# Warm-Up

## Solving Polynomial Inequalities

Express answers using interval notation.

$$x^3 - 3x^2 - 4x + 12 \le 0$$

where does the polynomial have negative y-values

$y = x^3 - 3x^2 - 4x + 12$ (Write as a polynomial function)

$y = (x^3 - 3x^2)(-4x + 12)$ (Factor by grouping)

$y = x^2(x-3) - 4(x-3)$

$y = (x-3)(x^2-4)$ (Factor using difference of squares)

$y = (x-3)(x-2)(x+2)$

Find the x-intercepts: $(y=0)$

$0 = (x-3)(x-2)(x+2)$

$x-3=0 \quad | \quad x-2=0 \quad | \quad x+2=0$

$x=3 \quad | \quad x=2 \quad | \quad x=-2$

$x = -2, 2, 3$

Create a number line and label your roots

− + − +

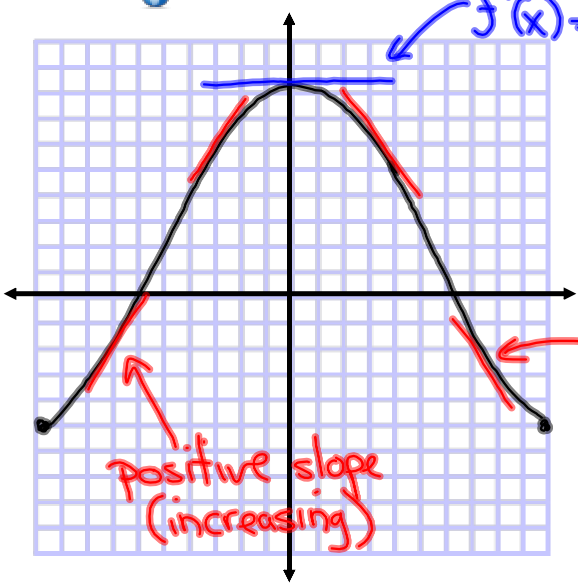
(-3) -2 (0) 2 (2.5) 3 (4)

State the intervals that satisfy the inequality

$$\boxed{x \in (-\infty, -2] \cup [2, 3]}$$

# Increasing and Decreasing Functions

$f'(x) = 0$ (horizontal line has a slope $= 0$)




negative slope (decreasing)

**Test for Increasing and Decreasing Functions**

1. If $f'(x) > 0$ for all $x$ in an interval $I$, then $f$ is increasing on $I$. $x \in (-10, 0)$
   positive
2. If $f'(x) < 0$ for all $x$ in an interval $I$, then $f$ is decreasing on $I$ $x \in (0, 10)$
   negative

Recall $f'(x)$ is the slope of your tangent to the curve

- positive slope:
- negative slope:

## *Example 1*

Find the intervals on which the function $f(x) = 1 - 5x + 4x^2$ is increasing and decreasing.

**Solution**

First we find the derivative of $f(x) = 1 - 5x + 4x^2$ and get

[redacted]

The function $f$ will be increasing when [redacted]

Thus $f$ will be increasing on the interval ________

**Similarly,**

The function $f$ will be decreasing when [redacted]

Thus $f$ will be decreasing on the interval ________

## *Example 2*

Where is the function $y = x^3 + 6x^2 + 9x + 2$ increasing?

where is $y' > 0$

**Solution**

First we compute the derivative and factor it:

$$y' = 3x^2 + 12x + 9$$
$$= 3(x^2 + 4x + 3)$$
$$= 3(x+1)(x+3)$$

Find critical numbers

$y' = 3(x+1)(x+3)$

$0 = 3(x+1)(x+3)$

$x+1=0 \mid x+3=0$

$x=-1 \mid x=-3$

The function $f$ will be increasing when $y' > 0$, so we have to solve the quadratic ineqaulity $(x+1)(x+3)>0$

Create a number line and label your critical #'s

$+ \quad - \quad +$

$(-4) \quad -3 \quad (-2) \quad -1 \quad (0)$

State the intervals where function is increasing.

$x \in (-\infty,-3) \cup (-1,\infty)$ or $\boxed{\text{increasing on } (-\infty,-3) \text{ and } (-1,\infty)}$

| Interval | (x+3) | (x+1) | $f'(x)$ | $f$ |
|---|---|---|---|---|
| $(-\infty,-3)$ (-4) | − | − | + | increasing |
| $(-3,-1)$ (-2) | + | − | − | decreasing |
| $(-1,\infty)$ (0) | + | + | + | increasing |

## *Example 3*

Find the intervals on which the function $f(x) = x^4 - 4x^3 - 8x^2 - 1$ is increasing and decreasing.

### Solution

First we compute the derivative and factor it:

$f'(x) = 4x^3 - 12x^2 - 16x$

$f'(x) = 4x(x^2 - 3x - 4)$

$f'(x) = 4x(x-4)(x+1)$

① Find Critical Numbers ($f'(x)=0$)

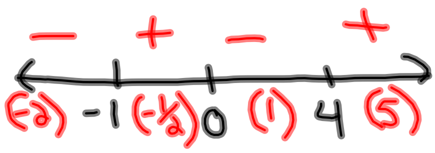
$0 = 4x(x-4)(x+1)$

$4x=0 \mid x-4=0 \mid x+1=0$

$x=0 \mid x=4 \mid x=-1$

$x = -1, 0, 4$

② Create a number line and label your critical #'s




③ Increasing on $(-1,0)$ + $(4,\infty)$

Decreasing on $(-\infty,-1)$ + $(0,4)$

---

| Interval | $4x$ | $(x-4)$ | $(x+1)$ | $f'(x)$ | |
|---|---|---|---|---|---|
| $(-\infty,-1)$ (-2) | – | – | – | – | decreasing |
| $(-1,0)$ (-0.5) | – | – | + | + | increasing |
| $(0,4)$ (1) | + | – | + | – | decreasing |
| $(4,\infty)$ (5) | + | + | + | + | increasing |

# Homework

④ a) $f(x) = 3x^2 - 18x + 1$

$f'(x) = 6x - 18$

$f'(x) = 6(x-3)$

Find Critical Numbers:

$0 = 6(x-3)$

$x - 3 = 0$

$x = 3$

Draw Number Line:

$-$ $+$

(0) 3 (4)

decreasing on $(-\infty, 3)$

increasing on $(3, \infty)$

h) $f(x) = (x^2 - 9)^{2/3}$

$f'(x) = \frac{2}{3}(x^2-9)^{-1/3}(2x) = \frac{4x}{3(x^2-9)^{1/3}} = \frac{4x}{3\sqrt[3]{x^2-9}}$

Critical Numbers:

$4x = 0$

$x = 0$

$3\sqrt[3]{x^2-9} = 0$

$x^2 - 9 = 0$

$x^2 = 9$

$x = \pm 3$

Number Line

$-$ $+$ $-$ $+$

(-4) -3 (-1) 0 (1) 3 (4)

decreases on $(-\infty,-3) + (0,3)$

increases on $(-3,0) + (3,0$