

Questions from Homework

$$\textcircled{2} \text{ c) } \left(\frac{1}{2}\right)^{2x} \text{ and } \left(\frac{1}{4}\right)^{x-1}$$

$$2^{-2x} \text{ and } 2^{-2(x-1)}$$

$$2^{-2x} \text{ and } 2^{-2x+2}$$

$$\textcircled{3} \text{ a) } (\sqrt{16})^2$$

$$(16^{\frac{1}{2}})^2$$

$$16 \rightarrow \frac{\log 16}{\log 4} = 2 \quad \left\{ \frac{\ln 16}{\ln 4} = 2 \right.$$

$$4^2$$

c) $\sqrt{16} (\sqrt[3]{64})^2$	$4(4)^2$
$(16^{\frac{1}{2}})(64^{\frac{2}{3}})^2$	$4(4^2)$
$(16^{\frac{1}{2}})(64^{\frac{4}{3}})$	4^3
$(4^2)^{\frac{1}{2}}(4^3)^{\frac{4}{3}}$	
$(4^1)(4^4)$	
4^5	

$$\textcircled{4} \text{ a) } 16^{2k-3} = 32^{k+3}$$

$$2^{4(2k-3)} = 2^{5(k+3)}$$

$$2^{8k-12} = 2^{5k+15}$$

$$8k-12 = 5k+15$$

$$3k = 27$$

$$k = 9$$

16^{15}	32^{18}
2^{60}	2^{60}

$k=9$ is a solution

Initial Amount = 2000
 Doubling \rightarrow Base 2
 Doubles every 0.75 hours.
 How long until bacteria count is 32,000

$$y = c^x$$

$$y = 2000(2)^{\frac{x}{0.75}}$$

$$\frac{32000}{2000} = \frac{2000(2)^{\frac{x}{0.75}}}{2000}$$

$$16 = 2^{\frac{x}{0.75}}$$

$$2^4 = 2^{\frac{x}{0.75}}$$

$$0.75 \cdot 4 = \frac{x}{0.75} \cdot 0.75$$

$$3 = x$$

Base function:

$$y = 2^x$$

For any exponential function of the form $y = c^x$

Domain: $\{x | x \in \mathbb{R}\}$

Range: $\{y | y > 0, y \in \mathbb{R}\}$

x int: none

y int: $(0, 1)$ or $y = 1$

horizontal asymptote: $y = 0$

where the curve crosses the y-axis

the equation of a horizontal line (on the x-axis)

$$y = c^x \longrightarrow y = a(c)^{b(x-h)} + k$$

Given:

$$y = -3(2)^{2x+2} + 4$$

$$y = -3(2)^{2(x+1)} + 4$$

- i) state the parameters and describe the corresponding transformations
- ii) create a table to show what happens to the given points under each transformation (mapping rule)
- iii) sketch the graph of the base function and the transformed function
- iv) describe the effects on the domain, range, equation of the horizontal asymptote, and intercepts

i) $a = -3$ $b = 2$ $h = -1$ $k = 4$

- a vertical stretch by a factor of 3 and a vertical reflection in the x-axis.
- a horizontal stretch by a factor of $\frac{1}{2}$
- translated 1 unit to the left and 4 up

(i) Mapping Rule: $(x, y) \longrightarrow (\frac{1}{2}x - 1, -3y + 4)$

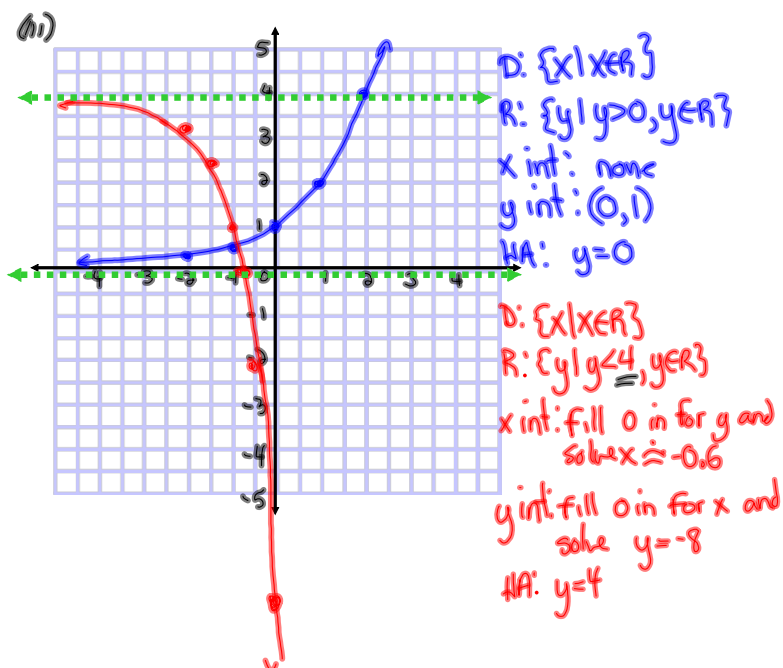
• $y = 2^x \longrightarrow y = -3(2)^{2(x+1)} + 4$

Old Points

x	y
-2	1/4
-1	1/2
0	1
1	2
2	4

New Points

x	y
-2	13/4
-3/2	5/2
-1	1
-1/2	-2
0	-8



Understanding Logarithms

Focus on...

