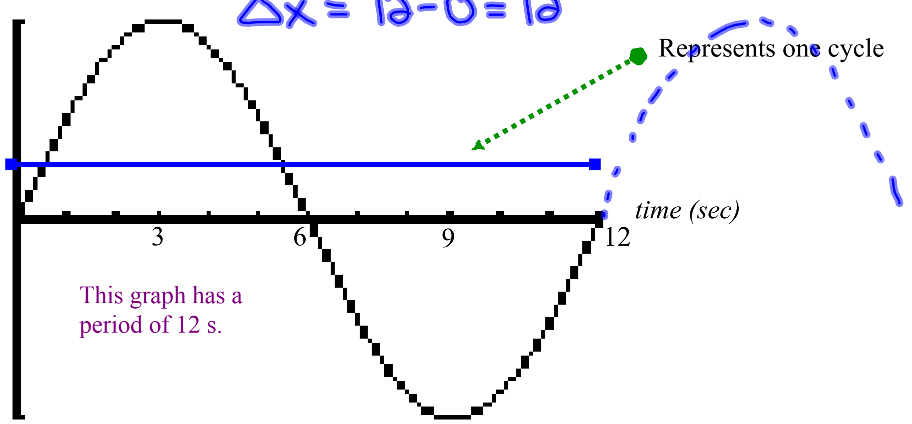


Vocabulary of Sinusoidal Functions

I. Period:

The change in x corresponding to one cycle. (max → max) (min → min)

$\Delta x = 12 - 0 = 12$



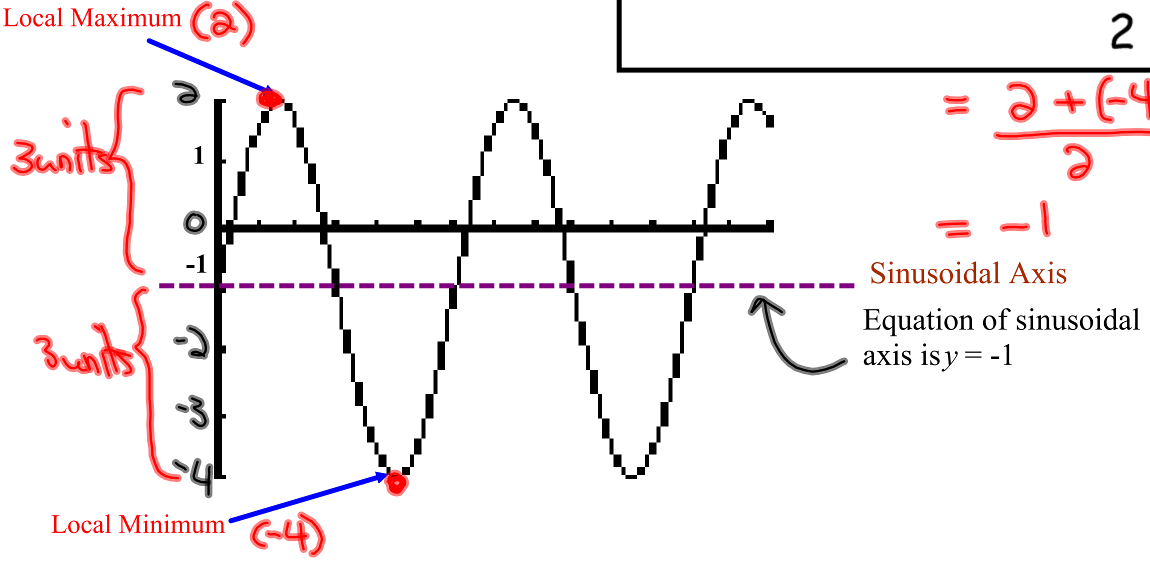
II. Sinusoidal Axis:

The y = horizontal line halfway between the local maximum (highest point) and local minimum (lowest).

$$\boxed{\text{Sinusoidal Axis} = \frac{\text{Max} + \text{Min}}{2}}$$

$$= \frac{2 + (-4)}{2}$$

$$= -1$$



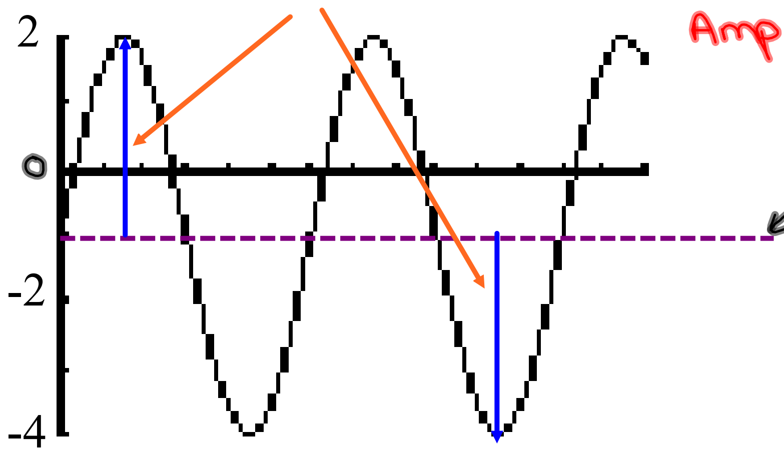
III. Amplitude:

