Equations in Standard Form

$$y = a\sin\left[b(x-c)\right]+d$$

a = ***Amplitude*** → influences how tall the sine curve is.

$b = \frac{360}{P}$ → influences how often the pattern repeats.

c = ***Horizontal Translation*** → Influences how far to the left or the right that the graph will shift.

- If C is positive → Shift Right
- If C is negative → Shift Left

d = ***Vertical Translation*** → influences how far up and down the graph will shift.

- If d is positive → Shift Up
- If d is negative → Shift Down

State ***a, b, c, d, and P*** from the following sinusoidal equations:

$$2y + 6 = 4\sin(4x + \frac{\pi}{2}) - 2$$

$$\frac{2y}{2} = \frac{4\sin(4x+\frac{\pi}{2})}{2} \frac{-8}{2}$$

$$y = 2\sin(4x+\frac{\pi}{2}) - 4 \quad \text{(Factor)}$$

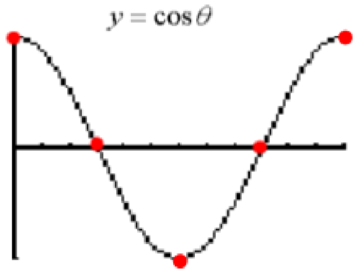
$$y = \underline{2}\sin[\underline{4}(x+\frac{\pi}{8})] \underline{-4}$$

a = 2 b = 4 c = $-\frac{\pi}{8}$ d = -4

P = $\frac{2\pi}{4} = \frac{\pi}{2}$

Solution to Assignment

$$y = 3\cos[2(\theta - 135^\circ)] + 2$$



Mapping:

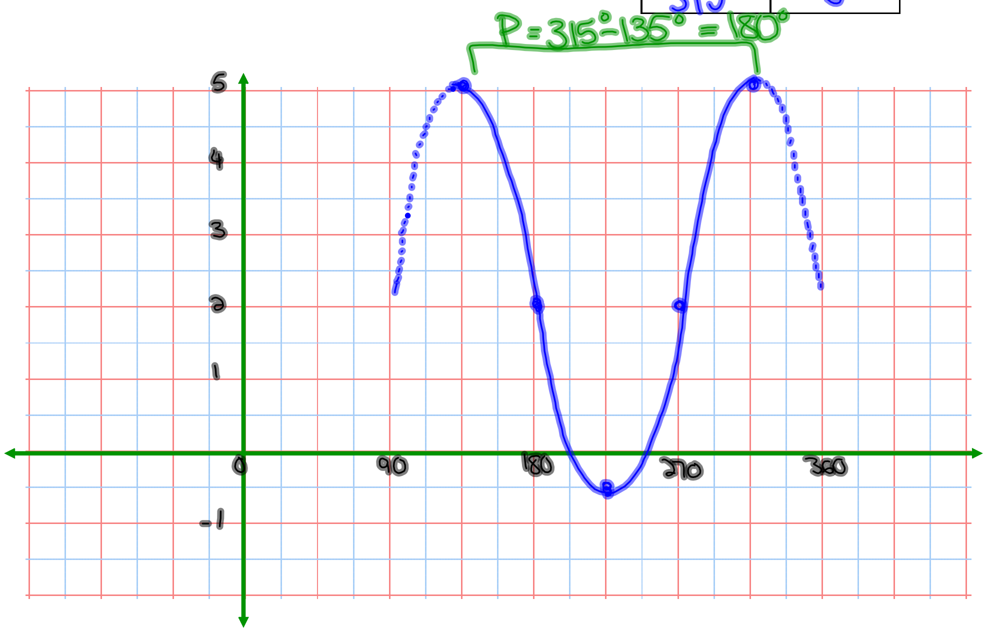
$$(x, y) \longrightarrow \left[\frac{1}{2}\theta + 135^\circ,\ 3y + 2\right]$$

$y = \cos\theta$

θ	y
0	1
90	0
180	-1
270	0
360	1

New points after mapping

θ	y
135°	5
180°	2
225°	-1
270°	2
315°	5



Domain	$\{\theta \mid \theta \in R\}$
Range	$\{y \mid -1 \le y \le 5, y \in R\}$
Amplitude	$a = 3$
Period	$P = \frac{360^\circ}{2} = 180^\circ$
Phase Shift	$c = 135^\circ$
Vertical Translation	$d = 2$
Equation of Sinusoidal Axis	$y = 2$

Sketching Sinusoidal Functions using Mapping

Development of a standard form for sinusoidal functions...

