# Questions from homework

④ f) $h(x) = \frac{x-1}{x+1}$

$$h'(x) = \frac{(x+1)(1) - (x-1)(1)}{(x+1)^2}$$

$$h'(x) = \frac{x+1-x+1}{(x+1)^2}$$

$$h'(x) = \frac{2}{(x+1)^2}$$

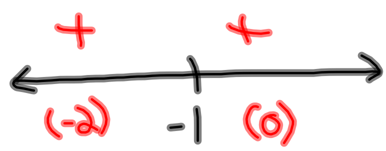
Critical Values:

$(x+1)^2 = 0$

$x+1 = 0$

$x = -1$

Make Number Line:



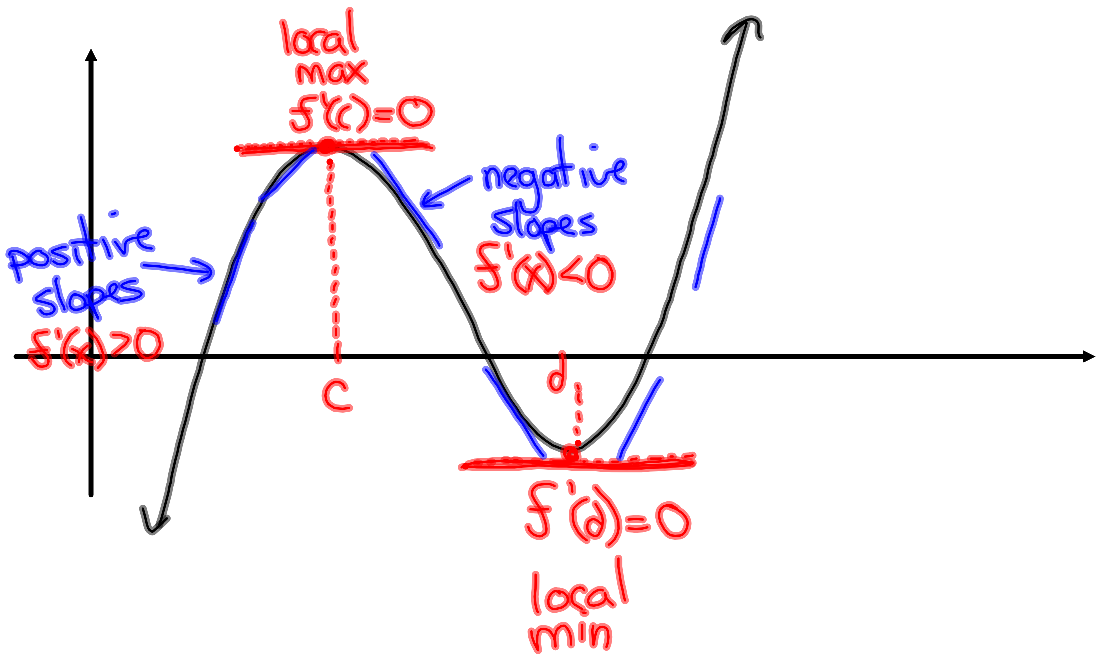

Increasing on $(-\infty, \infty)$

# The First Derivative Test

critical number

If $f$ has a local maximum or minimum at $c$, then $c$ must be a critical value of $f$ (Fermat's Theorem), but not all critical numbers give rise to a maximum or minimum. For instance, recall that 0 is a critical number of the function $y = x^3$ but this function has no maximum or minimum at a critical number.

One way of solving this is suggested by the figure below.
If $f$ is increasing to the left of a critical number $c$ and decreasing to the right of $c$, then $f$ has a local max at $c$.
If $f$ is decreasing to the left of a critical number $c$ and increasing to the right of $c$, then $f$ has a local min at $c$.



# The First Derivative Test

Let $c$ be a critical number of a continuous function $f$.

1. If $f'(x)$ changes from positive to negative at $c$, then $f$ has a local max at $c$.

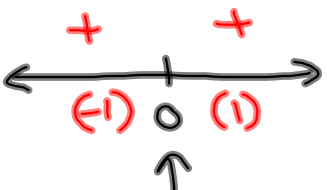
2. If $f'(x)$ changes from negative to positive at $c$, then $f$ has a local min at $c$.

3 If $f'(x)$ does not change signs at $c$, then $f$ has no max or min at $c$.

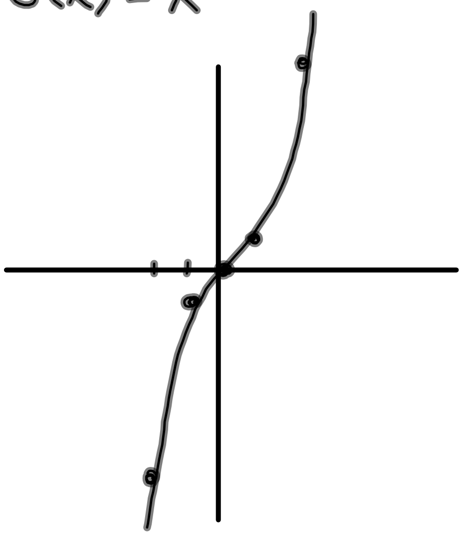
$f(x) = x^3$ $\longrightarrow$ $f(x) = x^3$