The vertical distance from the sinusoidal axis to a local maximum or local minimum. (Amplitude is always (+))

$y=-2\sin[3(x+30°)]-1$

Amp = 2

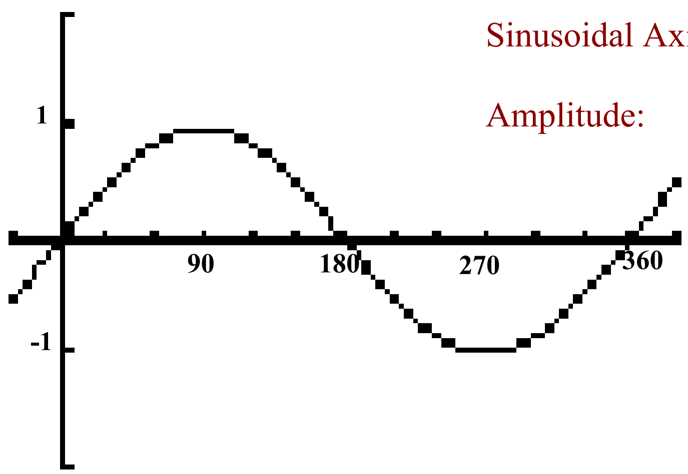
Amplitude would equal 3



sinusoidal axis: $\boxed{y=-1}$

Summarize...

Here is the graph of $y=\sin\theta$



Period :

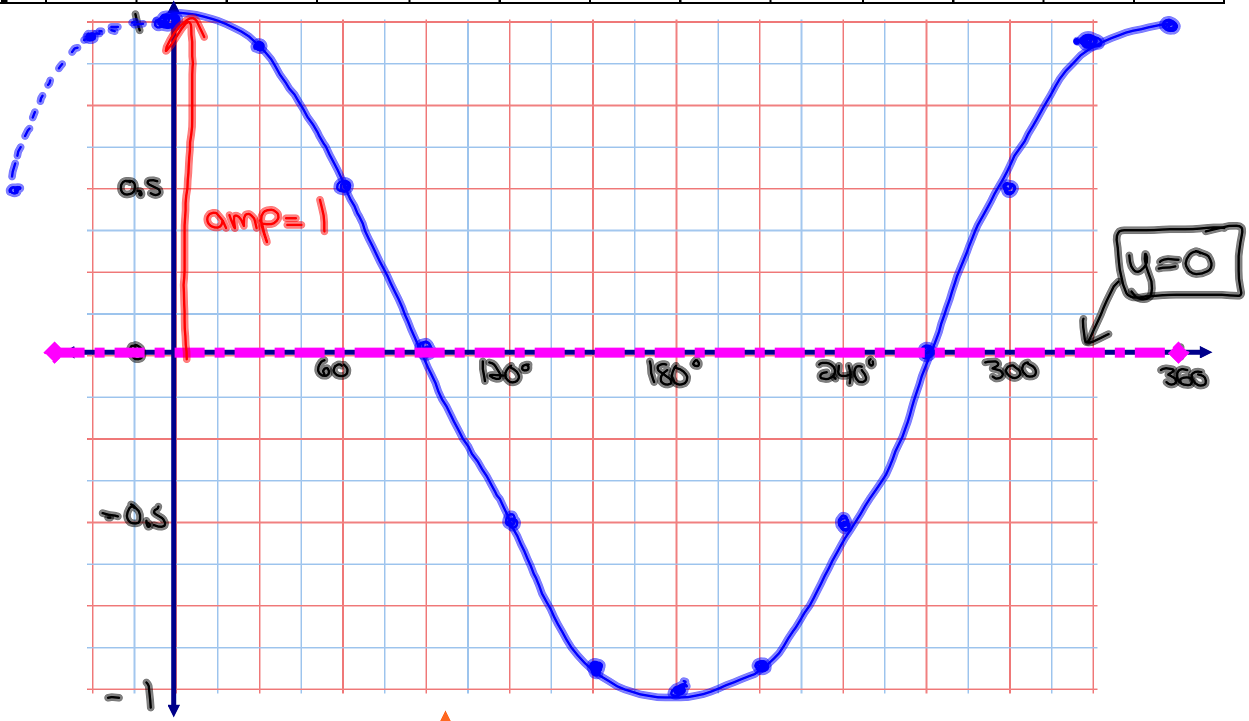
Sinusoidal Axis:

Amplitude:

What about $y = \cos\theta$?

Complete the table of values and sketch below

θ	$0°$	$30°$	$60°$	$90°$	$120°$	$150°$	$180°$	$210°$	$240°$	$270°$	$300°$	$330°$	$360°$
y	1	0.9	0.5	0	-0.5	-0.9	-1	-0.9	-0.5	0	0.5	0.9	1



Is this a sinusoidal function? Yes

What about the period, sinusoidal axis, and amplitude?