- demonstrating that a logarithmic function is the inverse of an exponential function
- sketching the graph of $y = \log_c x$, $c > 0$, $c \neq 1$
- determining the characteristics of the graph of $y = \log_c x$, $c > 0$, $c \neq 1$
- explaining the relationship between logarithms and exponents
- expressing a logarithmic function as an exponential function and vice versa
- evaluating logarithms using a variety of methods

For the exponential function $y = c^x$, the inverse is $x = c^y$. This inverse is also a function and is called a **logarithmic function**. It is written as $y = \log_c x$, where c is a positive number other than 1.

Logarithmic Form

Exponential Form



Since our number system is based on powers of 10, **logarithms** with base 10 are widely used and are called **common logarithms**. When you write a common logarithm, you do not need to write the base. For example, $\log 3$ means $\log_{10} 3$.

logarithmic function

- a function of the form $y = \log_c x$, where $c > 0$ and $c \neq 1$, that is the inverse of the exponential function $y = c^x$

logarithm

- an exponent
- in $x = c^y$, y is called the logarithm to base c of x

common logarithm

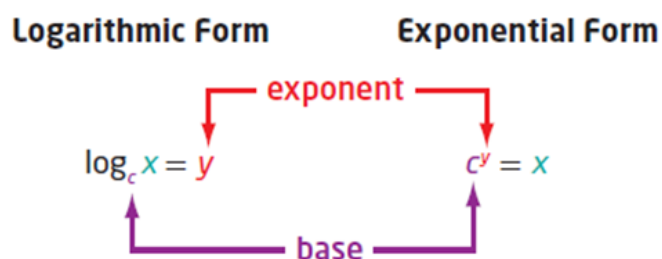
- a logarithm with base 10

Write each of the following in logarithmic form

a) $32 = 2^5$

b) $2^{-5} = \frac{1}{32}$

c) $x = 10^y$



Write each of the following in exponential form

a) $\log_4 16 = 2$

b) $\log_2 \left(\frac{1}{32} \right) = -5$

c) $\log 65 = 1.8129$

Example 1

Evaluating a Logarithm

Evaluate.

a) $\log_7 49$

b) $\log_6 1$

c) $\log 0.001$

d) $\log_2 \sqrt{8}$

Example 2

Determine an Unknown in an Expression in Logarithmic Form

Determine the value of x .

a) $\log_5 x = -3$

b) $\log_x 36 = 2$

c) $\log_{64} x = \frac{2}{3}$

Example 3



Graph the Inverse of an Exponential Function

- a) State the inverse of $f(x) = 3^x$.
- b) Sketch the graph of the inverse. Identify the following characteristics of the inverse graph:
 - the domain and range
 - the x -intercept, if it exists
 - the y -intercept, if it exists
 - the equations of any asymptotes

Solution

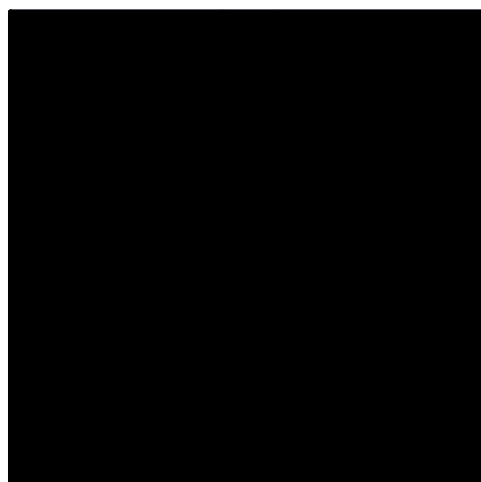
a) The inverse of $y = f(x) = 3^x$ is _____ or, _____ expressed in logarithmic form, _____. Since the inverse is a function, it can be written in function notation as _____

How do you know that $y = \log_3 x$ is a function?

b) Set up tables of values for both the exponential function, $f(x)$, and its inverse, $f^{-1}(x)$. Plot the points and join them with a smooth curve.

