Warm Up

2. Factor each of the following:

$x^{27} - 1 \longrightarrow (a^3 - b^3) = (a-b)(a^2+ab+b^2)$

$(x^9-1)(x^{18}+x^9+1)$

$(x^3-1)(x^6+x^3+1)(x^{18}+x^9+1)$

$\boxed{(x-1)(x^2+x+1)(x^6+x^3+1)(x^{18}+x^9+1)}$

$(x^2+1)^{\frac{1}{2}} + 3(x^2+1)^{-\frac{1}{2}}$ Common Factor of $(x^2+1)^{-1/2}$

$(x^2+1)^{-1/2}\left[(x^2+1)^1 + 3(x^2+1)^0\right]$

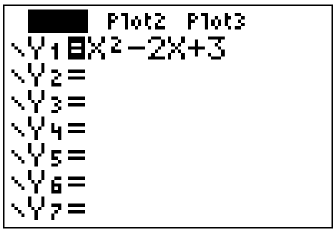
$(x^2+1)^{-1/2}(x^2+1+3)$

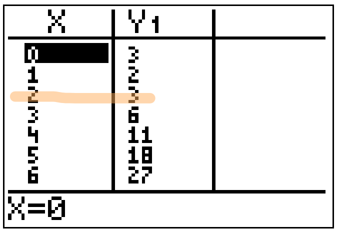
$(x^2+1)^{-1/2}(x^2+4)$

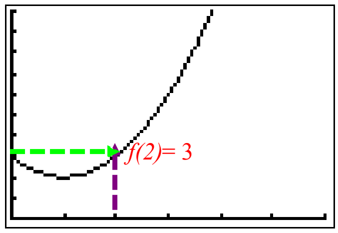
$\dfrac{x^2+4}{(x^2+1)^{1/2}}$ or $\dfrac{x^2+4}{\sqrt{x^2+1}}$

Limit of a Function

Let's examine the function $f(x) = x^2 - 2x + 3$







We can see that *f(2)*= 3...let's check the behaviour of *f* as we get closer and closer to $x = 2$.

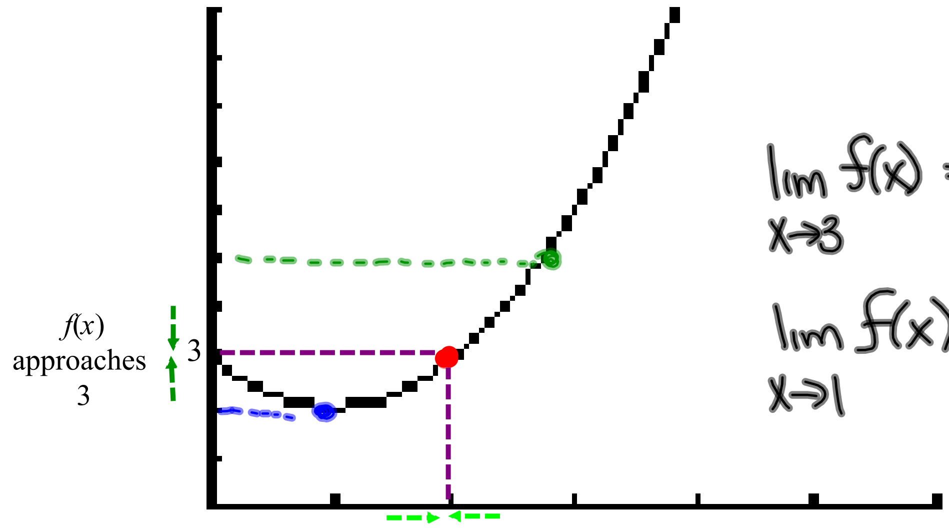
X	Y1
1.85	2.7225
1.9	2.81
1.95	2.9025
2	3
2.05	3.1025
2.1	3.21
2.15	3.3225

X=1.85

As x gets closer to 2 from the left y is getting closer to 3.

As x gets closer to 2 from the right y is getting closer to 3.

From the above, the notion of the limit of a function arises...



As x approaches 2

$\lim_{x\to 3} f(x) = 5$

$\lim_{x\to 1} f(x) = 2$

Notation: $\lim_{x\to 2} f(x) = 3$

"The limit of the function $f(x)$ as x approaches 2 is equal to 3."

The common sense definition of a limit...

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What *is* a limit?

A formal definition of a limit...