P= 360°

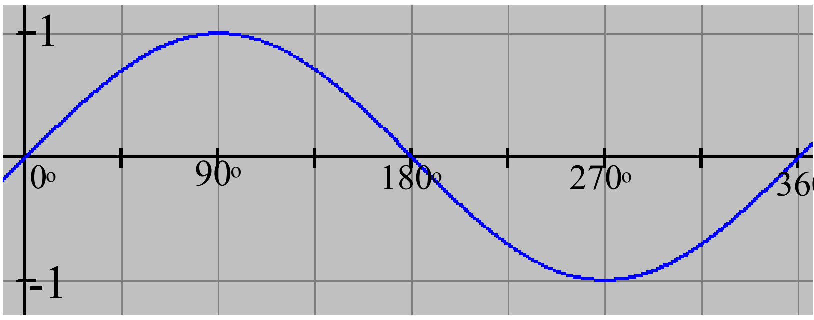
$$\text{sin axis} = \frac{1+(-1)}{2} = \frac{0}{2} = 0$$

$\boxed{y=0}$

Amp = 1

Basic Trig Graphs

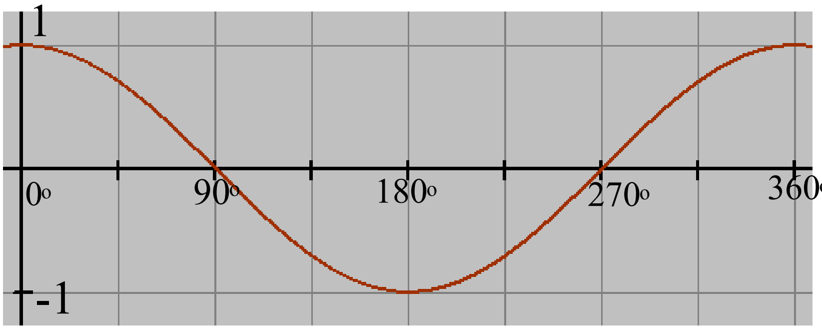
$y = \sin \theta$



Period = 360°
Amplitude = 1
Eq'n of Sinusoidal Axis: y = 0
Domain: $\{\theta \in R\}$
Range: $\{-1 \leq y \leq 1\}$

θ	y
0°	0
90°	1
180°	0
270°	−1
360°	0

$y = \cos \theta$



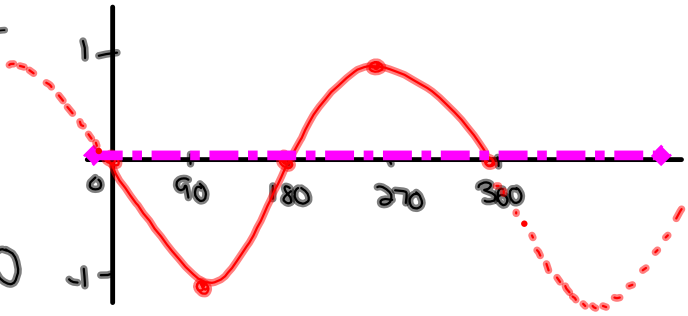
Period = 360°
Amplitude = 1
Eq'n of Sinusoidal Axis: y = 0
Domain: $\{\theta \in R\}$
Range: $\{-1 \leq y \leq 1\}$

θ	y
0°	1
90°	0
180°	−1
270°	0
360°	1

Homework

$y = -\sin\theta$ (Reflected in x-axis)

θ	y
0°	0
90°	-1
180°	0
270°	1
360°	0

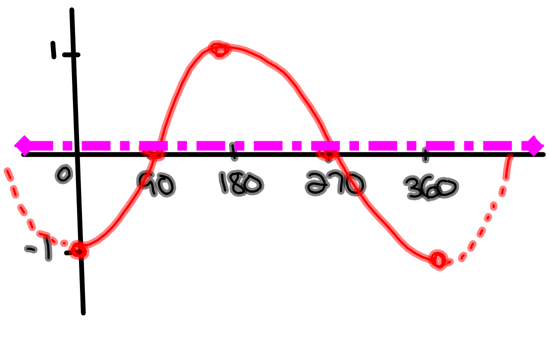


'Period': 360°
Amp: 1
equation of sin axis: $y=0$

D: $\{\theta \mid \theta \in R\}$
R: $\{y \mid -1 \leq y \leq 1, y \in R\}$

$y = -\cos\theta$

θ	y
0°	-1
90°	0
180°	1
270°	0
360°	-1



'Period': 360°
Amp: 1
equation of sin axis: $y=0$

D: $\{\theta \mid \theta \in R\}$
R: $\{y \mid -1 \leq y \leq 1, y \in R\}$

Transformations of the Sinusoidal Function

Recall... $y = -2(x-3)^2 + 4$ (Quadratic)

reflection in the x-axis (vertical)

