Equations in Standard Form

$$y = a\sin\left[b(x-c)\right]+d \quad \text{or} \quad y = a\cos[b(x-h)]+k$$

a = ***Amplitude*** → influences how tall the sine curve is. (always positive)

$b = \dfrac{360^\circ}{P}$ → influences how often the pattern repeats. $\left(P = \dfrac{360^\circ}{b}\right)$

Period

c = ***Horizontal Translation*** → Influences how far to the left or the right that the graph will shift.

- If C is positive → Shift Left
- If C is negative → Shift Right

} Inside Brackets

d = ***Vertical Translation*** → influences how far up and down the graph will shift.

- If d is positive → Shift Up
- If d is negative → Shift Down
- equal to the sinusoidal axis:
  ↳ equation of sinusoidal axis: $y=d$

Example:

$$2y+5 = -6\sin\left(\tfrac{1}{3}x-30^\circ\right)-3$$

$$\frac{2y}{2} = \frac{-6}{2}\sin\left(\tfrac{1}{3}x-30^\circ\right)\frac{-8}{2}$$ (Subtract 5 from both sides)

$$y = -3\sin\left(\tfrac{1}{3}x-30^\circ\right)-4$$ (Divide by 2)

$$y = \underline{-3}\sin\left[\tfrac{1}{3}(x-\underline{90^\circ})\right]\underline{-4}$$ (Factor out a $\tfrac{1}{3}$)

$a = 3$ $\quad b = \tfrac{1}{3}$ $\quad c = 90^\circ$ $\quad d = -4$

$P = \dfrac{360^\circ}{b} = \dfrac{360^\circ}{\tfrac{1}{3}} = 1080^\circ$

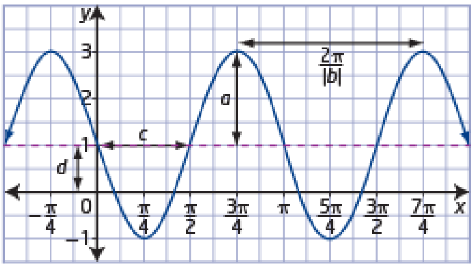
equation of sinusoidal axis: $y=-4$

Key Ideas

- You can determine the amplitude, period, phase shift, and vertical displacement of sinusoidal functions when the equation of the function is given in the form $y = a \sin b(x - c) + d$ or $y = a \cos b(x - c) + d$.

  For: $y = a \sin b(x - c) + d$
  $y = a \cos b(x - c) + d$

  How does changing each parameter affect the graph of a function?

  Vertical stretch by a factor of $|a|$
  - changes the amplitude to $|a|$
  - reflected in the x-axis if $a < 0$

  Horizontal stretch by a factor of $\frac{1}{|b|}$
  - changes the period to $\frac{360°}{|b|}$ (in degrees) or $\frac{2\pi}{|b|}$ (in radians)
  - reflected in the y-axis if $b < 0$

  Horizontal phase shift represented by c
  - to right if $c > 0$
  - to left if $c < 0$

  Vertical displacement represented by d
  - up if $d > 0$
  - down if $d < 0$

  $d = \frac{\text{maximum value} + \text{minimum value}}{2}$

- You can determine the equation of a sinusoidal function given its properties or its graph.

Sketching Sinusoidal Functions using Mapping

Development of a standard form for sinusoidal functions...

Standard Form ⟶ $y = a\sin\left[b(x-c)\right]+d$

1. Reflection: If a < 0 the graph will be reflected in the x-axis.

2. Amplitude: The amplitude of the graph will be equal to | a | . always stated as a positive

3. Period: The period of the graph will be equal to $\frac{360°}{b}$ or $\frac{2\pi}{b}$

4. Horizontal Phase Shift: The graph will shift "c" units to the right.
(Change the sign when you remove it from brackets)

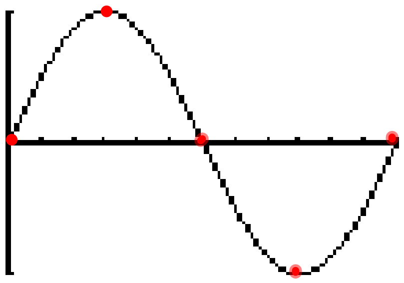
5. Vertical Translation: The graph will shift "d" units up.