Standard Form ⟶ $$y = a\sin\left[b(x-c)\right]+d$$

1. Reflection: If a < 0 the graph will be reflected in the x-axis.

2. Amplitude: The amplitude of the graph will be equal to | a | .

3. Period: The period of the graph will be equal to $\frac{360°}{b}$

4. Horizontal Phase Shift: The graph will shift "c" units to the right.

5. Vertical Translation: The graph will shift "d" units up.

The Mapping Rule: $$(x, y) \rightarrow \left[\frac{x}{b}+c, ay+d\right]$$

Use Mapping to Graph

$$\frac{1}{2}(y+1) = 3\sin\left(\frac{1}{2}\theta - 90^\circ\right) + 2$$

$\cdot 2 \quad \cdot 2 \quad \cdot 2$

$$y+1 = 6\sin\left(\frac{1}{2}\theta - 90^\circ\right) + 4$$

$$y = 6\sin\left(\frac{1}{2}\theta - 90^\circ\right) + 3$$

$$y = 6\sin\left[\frac{1}{2}\left(\theta - 180^\circ\right)\right] + 3$$

Remember...Put in standard form first!!

(Factor)

a = 6 b = $\frac{1}{2}$ c = 180° d = 3

P = 720°

$y = \sin\theta$

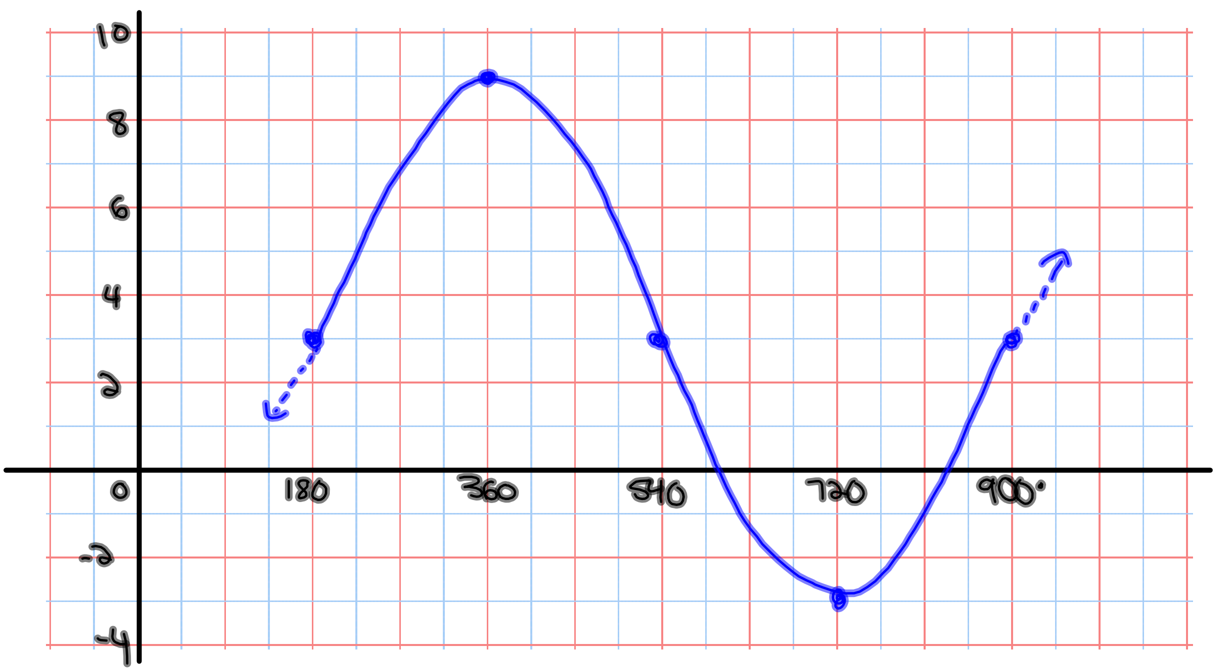
θ	y
0	0
90	1
180	0
270	-1
360	0

$$(x, y) \rightarrow \left[\frac{x}{b} + c, ay + d\right]$$

New points after mapping

$$(x, y) \rightarrow \left[2x + 180^\circ, 6y + 3\right]$$

θ	y
180°	3
360°	9
540°	3
720°	-3
900°	3



Use Mapping to Graph

$$\frac{3y}{3} = \frac{-6}{3}\cos(3x-\pi) - \frac{9}{3}$$

$$y = -2\cos\left[3\left(x - \frac{\pi}{3}\right)\right] - 3$$

a = 2 b = 3 c = $\frac{\pi}{3}$ d = -3

P = $\frac{2\pi}{3}$

$y = \cos\theta$

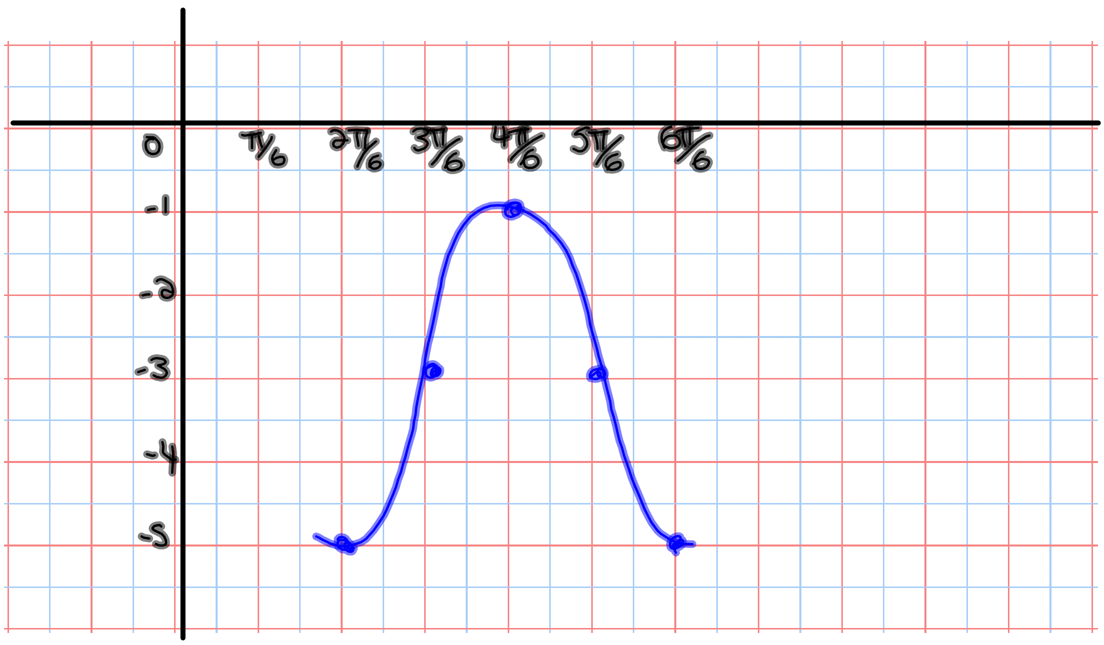