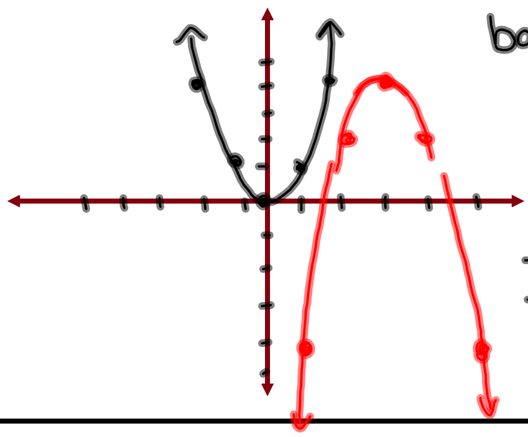
vertical stretch by a factor of 2
$a=-2$

horizontal translation
$h=3$
(right 3)

vertical translation
$k=4$
(up 4)

(h,k)
Vertex $\Rightarrow$ (3, 4)

Sketch $\Rightarrow$



base: $y=x^2$

$y=-2(x-3)^2+4$

x	y
-2	4
-1	1
0	0
1	1
2	4

x	y
1	-4
2	2
3	4
4	2
5	-4

Now, let's look at a sinusoidal function...

$$y = -2\sin[3(\theta - 60^\circ)] - 1$$

Reflection

Amplitude (v. stretch)
$a=2$

Horizontal Stretch
$b=3$

Phase Shift (h. translation)
$h=60^\circ$

Vertical Translation
$k=-1$

Equations in Standard Form

$$y = a\sin[b(x-c)]+d \quad \text{or} \quad y = a\cos[b(x-h)]+k$$

a = ***Amplitude*** → influences how tall the sine curve is.(always positive)

$b = \dfrac{360^\circ}{P}$ → influences how often the pattern repeats. $\left(P = \dfrac{360^\circ}{b}\right)$

Period

c = ***Horizontal Translation*** → Influences how far to the left or the right that the graph will shift.

- If C is positive → Shift Left
- If C is negative → Shift Right

} Inside Brackets

d = ***Vertical Translation*** → influences how far up and down the graph will shift.

- If d is positive → Shift Up
- If d is negative → Shift Down
- equal to the sinusoidal axis:
  ↳ equation of sinusoidal axis: $y=d$

Example:

$$2y+5 = -6\sin\left(\tfrac{1}{3}x-30^\circ\right)-3$$

$$\frac{2y}{2} = \frac{-6}{2}\sin\left(\tfrac{1}{3}x-30^\circ\right)\frac{-8}{2}$$ (Subtract 5 from both sides)

$$y = -3\sin\left(\tfrac{1}{3}x-30^\circ\right)-4$$ (Divide by 2)

$$y = -3\sin\left[\tfrac{1}{3}(x-90^\circ)\right]-4$$ (Factor out a $\tfrac{1}{3}$)

$a = 3$ $\quad b = \tfrac{1}{3}$ $\quad c = 90^\circ$ $\quad d = -4$

$P = \dfrac{360^\circ}{b} = \dfrac{360^\circ}{\tfrac{1}{3}} = 1080^\circ$

equation of sinusoidal axis: $y=-4$

Homework

Page 233 #1-9

g) $y+5 = -2\sin\left(4x+\frac{\pi}{3}\right)$

$y = -2\sin\left(4x+\frac{\pi}{3}\right)-5$

$y = -2\sin\left[4\left(x+\frac{\pi}{12}\right)\right]-5$

$a=2$ $\quad c=-\frac{\pi}{12}$ $\quad$ equation of sin axis: $y=-5$

$b=4$ $\quad d=-5$ $\quad P=\frac{360}{4}=90^\circ$

Warm-up

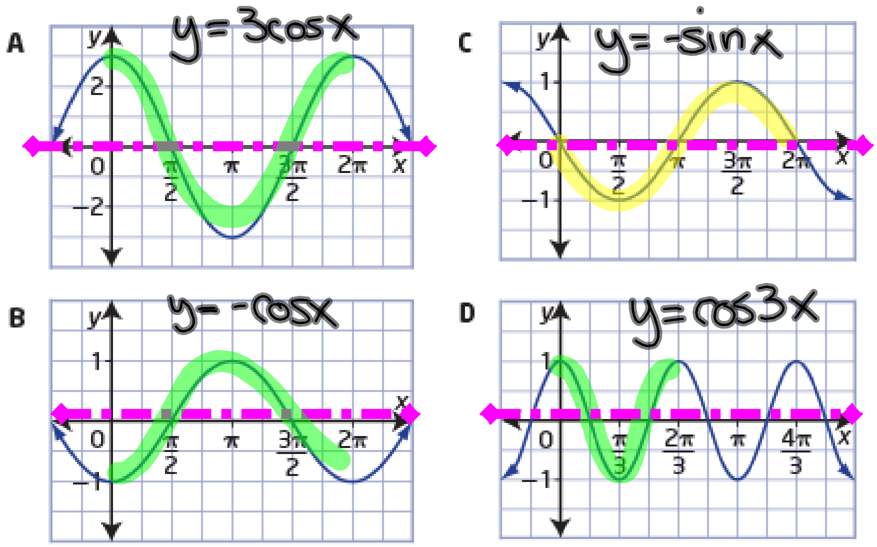
Match each function with its graph.