$f(x) = 3^x$	
x	y
-3	
-2	
-1	
0	
1	
2	
3	

$f^{-1}(x) = \log_3 x$	
x	y



The graph of the inverse, $f^{-1}(x) = \log_3 x$, is a reflection of the graph

of $f(x) = 3^x$ about the line $y = x$. For $f^{-1}(x) = \log_3 x$,

- the domain is _____ and the range is _____
- the x-intercept is _____
- there is no y-intercept
- the vertical asymptote, the _____ axis, has equation _____ there is no _____ asymptote

How do the characteristics of $f^{-1}(x) = \log_3 x$ compare to the characteristics of $f(x) = 3^x$?

Key Ideas

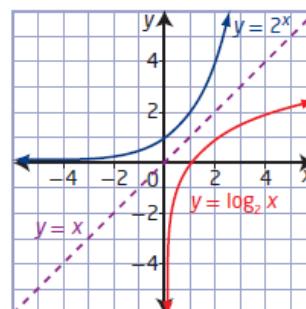
- A logarithm is an exponent.
- Equations in exponential form can be written in logarithmic form and vice versa.

Exponential Form **Logarithmic Form**

$$x = c^y \qquad y = \log_c x$$

- The inverse of the exponential function $y = c^x$, $c > 0$, $c \neq 1$, is $x = c^y$ or, in logarithmic form, $y = \log_c x$. Conversely, the inverse of the logarithmic function $y = \log_c x$, $c > 0$, $c \neq 1$, is $x = \log_c y$ or, in exponential form, $y = c^x$.
- The graphs of an exponential function and its inverse logarithmic function are reflections of each other in the line $y = x$, as shown.
- For the logarithmic function $y = \log_c x$, $c > 0$, $c \neq 1$,
 - the domain is $\{x \mid x > 0, x \in \mathbb{R}\}$
 - the range is $\{y \mid y \in \mathbb{R}\}$
 - the x-intercept is 1
 - the vertical asymptote is $x = 0$, or the y-axis
- A common logarithm has base 10. It is not necessary to write the base for common logarithms:

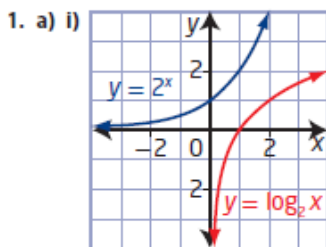
$$\log_{10} x = \log x$$



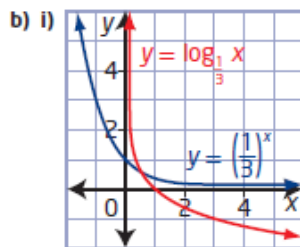
Homework

#1-5, 8, 10, 12, 13, 17 on page 380

8.1 Understanding Logarithms, pages 380 to 382

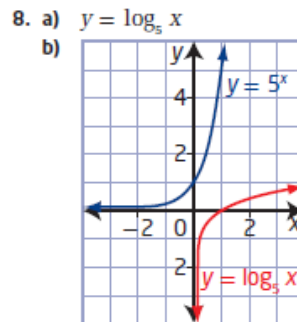


ii) $y = \log_2 x$
 iii) domain $\{x \mid x > 0, x \in \mathbb{R}\}$,
 range $\{y \mid y \in \mathbb{R}\}$,
 x-intercept 1, no y-intercept,
 vertical asymptote $x = 0$



ii) $y = \log_3 x$
 iii) domain $\{x \mid x > 0, x \in \mathbb{R}\}$,
 range $\{y \mid y \in \mathbb{R}\}$,
 x-intercept 1, no y-intercept,
 vertical asymptote $x = 0$

2. a) $\log_{12} 144 = 2$ b) $\log_8 2 = \frac{1}{3}$
 c) $\log_{10} 0.000\ 01 = -5$ d) $\log_7 (y + 3) = 2x$
3. a) $5^2 = 25$ b) $8^{\frac{2}{3}} = 4$
 c) $10^6 = 1\ 000\ 000$ d) $11^y = x + 3$
4. a) 3 b) 0 c) $\frac{1}{3}$ d) -3
5. $a = 4; b = 5$



domain $\{x \mid x > 0, x \in \mathbb{R}\}$,
 range $\{y \mid y \in \mathbb{R}\}$,
 x-intercept 1,
 no y-intercept,
 vertical asymptote $x = 0$

10. They are reflections of each other in the line $y = x$.
11. a) They have the exact same shape.
 b) One of them is increasing and the other is decreasing.
12. a) 216 b) 81 c) 64 d) 8
13. a) 7 b) 6
14. a) 0 b) 1
15. -1
16. 16
17. a) $t = \log_{0.11} N$ b) 145 days
18. The larger asteroid had a relative risk that was 1479 times as dangerous.
19. 1000 times as great
20. 5
21. $m = 14, n = 13$
22. $4n$
23. $y = 3^{2^x}$