The Mapping Rule: $(x,y) \rightarrow \left[\frac{x}{b}+c, ay+d\right]$

$$f(\theta) = -2\sin 3(\theta + 30°) - 2$$

$a = 2$ $b = 3$ $c = -30°$ $d = -2$

$y = \sin\theta$



Mapping:

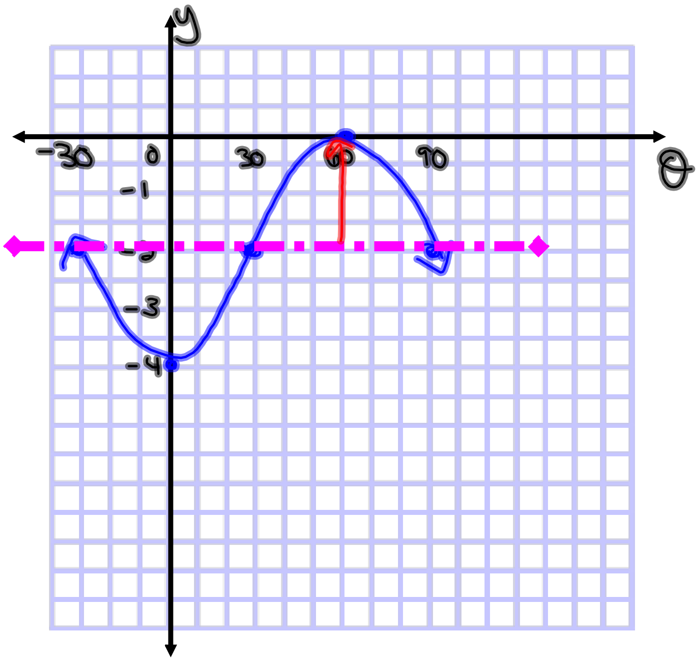
$$(\theta, y) \longrightarrow \left[\frac{1}{3}(\theta) - 30°, -2y - 2\right]$$

$y = \sin\theta$

θ	y
0	0
90	1
180	0
270	-1
360	0

New points after mapping →

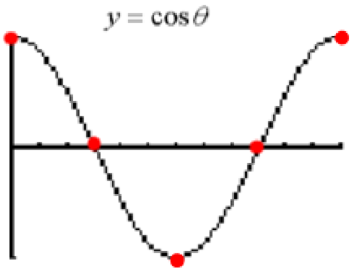
θ	y
-30°	-2
0°	-4
30°	-2
60°	0
90°	-2



Domain	$\{\theta \mid \theta \in R\}$
Range	$\{y \mid -4 \le y \le 0, y \in R\}$
Amplitude	$a = 2$
Period	$P = \frac{360°}{3} = 120°$
Phase Shift	$c = -30°$
Vertical Translation	$d = -2$
Equation of Sinusoidal Axis	$y = -2$

Solution to Assignment

$$y = 3\cos[2(\theta - 135°)] + 2$$



Mapping:

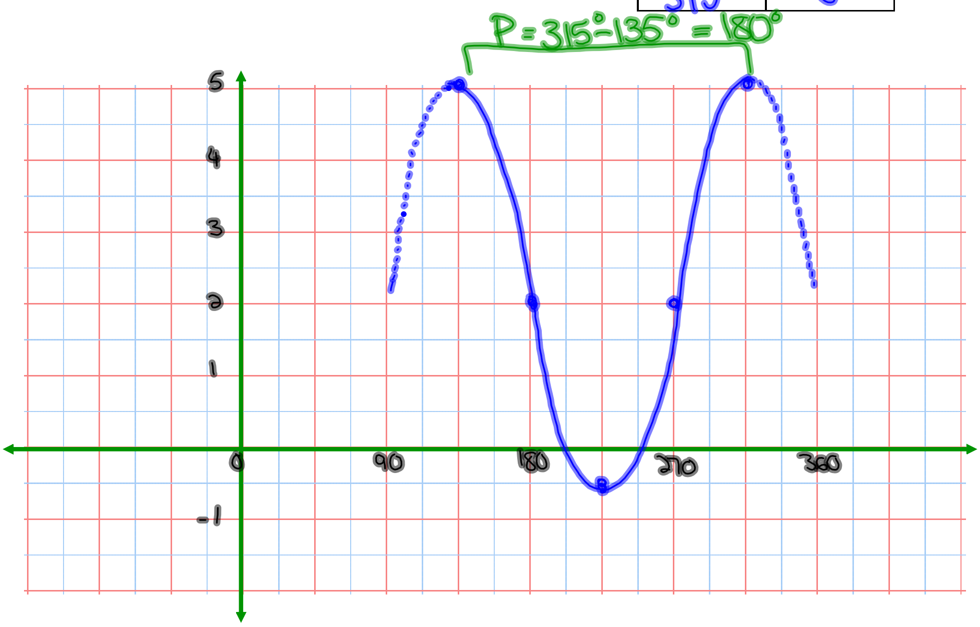
$$(x, y) \longrightarrow \left[\frac{1}{2}\theta + 135°,\ 3y + 2\right]$$

$y = \cos\theta$

θ	y
0	1
90	0
180	-1
270	0
360	1

New points after mapping

θ	y
135°	5
180°	2
225°	-1
270°	2
315°	5



DOMAIN	$\{\theta \mid \theta \in R\}$
RANGE	$\{y \mid -1 \le y \le 5, y \in R\}$
AMPLITUDE	$a = 3$
PERIOD	$P = \frac{360°}{2} = 180°$
PHASE SHIFT	$c = 135°$
VERTICAL TRANSLATION	$d = 2$
EQUATION OF SINUSOIDAL AXIS	$y = 2$

Use Mapping to Graph

$$\frac{1}{2}(y+1) = 3\sin\left(\frac{1}{2}\theta - 90°\right) + 2$$

Remember...Put in standard form first!!

$y+1 = 6\sin\left(\frac{1}{2}\theta - 90°\right) + 4$

$y = 6\sin\left(\frac{1}{2}\theta - 90°\right) + 3$

(Factor)