a) $y = 3\cos x$

b) $y = \cos 3x$

c) $y = -\sin x$

d) $y = -\cos x$



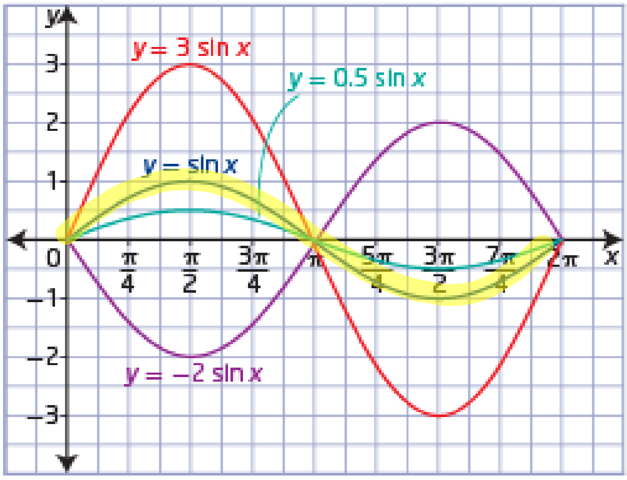
The Value of "a" applies a vertical stretch by a factor of |a|

(Page 286) (Multiply y coordinates by "a")

For the graph of $y = 3 \sin x$, apply a vertical stretch by a factor of 3. (a=3)

For the graph of $y = 0.5 \sin x$, apply a vertical stretch by a factor of 0.5. (a=0.5)

For the graph of $y = -2 \sin x$, reflect in the x-axis and apply a vertical stretch by a factor of 2. (a=2)



The Value of b effects the period of the graph and applies a horizontal stretch by a factor of $\frac{1}{|b|}$

(Multiply x coordinates by $\frac{1}{b}$)

Thus, the period for $y = \sin bx$ or $y = \cos bx$ is $\frac{2\pi}{|b|}$, in radians, or $\frac{360°}{|b|}$, in degrees.

$P = \frac{2\pi}{|b|}$ or $P = \frac{360°}{|b|}$

Find the period of the following functions in both radians and degrees.

$y = \sin 4x$

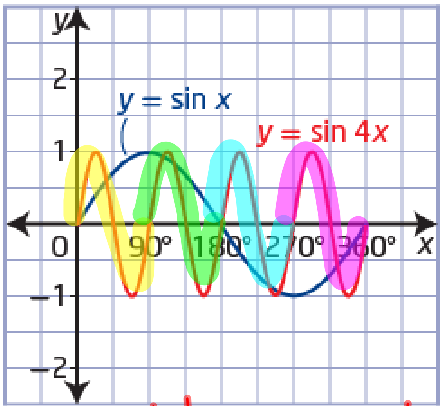
$b = 4$ $P = \frac{360°}{4} = 90°$

$P = \frac{2\pi}{4} = \frac{\pi}{2}$

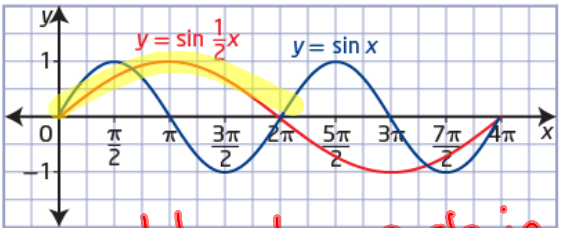
$y = \sin \frac{1}{2}x$

$b = \frac{1}{2}$ $P = \frac{360°}{1/2} = 720°$

$P = \frac{2\pi}{1/2} = 4\pi$



completes 4 cycles in 360° or 2π rads



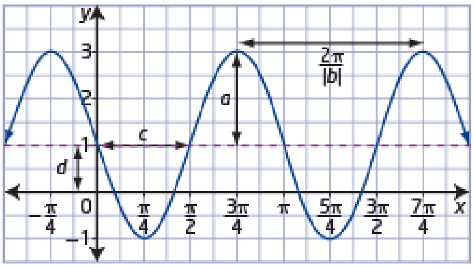
completes ½ a cycle in 360° or 2π rads

Key Ideas

- You can determine the amplitude, period, phase shift, and vertical displacement of sinusoidal functions when the equation of the function is given in the form $y = a \sin b(x - c) + d$ or $y = a \cos b(x - c) + d$.

  For: $y = a \sin b(x - c) + d$
  $y = a \cos b(x - c) + d$

  How does changing each parameter affect the graph of a function?