We write $\lim_{x \to a} f(x) = L$ if we can make the values of $f(x)$ arbitrarily close to L

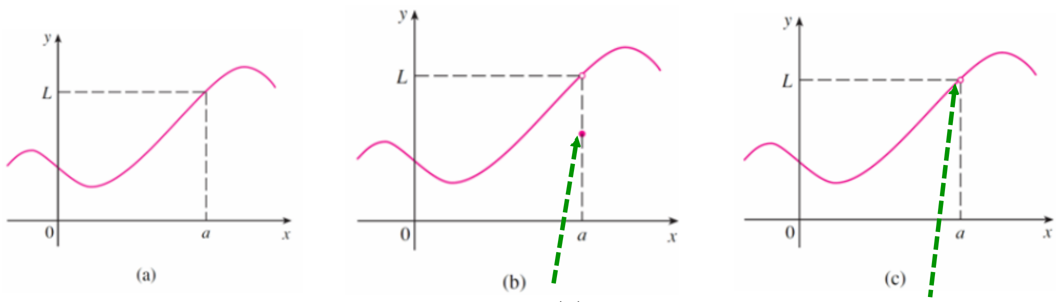
- (as close to L as we like)

by taking x to be sufficiently close to a

- (on either side of a)

but not equal to a.

Look at the graphs of these three functions...



Notice $f(a) \neq L$

Notice $f(a)$ is undefined

But in each case, regardless of what happens at a, it is true that

$$\lim_{x \to a} f(x) = L$$

Evaluating Limits

I. Using a Graph:

- We looked at this in the previous two examples

II. Algebraically:

- Direct Substitution...

Examples:

$$\lim_{x\to -2}\frac{x^2-2x+1}{x+3}$$

$$\lim_{x\to -2}\frac{(-2)^2-2(-2)+1}{(-2)+3}=\frac{4+4+1}{1}=\boxed{9}$$

$$\lim_{x\to 3}\left(16-x^2\right)$$

$$\lim_{x\to 3}\left(16-(3)^2\right)=16-9=\boxed{7}$$

- Indeterminate limits... $\Rightarrow$ Direct substitution leads to $\frac{0}{0}$

  $\Rightarrow$ Factor
  $\Rightarrow$ Rationalize
  $\Rightarrow$ Expand
  $\Rightarrow$ Find Common Denominators

Examples:

$$\lim_{x\to 4}\frac{x^2-16}{x-4}$$

$$\lim_{x\to 4}\frac{(x+4)\cancel{(x-4)}}{\cancel{(x-4)}}$$

$$\lim_{x\to 4} x+4 = 4+4=\boxed{8}$$

$$\lim_{h\to 0}\frac{\left(\sqrt{4+h}-2\right)}{h}\cdot\frac{\left(\sqrt{4+h}+2\right)}{\left(\sqrt{4+h}+2\right)}$$

$$\lim_{h\to 0}\frac{4+h-4}{h\left(\sqrt{4+h}+2\right)}$$

$$\lim_{h\to 0}\frac{\cancel{h}}{\cancel{h}\left(\sqrt{4+h}+2\right)}=\boxed{\frac{1}{4}}$$

Try these...remember to use your algebra skills to try and eliminate the **indeterminate form**.

$$\lim_{x\to 2}\frac{(x+2)^2-16}{x^2-4}$$

Expand

Factor

$$\lim_{x\to 2}\frac{x^2+4x+4-16}{(x-2)(x+2)}$$

$$\lim_{x\to 2}\frac{x^2+4x-12}{(x-2)(x+2)}$$

Factor

$$\lim_{x\to 2}\frac{(x+6)\cancel{(x-2)}}{\cancel{(x-2)}(x+2)}=\frac{8}{4}=\boxed{2}$$

$$\lim_{x\to -2}\frac{x^4-16}{x^3+8}$$

Factor

$$\lim_{x\to -2}\frac{(x^2-4)(x^2+4)}{(x+2)(x^2-2x+4)}$$

$$\lim_{x\to -2}\frac{\cancel{(x+2)}(x-2)(x^2+4)}{\cancel{(x+2)}(x^2-2x+4)}$$

$$=\frac{(-4)(8)}{12}=-\frac{32}{12}=\boxed{-\frac{8}{3}}$$

$$\lim_{x\to 0}\frac{x^2+3x}{(x+2)^2-(x-2)^2}$$

Common Factor

Expand

$$\lim_{x\to 0}\frac{x(x+3)}{x^2+4x+4-(x^2-4x+4)}$$

$$\lim_{x\to 0}\frac{x(x+3)}{x^2+4x+4-x^2+4x-4}$$

$$\lim_{x\to 0}\frac{\cancel{x}(x+3)}{8\cancel{x}}$$

$$\lim_{x\to 0}\frac{x+3}{8}=\boxed{\frac{3}{8}}$$

CD: $2x$

$$\lim_{x\to 2}\frac{2x\cdot\frac{1}{x}-\frac{1}{2}\cdot 2x}{(x-2)\,2x}$$

$$\lim_{x\to 2}\frac{\cancel{(2-x)}^{-1}}{2x\cancel{(x-2)}^{1}}$$

$$\lim_{x\to 2}\frac{-1}{2x}=\boxed{-\frac{1}{4}}$$

Homework