$y = 6\sin\left[\frac{1}{2}(\theta - 180°)\right] + 3$

a = 6 b = $\frac{1}{2}$ c = 180° d = 3

P = 720°

$y = \sin\theta$

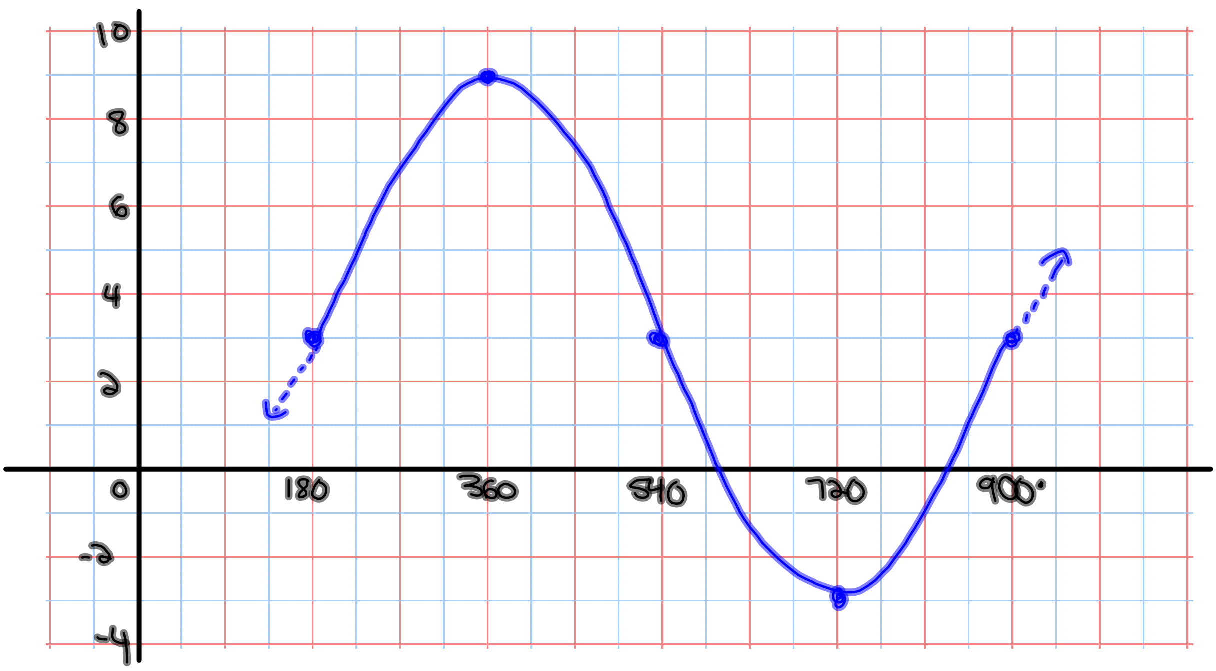
θ	y
0	0
90	1
180	0
270	-1
360	0

$$(x,y) \rightarrow \left[\frac{x}{b} + c, ay + d\right]$$

New points after mapping

$(x,y) \rightarrow [2x + 180°, 6y + 3]$

θ	y
180°	3
360°	9
540°	3
720°	-3
900°	3



Use Mapping to Graph

$$\frac{3y}{3} = \frac{-6}{3}\cos(3x - \pi) - \frac{9}{3}$$

$$y = -2\cos\left[3\left(x - \frac{\pi}{3}\right)\right] - 3$$

a = 2 b = 3 c = $\frac{\pi}{3}$ d = -3

P = $\frac{2\pi}{3}$

$y = \cos\theta$

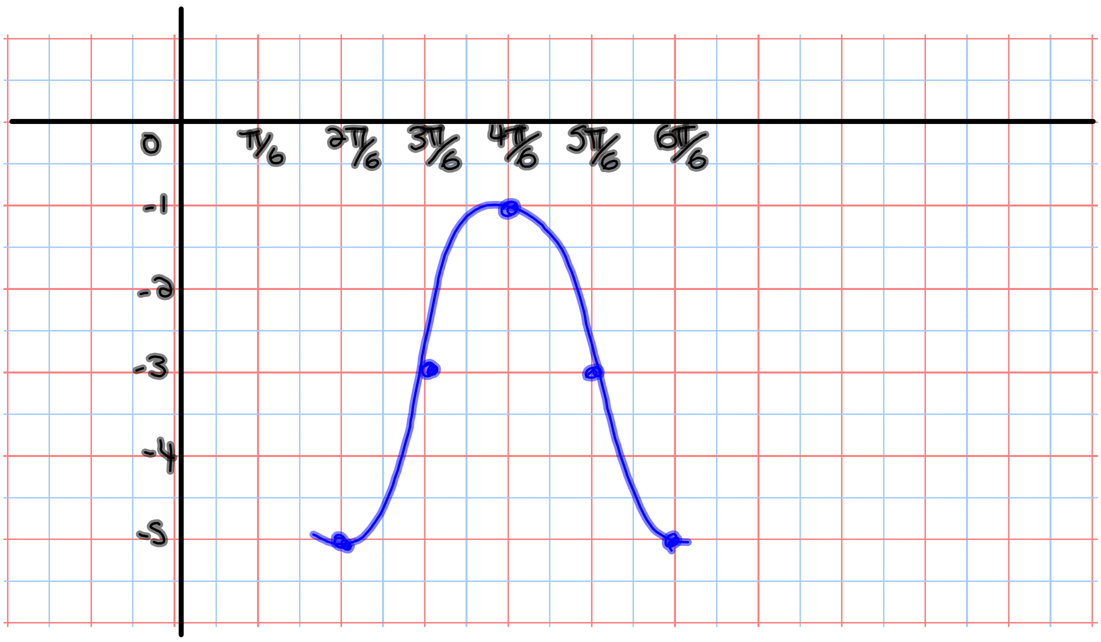
θ	y
0	1
$\pi/2$	0
π	-1
$3\pi/2$	0
2π	1

$$(x, y) \rightarrow \left[\frac{x}{b} + c, ay + d\right]$$

New points after mapping

$$(x, y) \rightarrow \left[\frac{1}{3}x + \frac{\pi}{3}, -2y - 3\right]$$

θ	y
($2\pi/6$) $\pi/3$	-5
($3\pi/6$) $\pi/2$	-3
($4\pi/6$) $2\pi/3$	-1
$5\pi/6$	-3
($6\pi/6$) π	-5



Questions from Homework

Example...