  Vertical stretch by a factor of $|a|$
  - changes the amplitude to $|a|$
  - reflected in the x-axis if $a < 0$

  Horizontal stretch by a factor of $\frac{1}{|b|}$
  - changes the period to $\frac{360°}{|b|}$ (in degrees) or $\frac{2\pi}{|b|}$ (in radians)
  - reflected in the y-axis if $b < 0$

  Horizontal phase shift represented by c
  - to right if $c > 0$
  - to left if $c < 0$

  Vertical displacement represented by d
  - up if $d > 0$
  - down if $d < 0$

  $$d = \frac{\text{maximum value} + \text{minimum value}}{2}$$

- You can determine the equation of a sinusoidal function given its properties or its graph.

Sketching Sinusoidal Functions using Mapping

Development of a standard form for sinusoidal functions...

Standard Form ⟶ $y = a\sin\left[b(x-c)\right]+d$

1. Reflection: If a < 0 the graph will be reflected in the x-axis.

2. Amplitude: The amplitude of the graph will be equal to | a | . always stated as a positive

3. Period: The period of the graph will be equal to $\frac{360°}{b}$ or $\frac{2\pi}{b}$

4. Horizontal Phase Shift: The graph will shift "c" units to the right.
(Change the sign when you remove it from brackets)

5. Vertical Translation: The graph will shift "d" units up.

The Mapping Rule: $(x, y) \rightarrow \left[\frac{x}{b}+c, ay+d\right]$

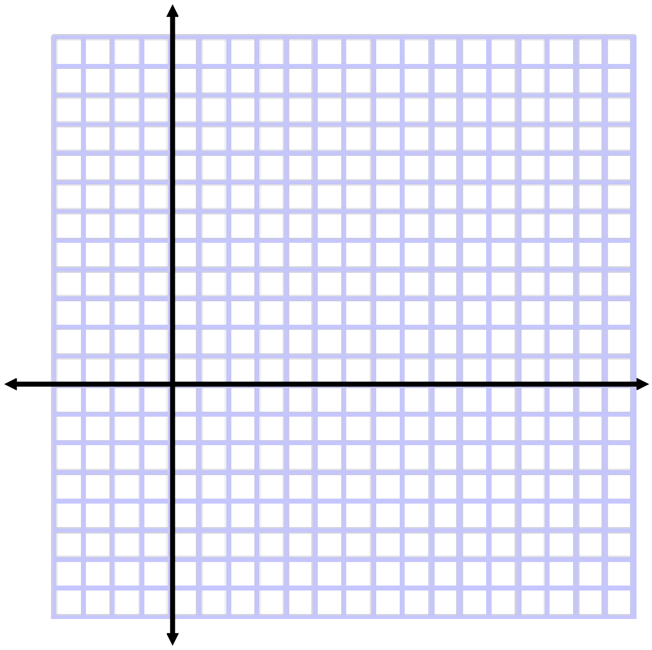
EXAMPLE #1

Now let's sketch a graph of $f(\theta) = -2\sin 3(\theta + 30°) - 2$

" THINK: RST "

Sketching using transformations:

- *Apply the reflections and stretches first*
- *Apply phase shift and vertical translation second*



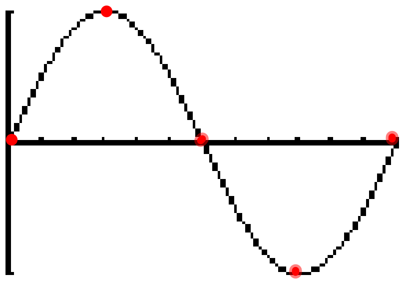
DOMAIN	
RANGE	
AMPLITUDE	
PERIOD	
PHASE SHIFT	
VERTICAL TRANSLATION	
EQUATION OF SINUSOIDAL AXIS	

Check our graph using a graphing calculator

$$f(\theta) = -2\sin 3(\theta + 30°) - 2$$

$a = 2$ $b = 3$ $c = -30°$ $d = -2$

$y = \sin\theta$



Mapping:

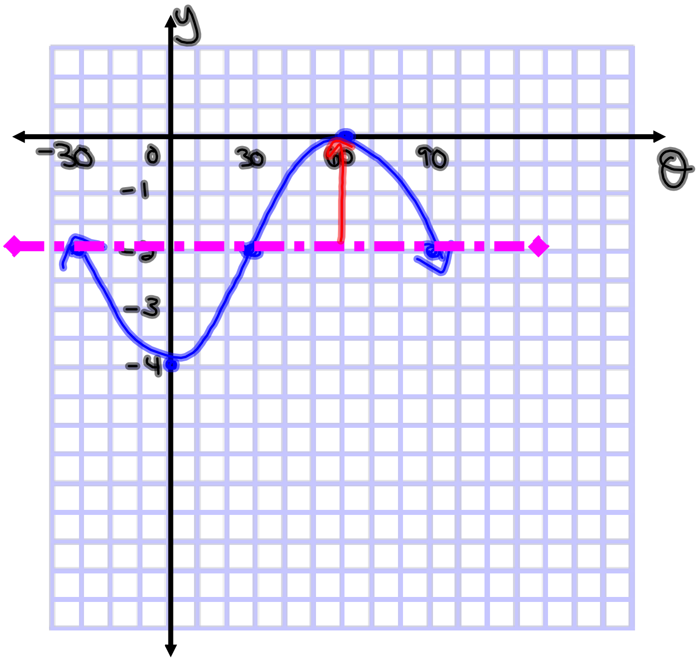
$$(\theta, y) \longrightarrow \left[\tfrac{1}{3}(\theta) - 30°, -2y - 2\right]$$