$f'(x) = 3x^2$

CV: $x = 0$




Neither a max or min



**Example 1**

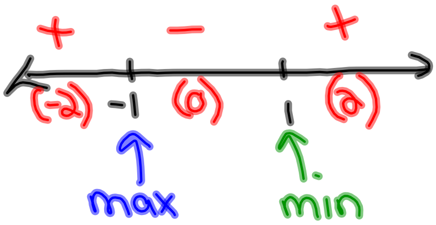
Find the local maximum and minimum values of
$f(x) = x^3 - 3x + 1$

$f'(x) = 3x^2 - 3$

$f'(x) = 3(x^2 - 1)$

$f'(x) = 3(x+1)(x-1)$

CV: $x = \pm 1$




Local max: $f(-1) = (-1)^3 - 3(-1) + 1$
$= -1 + 3 + 1$
$= 3$ $(-1, 3)$

Local min: $f(1) = (1)^3 - 3(1) + 1$
$= 1 - 3 + 1$
$= -1$ $(1, -1)$

Increasing on $(-\infty, -1) + (1, \infty)$
Decreasing on $(-1, 1)$

## Example 2

Find the local maximum and minimum values of
***g(x) = x⁴ - 4x³ - 8x² - 1.*** Use this information to sketch the graph of g.

$g'(x) = 4x^3 - 12x^2 - 16x$

$g'(x) = 4x(x^2 - 3x - 4)$

$g'(x) = 4x(x-4)(x+1)$

CV: $x = -1, 0, 4$

$-$ $+$ $-$ $+$

(-2) -1 (-0.5) 0 (1) 4 (5)

min max min

Local mins:

$g(-1) = (-1)^4 - 4(-1)^3 - 8(-1)^2 - 1 = -4$

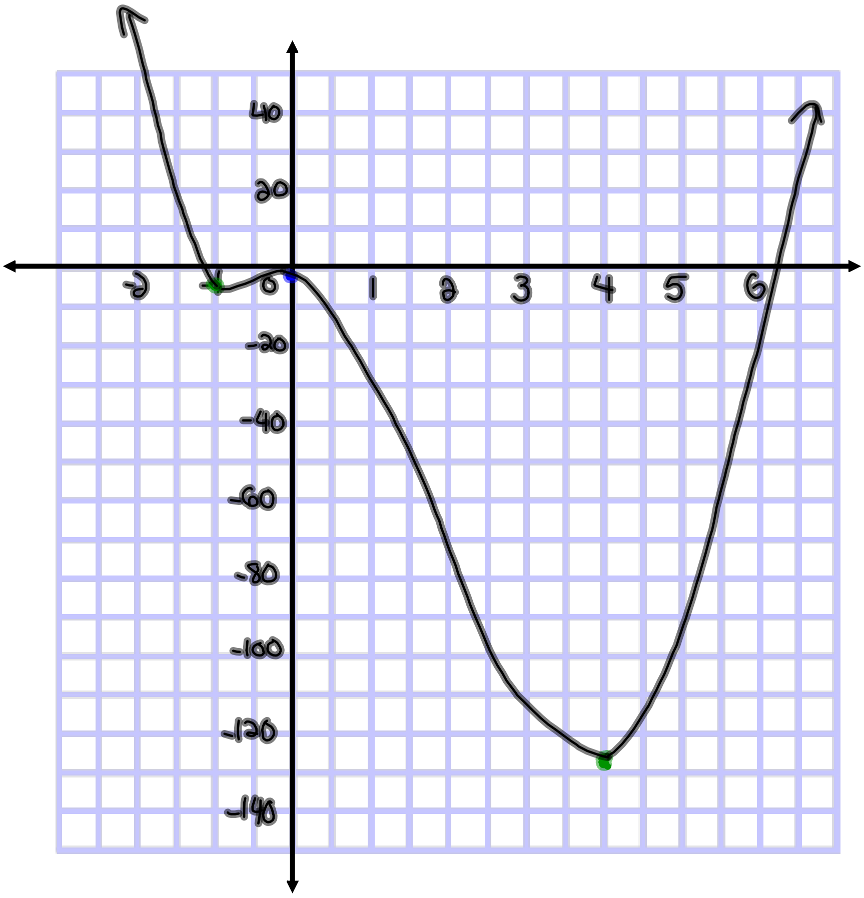
$g(4) = (4)^4 - 4(4)^3 - 8(4)^2 - 1 = -129$

$(-1, -4)$ & $(4, -129)$

Local Max:

$g(0) = (0)^4 - 4(0)^3 - 8(0)^2 - 1 = -1$

$(0, -1)$



# The First Derivative Test

(for absolute extreme values)

Let $c$ be a critical number of a continuous function $f$.

1. If $f'(x)$ is positive for all $x < c$ and $f'(x)$ is negative for all $x > c$, then $f(c)$ is the absolute maximum value.

2. If $f'(x)$ is negative for all $x < c$ and $f'(x)$ is positive for all $x > c$, then $f(c)$ is the absolute minimum value.

# Homework