Graph the equation $y = -3\sin(2\theta + \pi) + 1$ using mapping notation.

$y = -3\sin\left[2\left(\theta + \frac{\pi}{2}\right)\right] + 1$ $\quad (\theta, y) \to \left[\frac{1}{2}\theta - \frac{\pi}{2}, -3y + 1\right]$

AMPLITUDE	$a = 3$
PERIOD	$P = \frac{2\pi}{2} = \pi$
PHASE SHIFT	$c = -\frac{\pi}{2}$
VERTICAL TRANSLATION	$d = 1$
EQUATION OF SINUSOIDAL AXIS	$y = 1$

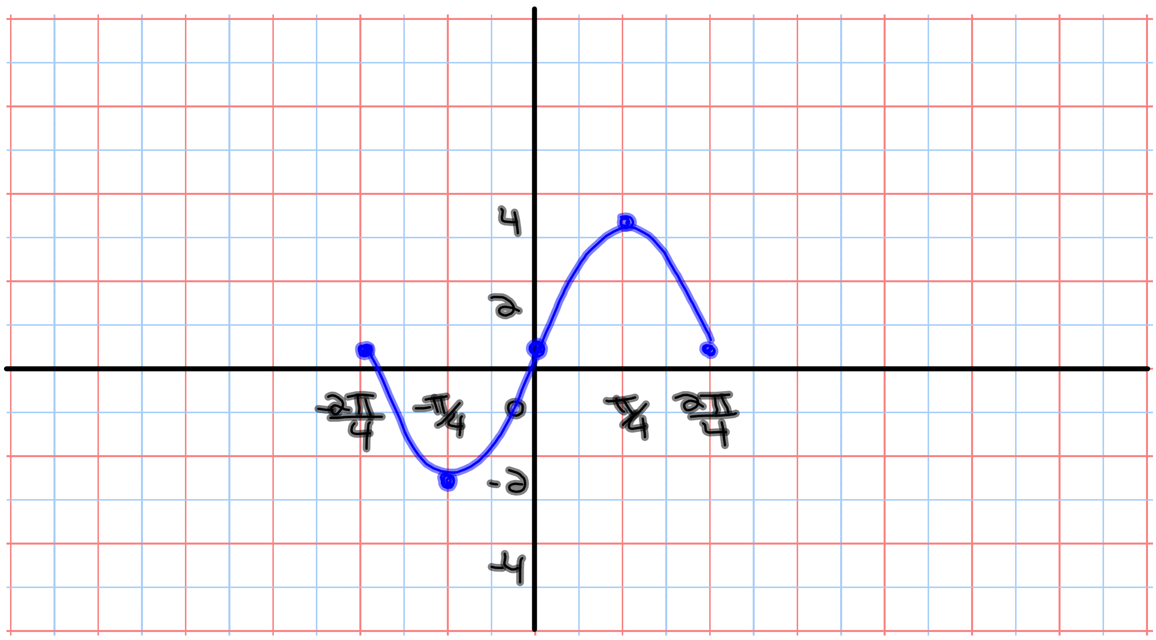
$y = \sin\theta$

θ	y
0	0
$\pi/2$	1
π	0
$3\pi/2$	-1
2π	0

New points after mapping

θ	y
$-\pi/2$	1
$-\pi/4$	-2
0	1
$\pi/4$	4
$\pi/2$	1

D: $\{\theta \mid \theta \in R\}$
R: $\{y \mid -2 \le y \le 4, y \in R\}$



Homework

Worksheet - Sketching Trigonometric Functions.doc

Solutions to the homework

① $y = 3\cos(x) - 2$

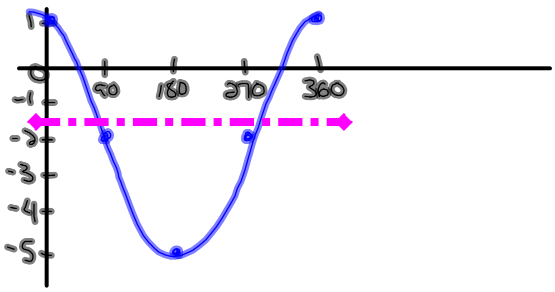
$A = 3$ $\quad b = 1$ $\quad C = 0$ $\quad D = -2$ $\quad P = 360°$

$y = \cos x$

x	y
0	1
90	0
180	-1
270	0
360	1

→

x	y
0	1
90	-2
180	-5
270	-2
360	1



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② $y = -\sin(2x - \frac{\pi}{6})$