$y = \sin\theta$

θ	y
0	0
90	1
180	0
270	-1
360	0

New points after mapping

θ	y
$-30°$	-2
$0°$	-4
$30°$	-2
$60°$	0
$90°$	-2



DOMAIN	$\{\theta \mid \theta \in R\}$
RANGE	$\{y \mid -4 \le y \le 0, y \in R\}$
AMPLITUDE	$a = 2$
PERIOD	$P = \frac{360°}{3} = 120°$
PHASE SHIFT	$c = -30°$
VERTICAL TRANSLATION	$d = -2$
EQUATION OF SINUSOIDAL AXIS	$y = -2$

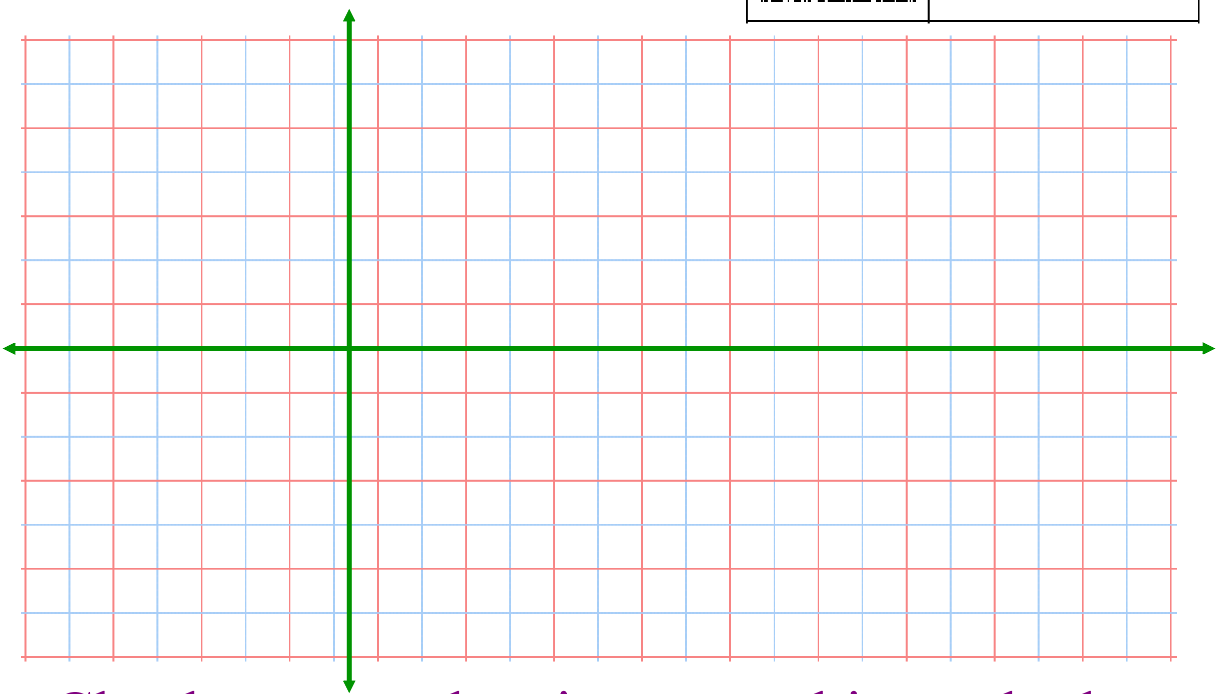
EXAMPLE #2

Now let's sketch a graph of $y = 3\cos[2(\theta - 135°)] + 2$

Sketching using transformations:

- *Apply the reflections and stretches first*
- *Apply phase shift and vertical translation second*

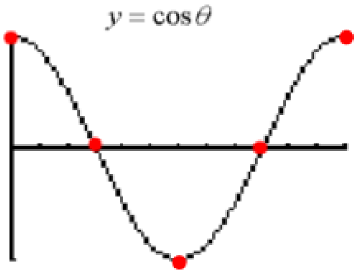
DOMAIN	
RANGE	
AMPLITUDE	
PERIOD	
PHASE SHIFT	
VERTICAL TRANSLATION	
EQUATION OF SINUSOIDAL AXIS	



