Warm Up

Evaluate the following limits, if they exist:

1. $\lim\limits_{x\to2}\dfrac{x-2}{x^3-8}$

$$\lim_{x\to2}\frac{\cancel{(x-2)}}{\cancel{(x-2)}(x^2+2x+4)}$$

$$\lim_{x\to2}\frac{1}{((2)^2+2(2)+4)}=\boxed{\frac{1}{12}}$$

2. $\lim\limits_{x\to7}\dfrac{\sqrt{x+2}-3}{x-7}\cdot\dfrac{(\sqrt{x+2}+3)}{(\sqrt{x+2}+3)}$

$$\lim_{x\to7}\frac{x+2-9}{(x-7)(\sqrt{x+2}+3)}$$

$$\lim_{x\to7}\frac{\cancel{(x-7)}}{\cancel{(x-7)}(\sqrt{x+2}+3)}$$

$$\lim_{x\to7}\frac{1}{\sqrt{7+2}+3}=\boxed{\frac{1}{6}}$$

3. $\lim\limits_{h\to0}\dfrac{(a+h)^2-a^2}{h}$

$$\lim_{h\to0}\frac{(a+h-a)(a+h+a)}{h}$$

$$\lim_{h\to0}\frac{\cancel{(h)}(2a+h)}{\cancel{h}}$$

$$\lim_{h\to0}2a+0=\boxed{2a}$$

Sum of Cubes: $a^3+b^3=(a+b)(a^2-ab+b^2)$

Ex: x^3+8

$(x+2)(x^2-2x+4)$

Difference of Cubes: $a^3-b^3=(a-b)(a^2+ab+b^2)$

Ex: $(x-4)^3-64$

$((x-4)-4)((x-4)^2+4(x-4)+16)$

Questions from Homework

4 g) $\lim_{x\to 9} \frac{(x-9)(\sqrt{x}+3)}{(\sqrt{x}-3)(\sqrt{x}+3)}$

$\lim_{x\to 9} \frac{\cancel{(x-9)}(\sqrt{x}+3)}{\cancel{(x-9)}} = \boxed{6}$

5 a) $\lim_{h\to 0} \frac{(4+h)^3-64}{h}$

$\lim_{h\to 0} \frac{((4+h)-4)((4+h)^2+4(4+h)+16)}{h}$

$\lim_{h\to 0} \frac{\cancel{h}[(4+h)^2+4(4+h)+16]}{\cancel{h}} = \boxed{48}$

5 c) $\lim_{h\to 0} \frac{\frac{1}{1+h}-\frac{1}{1}}{h}$ $CD = (1+h)$

$\lim_{h\to 0} \frac{\frac{1}{1+h}-\frac{1+h}{1+h}}{h}$

$\lim_{h\to 0} \frac{* \frac{1-(1+h)}{1+h}}{h}$ $* \frac{1-1-h}{1+h}$

$\lim_{h\to 0} \frac{-\cancel{h}}{1+h} \times \frac{1}{\cancel{h}} = \frac{-1}{1+h} = \boxed{-1}$

f) $\lim_{h\to 0} \frac{\frac{1}{(2+h)^2}-\frac{1}{4}}{h}$ $CD = 4(2+h)^2$

$\lim_{h\to 0} \frac{4-(2+h)^2}{4h(2+h)^2}$ ← Difference of Squares

$\lim_{h\to 0} \frac{(2-(2+h))(2+(2+h))}{4h(2+h)^2}$

$\lim_{h\to 0} \frac{(2-2-h)(2+2+h)}{4h(2+h)^2}$

$\lim_{h\to 0} \frac{(-\cancel{h})(4+h)}{4\cancel{h}(2+h)^2} = \frac{-4}{16} = \frac{-1}{4}$

6 c) $\lim_{x\to 1} \frac{x^4-1}{x-1}$

$\lim_{x\to 1} \frac{(x^2-1)(x^2+1)}{(x-1)}$

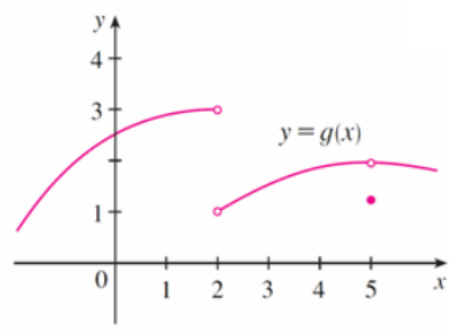
$\lim_{x\to 1} \frac{\cancel{(x-1)}(x+1)(x^2+1)}{\cancel{(x-1)}} = 2(2) = \boxed{4}$

The common sense definition of a limit...

When does a limit exist?

One-sided limits

Use the graph shown below to evaluate the following limits:



1. $\lim_{x \to 2^-} g(x) = \square$

"as x approaches 2 from the left"

2. $\lim_{x \to 2^+} g(x) = \square$

"as x approaches 2 from the right"

3. $\lim_{x \to 2} g(x) = \square$

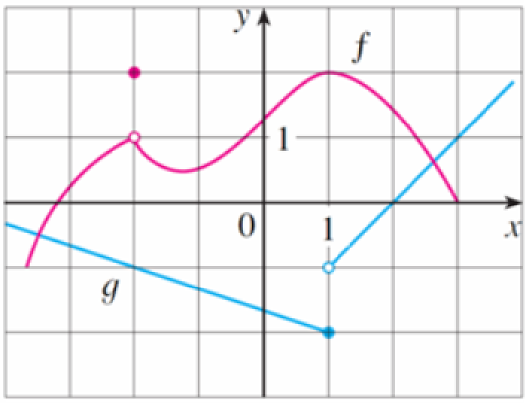
4. $\lim_{x \to 5^-} g(x) = \square$

5. $\lim_{x \to 5^+} g(x) = \square$

6. $\lim_{x \to 5} g(x) = \square$

Notice... $g(5) =$

Example:



Evaluate each of the following:

$f(-2) =$ $\lim_{x \to 1^-} g(x) =$ $g(1) =$

$\lim_{x \to 1^+} g(x) =$ $\lim_{x \to 1} g(x) =$ $\lim_{x \to 1} f(x) =$

$\lim_{x \to -2} f(x) =$

Homework

Page 27

1, 2, 3