$y = -\sin[2(x - \frac{\pi}{12})]$

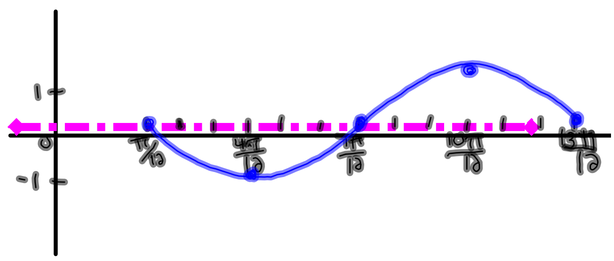
$A = 1$ $\quad b = 2$ $\quad C = \frac{\pi}{12}$ $\quad D = 0$ $\quad P = \pi$

$y = \sin x$

x	y
0	0
$\pi/2$	1
π	0
$\frac{3\pi}{2}$	-1
2π	0

→

x	y
$\pi/12$	0
$\pi/3$	-1
$\frac{7\pi}{12}$	0
$5\pi/6$	1
$13\pi/12$	0



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③ $y = 4\sin(3x - 180°) + 2$

$y = 4\sin[3(x - 60°)] + 2$

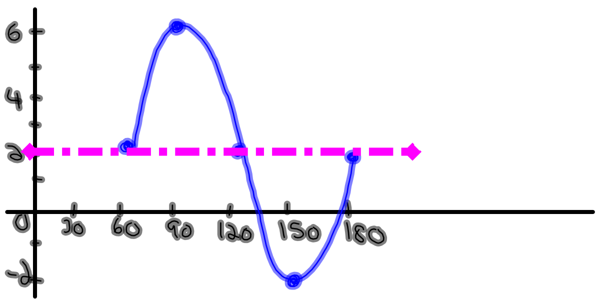
$A = 4$ $\quad b = 3$ $\quad c = 60$ $\quad D = 2$ $\quad P = 120°$

$y = \sin x$

x	y
0	0
90	1
180	0
270	-1
360	0

→

x	y
60	2
90	6
120	2
150	-2
180	2



⑤ $2y+3 = -4\sin(4x-\frac{\pi}{3})-3$

$2y = -4\sin[4(x-\frac{\pi}{12})]-6$

$y = -2\sin[4(x-\frac{\pi}{12})]-3$

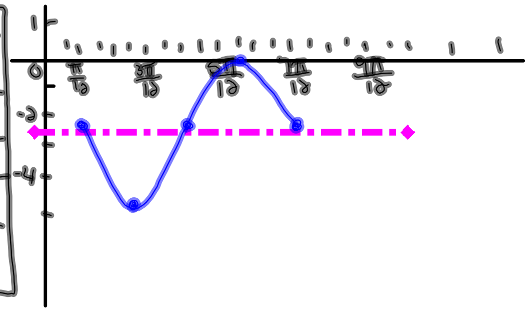
$A=2$ $b=4$ $C=\frac{\pi}{12}$ $D=-3$ $P=\frac{\pi}{2}$

y= sinx

x	y
0	0
$\pi/2$	1
π	0
$3\pi/2$	-1
2π	0

→

x	y
$\pi/12$	-3
$\frac{5\pi}{24}$	-5
$\pi/3$	-3
$11\pi/24$	-1
$\frac{7\pi}{12}$	-3