θ	y
0	1
$\pi/2$	0
π	-1
$3\pi/2$	0
2π	1

$$(x,y) \to \left[\frac{x}{b} + c, ay + d\right]$$

New points after mapping

$$(x,y) \to \left[\frac{1}{3}x + \frac{\pi}{3}, -2y - 3\right]$$

θ	y
$(2\pi/6)$ $\pi/3$	-5
$(3\pi/6)$ $\pi/2$	-3
$(4\pi/6)$ $2\pi/3$	-1
$5\pi/6$	-3
$(6\pi/6)$ π	-5

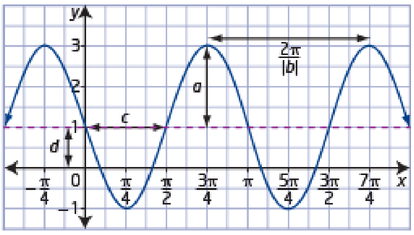


Key Ideas

- You can determine the amplitude, period, phase shift, and vertical displacement of sinusoidal functions when the equation of the function is given in the form $y = a \sin b(x - c) + d$ or $y = a \cos b(x - c) + d$.

For: $y = a \sin b(x - c) + d$
$y = a \cos b(x - c) + d$

How does changing each parameter affect the graph of a function?



Vertical stretch by a factor of $|a|$
- changes the amplitude to $|a|$
- reflected in the x-axis if $a < 0$

Horizontal stretch by a factor of $\frac{1}{|b|}$
- changes the period to $\frac{360°}{|b|}$ (in degrees) or $\frac{2\pi}{|b|}$ (in radians)
- reflected in the y-axis if $b < 0$

Horizontal phase shift represented by c
- to right if $c > 0$
- to left if $c < 0$

Vertical displacement represented by d
- up if $d > 0$
- down if $d < 0$

$$d = \frac{\text{maximum value} + \text{minimum value}}{2}$$

- You can determine the equation of a sinusoidal function given its properties or its graph.

Homework

Finish worksheet

$y = 3\sin\theta + 2$

$y = 3\sin[1(\theta + 0)] + 2$

Attachments

Sketching Sinusoidal Functions.pdf