Check our graph using a graphing calculator

This time we will graph the same function using a mapping:

$$y = 3\cos[2(\theta - 135°)] + 2$$



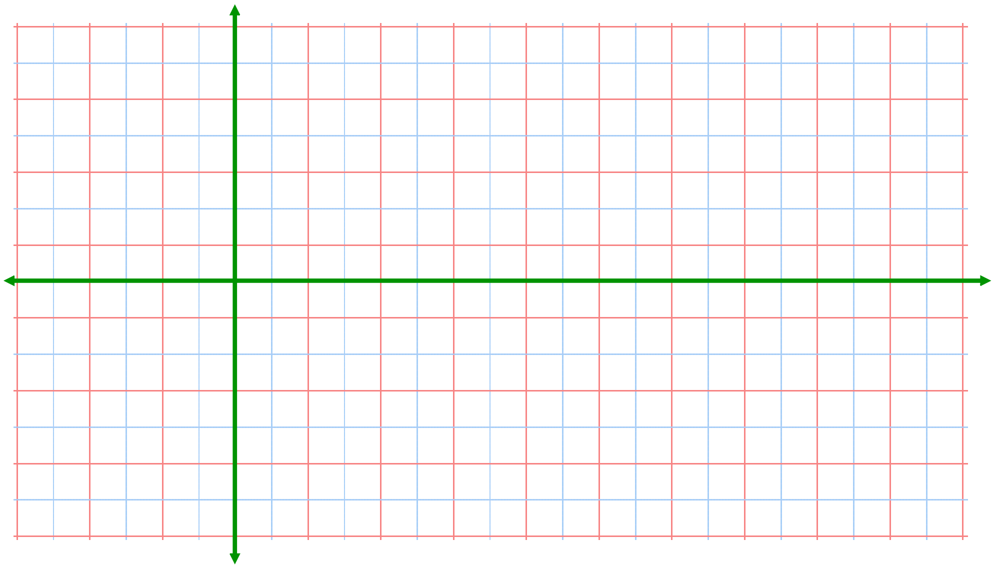
Mapping:

$y = \cos\theta$

θ	y
0	
90	
180	
270	
360	

New points after mapping

θ	y



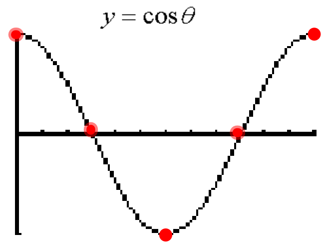
DOMAIN	
RANGE	
AMPLITUDE	
PERIOD	
PHASE SHIFT	
VERTICAL TRANSLATION	
EQUATION OF SINUSOIDAL AXIS	

Hopefully you are not too puzzled for this one...

$$\frac{1}{2}(y+1) = 3\cos\left(\frac{1}{2}\theta - 90°\right) + 2$$

Remember...Put in standard form first!!

Remember what the graph of cosine looks like ??



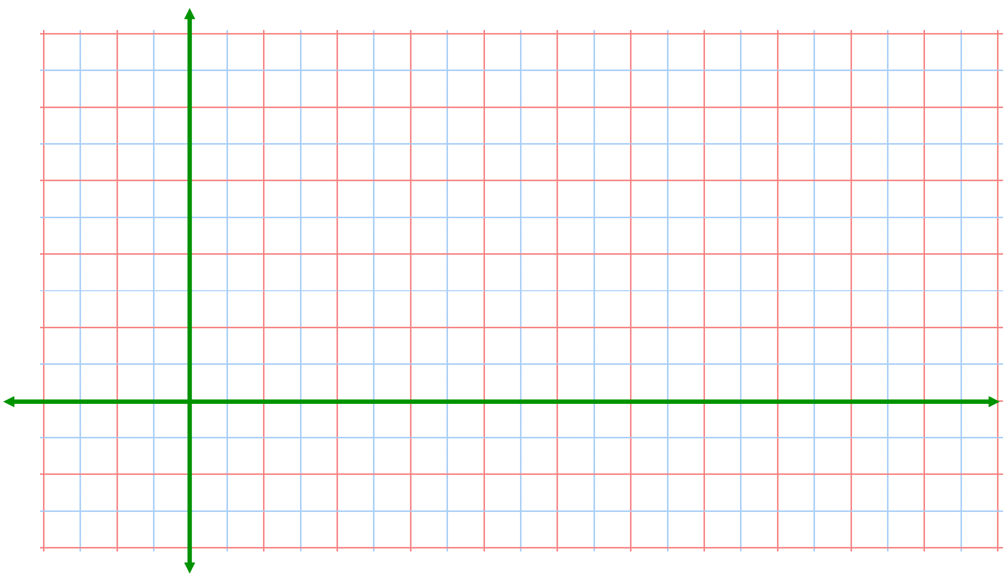
Mapping:

θ	y
0	
90	
180	
270	
360	

New points after mapping

θ	y

DOMAIN	
RANGE	
AMPLITUDE	
PERIOD	
PHASE SHIFT	
VERTICAL TRANSLATION	
EQUATION OF SINUSOIDAL AXIS	



Attachments

worksheet-sketching in radian measure.doc

Worksheet - Finding the Equation.doc

Worksheet - Sketching Trigonometric Functions.doc

Worksheet Solns - Sketching Sinusoidal Relations.doc

Worksheet - Sketching Sinusoidal relations (sept06).pdf

Bonus Soln - Fox Population.doc

Worksheet Solns - Applications of Sinusoidal Relations.doc

Review - Practice Test for Sinusoidal Functions.doc

Review - Trigonometric Functions(3)(4).doc