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⑥ $2\cdot\frac{y-1}{\cancel{2}} = 2\cdot 2\cos(\theta+45°)+0\cdot 2$

$y-1 = 4\cos(\theta+45°)+0 +1$

$\boxed{y = 4\cos(\theta+45°)+1}$

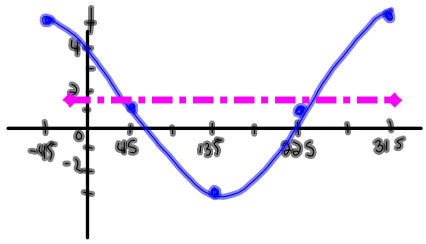
$A=4$ $b=1$ $C=-45$ $D=1$ $P=360$

y = cosx

x	y
0	1
90	0
180	-1
270	0
360	1

→

x	y
-45	5
45	1
135	-3
225	1
315	5



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⑦ $\frac{1}{2}y-1 = \sin[2(x+\frac{\pi}{4})]$

$2\cdot\frac{1}{2}y = 2\cdot\sin[2(x+\frac{\pi}{4})]+1\cdot 2$

$y = 2\sin[2(x+\frac{\pi}{4})]+2$

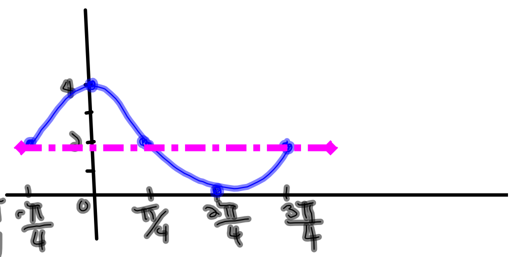
$A=2$ $b=2$ $C=-\frac{\pi}{4}$ $D=2$ $P=\pi$

y= sinx

x	y
0	0
$\pi/2$	1
π	0
$3\pi/2$	-1
2π	0

→

x	y
$-\pi/4$	2
0	4
$\pi/4$	2
$\pi/2$	0
$\frac{3\pi}{4}$	2



⑧ $y = -4\cos(3x + 90^\circ) - 2$

$y = -4\cos[3(x + 30)] - 2$

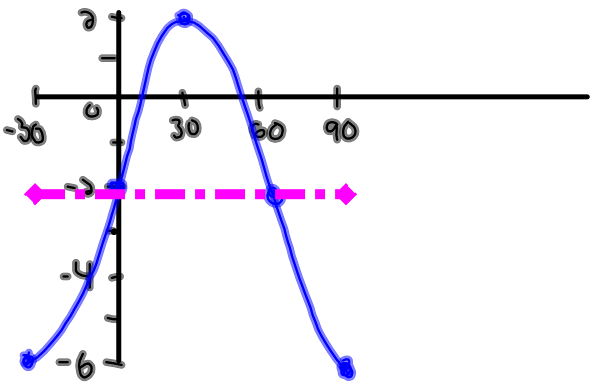
$A = 4 \quad b = 3 \quad c = -30 \quad D = -2 \quad P = 120$

$y = \cos x$

x	y
0	1
90	0
180	-1
270	0
360	1

→

x	y
-30	-6
0	-2
30	2
60	-2
90	-6



Attachments

- worksheet-sketching in radian measure.doc
- Worksheet - Finding the Equation.doc
- Worksheet - Sketching Trigonometric Functions.doc
- Worksheet Solns - Sketching Sinusoidal Relations.doc
- Worksheet - Sketching Sinusoidal relations (sept06).pdf
- Bonus Soln - Fox Population.doc
- Worksheet Solns - Applications of Sinusoidal Relations.doc
- Review - Practice Test for Sinusoidal Functions.doc
- Review - Trigonometric Functions(3)(4).doc
- Sketching Sinusoidal Functions #2.pdf
- Sketching Sinusoidal Functions #2.doc